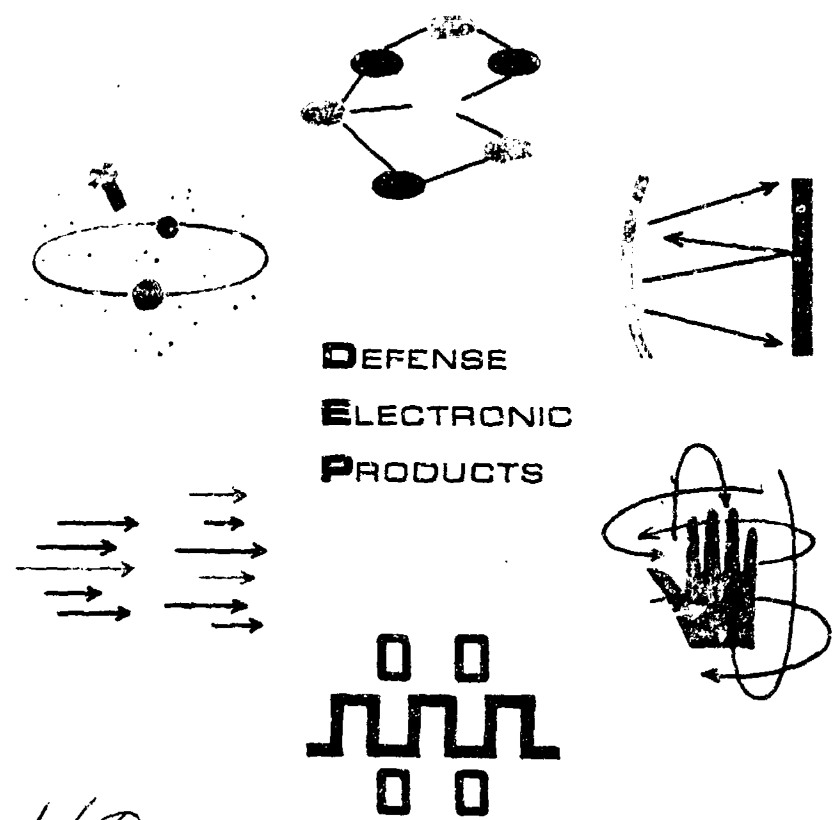
AD641391

CONTRACT NO.
NOnr-4649(00)(X)

FINAL REPORT
OPTIMUM WAVEFORM STUDY FOR COHERENT PULSE DOPPLER



405,690

(2)

PREPARED FOR:

OFFICE OF NAVAL RESEARCH
DEPARTMENT OF THE NAVY
WASHINGTON, D. C.

RADIO CORPORATION OF AMERICA ■ DEFENSE ELECTRONIC PRODUCTS
MISSILE AND SURFACE RADAR DIVISION, MOORESTOWN, N. J.

FINAL REPORT

OPTIMUM WAVEFORM STUDY

for

COHERENT PULSE DOPPLER

Prepared for: Office of Naval Research
Department of Navy
Washington 25, D. C.

Prepared by: Radio Corporation of America
Defense Electronics Products
Missile and Surface Radar Division
Moorestown, New Jersey

Contract Number: NOnr 4649(00)(X)

28 February 1965

ABSTRACT

This report contains the results of a theoretical investigation under contract NOnr-4649(00)(X) directed towards improving the sidelobe characteristics with respect to doppler of a swept frequency modulated signal. The methods of suppressing the time sidelobes analyzed were (a) Taylor frequency weighting in the receiver, (b) tone injection to predistort the phase of transmitted signal, (c) spectrum weighting by nonlinear FM of transmitted signal, (d) spectrum weighting by nonlinear FM in transmitter and frequency weighting in receiver, and (e) step-amplitude weighting in the transmitter. For each of the cases studied, where appreciable, normalized curves of the compressed signal time response are presented to illustrate the results. It has been found that nonlinear FM of the transmitted signal results in deterioration of the sidelobe performance with doppler due to phase mismatch in the receiver. A method that removes the effects of doppler on the received signal is frequency weighting of the transmitted signal. Step-amplitude modulation with a small number of steps is one method by which this weighting may be accomplished.

Based on the results of this study, it is recommended that experimental simulation be implemented for the case of amplitude-weighted, frequency coded transmitted signals with frequency weighting in the receiver. Since this system is a partially tracking function, the sidelobe behavior with doppler should be better than the linear FM system with a Taylor weighted receiver. Also, since the system is closer to a matched system, the signal-to-noise loss should be less than the Taylor receiver case.

TABLE OF CONTENTS

FIGURES

OPTIMUM WAVEFORM STUDY for COHERENT PULSE DOPPLER

1. INTRODUCTION

The overall purpose of the Optimum Waveform Study was to determine some type of modification of the normal linear FM pulse to reduce the effects of doppler on the amplitude of the range sidelobes. Various methods of suppressing the sidelobes with doppler have been analyzed and results of the study are presented. The methods studied were

(a) Taylor Frequency Weighting in the Receiver,

(b) Tone Injection to Predistort the Phase of Transmitted Signal,

(c) Spectrum Weighting by Nonlinear FM of Transmitted Signal,

(d) Spectrum Weighting by Nonlinear FM in Transmitted Signal and Weighted Filtering in Receiver,

(e) Step Amplitude Modulation of Transmitted Signal.

Each of the above techniques will be briefly discussed to clarify and summarize the contents of this report.

It has been recognized that application of the Taylor type weighting function to the signal spectrum in the receiver can produce very low time

sidelobes when the target return contains no doppler shift. When doppler shifts place the return spectrum "off center" with respect to the weighting function, the amplitude of the sidelobes increase. This type of system is illustrated in Figure 1.1(a) with the corresponding performance characteristics.

If the weighting function could be made to track in doppler, the effects in doppler could be greatly reduced. To avoid amplitude modulation of the transmitted signal, it was believed that some sort of phase modulation could be applied to the transmitted signal to effectively create a spectrum whose modulus was properly weighted. It is known that small sinusoidal phase modulations, added to a signal of a given spectrum form, produce replica spectra spaced by the modulation rate about the primary spectrum. Based on this, it was thought that such replicas could emerge from a compression filter at various increments in time to provide the time domain equivalent of a spectral weighting function. This method of predistorting the phase of the transmitted signal is shown not to produce the desired result since the phase of the replica spectra cannot be made the same as that of the main spectrum. A system using this approach is summarized in Figure 1.1(b).

From the synthesis methods of Key, Fowle and Haggarty[1] it is evident that some function exists that has a rectangular time envelope and a spectral modulus that is Taylor shaped. Since the signal sidelobes at zero doppler depend on the spectral modulus assuming a receiver matched in phase to the signal, the sidelobes of zero doppler will be suppressed.

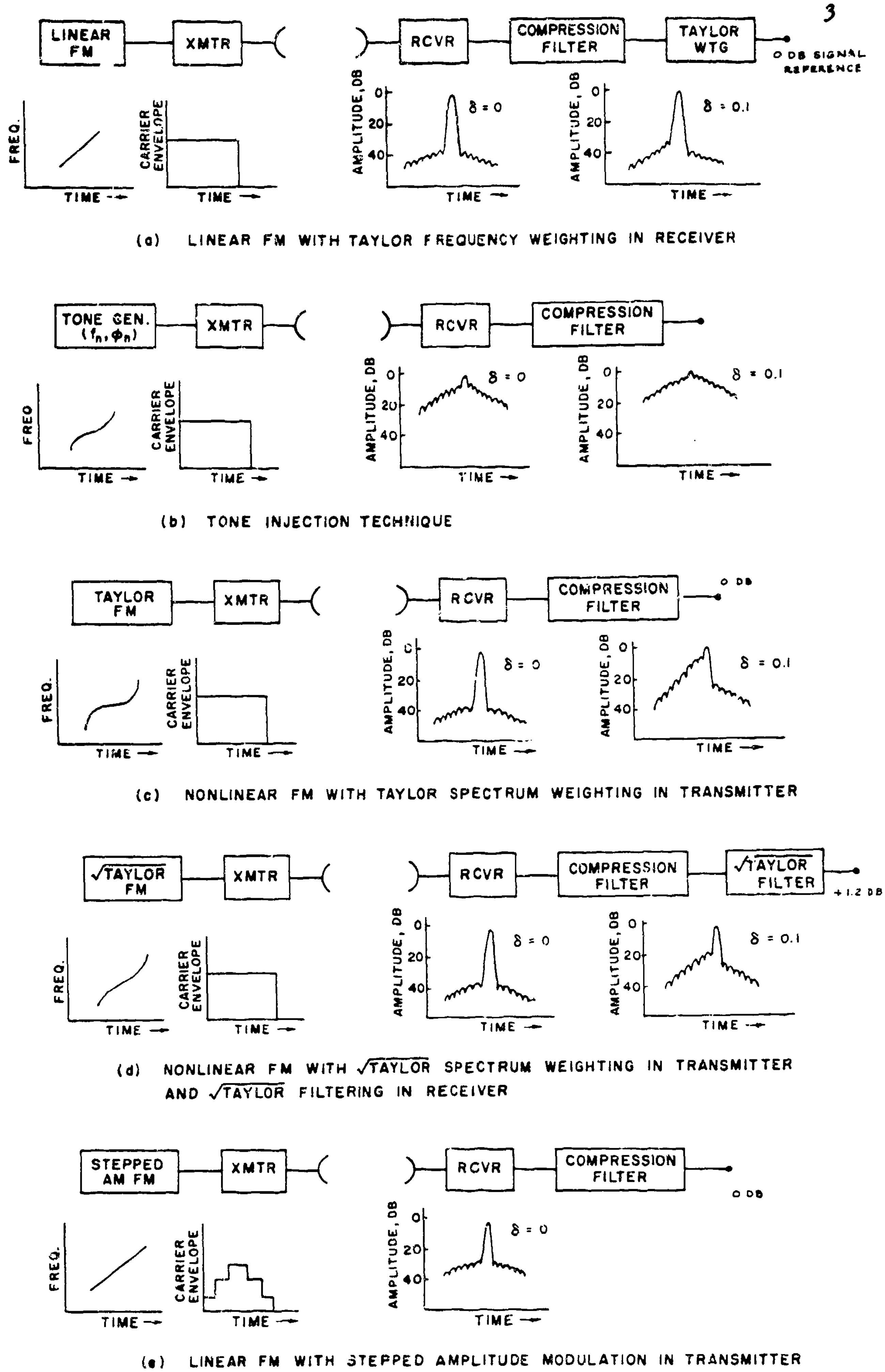


FIG. 1.1 SUMMARY OF OPTIMUM WAVEFORM STUDY PROGRAM

The behavior of the time sidelobes with doppler shift is examined for the pseudo-matched and matched systems. These are illustrated in Figures 1.1(c) and 1.1(d).

Another method of obtaining a tracking weighting function is to amplitude modulate the transmitted linear FM signal. The type of amplitude modulation considered is rectangular AM of the carrier. This technique requires the use of amplitron type of tubes or multiple tubes and is shown in Figure 1.1(e). In the following sections each of the above techniques is analyzed and discussed in detail.

2. Linear FM with Taylor Frequency Weighting in Receiver

The post compression spectrum of the linear FM pulse approaches a rectangular amplitude shape with flat phase response for arbitrarily large values of the time-bandwidth product TW. Taylor[2] has developed a weighting function which, when applied to the rectangular spectrum, will result in a time response with low sidelobes. The Taylor weighting function for sidelobes which are 40 db below the peak response is described by the finite series

$$H_d(\omega) = 1 + 2 \sum_{m=1}^{5} F_m \cos \frac{m}{W} (\omega - \omega_o) \qquad (2.1)$$

where ω_o = angular frequency about which the weighting function is centered,

W = frequency deviation in cps,

F_1 = 0.3891154

F_2 = -0.0094523

F_3 = 0.0048819

F_4 = -0.0016105

F_5 = 0.0003474

for 40 db sidelobes.

In the system under consideration, the receiver frequency characteristic is assumed to be Taylor weighted in amplitude and matched in phase to the transmitted signal for zero doppler. The received doppler shifted spectrum for a linear FM waveform can be expressed in the form[3]

$$F(\Delta) = \frac{1}{\sqrt{2}} \left[Z(u_1) - Z(u_2) \right] \cdot \exp \left[-j\pi D(\Delta - \delta)^2 \right] \qquad (2.2)$$

where $Z(u)$ is the complex Fresnel integral

$$Z(u) = C(u) + j\,S(u) = \int_0^u \exp\,(j\pi\alpha^2/2)\;d\alpha \tag{2.3}$$

D = Time-bandwidth product, TW

and the arguments u_1 and u_2 are defined by

$$u_1 = \sqrt{2D}\,(\,1/2 + \Delta - \delta\,) \tag{2.4}$$

$$u_2 = \sqrt{2D}\,(\,1/2 - \Delta + \delta\,) \tag{2.5}$$

with

Δ = normalized frequency variable, $(\,f - f_o)/W$

W = frequency modulation deviation in cps,

δ = normalized doppler offset $\dot{\phi}/W$

ϕ = doppler frequency cps.

Thus for a receiver matched in phase to the transmitted signal at zero doppler, the Taylor weighted receiver characteristic is expressed as

$$H(\Delta) = 1 + 2\sum_{m=1}^{5} F_m \cos\,(2\pi m\Delta)\;\exp(j\pi D\,\Delta^2), \qquad -1/2 \le \Delta \le 1/2 \tag{2.6}$$

$$= 0 \qquad\qquad |\Delta| > 1/2$$

Using equations (2.2) and (2.6) for the input and receiver spectra, the output compressed pulse is given by the inverse Fourier transform

$$s(t) = \int_{-\infty}^{\infty} F(\Delta)\,H(\Delta)\,\exp\,(j2\pi t\Delta)\;d\Delta \tag{2.7}$$

Equation (2.7) represents the response of an idealized system where zero rise and fall times have been assumed for the expanded pulse envelope. The system model assumes that the receiver compression filter has a linear time delay versus frequency relationship for all frequencies. It is also assumed that there are no amplitude or phase distortions in the system.

To provide a reference to serve as a standard of comparison of the various techniques of time sidelobe reduction with doppler, the response function given by equation (2.7) was calculated by use of the IBM 7090 computer. The post compression spectrum was also computed using equation (2.2). Computations were made for a number of compression ratios and a number of relative dopplers at each ratio. The computed results have been plotted in Figures 2.1 A, B, C, D, E, F, G to Figures 2.11 A, B, C, D, E, F, G, inclusive. Table 2.1 gives the parameters for the different figures. Figure 2.12 shows the sidelobe level as a function of doppler for various dispersion ratios. To make the sidelobe level comparisons, each plot was normalized to its zero doppler peak amplitude. A curve of the zero doppler sidelobe level as a function of the compression ratio is shown in Figure 2.13. In this plot, the peak sidelobe was taken at $\pm$ 2.0 normalized time units. The results for a tracking weighting function in the receiver are plotted in Figures 2.14 A, B, C, D, E, and F for the case where D = 126 and the normalized doppler δ = 0, .1, .2, .3, .4, .5. In this case the receiver is mismatched only in phase as the doppler is varied.

TABLE 2.1

KEY TO RESULTS

Time Bandwidth Product D	Post Compression Spectrum	COMPRESSED PULSE					
		$\delta = 0$	$\delta = 0.1$	$\delta = 0.2$	$\delta = 0.3$	$\delta = 0.4$	$\delta = 0.5$
25	Fig. 2.1A	2.1B	2.1C	2.1D	2.1E	2.1F	2.1G
50	2.2A	2.2B	2.2C	2.2D	2.2E	2.2F	2.2G
75	2.3A	2.3B	2.3C	2.3D	2.3E	2.3F	2.3G
100	2.4A	2.4B	2.4C	2.4D	2.4E	2.4F	2.4G
126	2.5A	2.5B	2.5C	2.5D	2.5E	2.5F	2.5G
150	2.6A	2.6B	2.6C	2.6D	2.6E	2.6F	2.6G
200	2.7A	2.7B	2.7C	2.7D	2.7E	2.7F	2.7G
250	2.8A	2.8B	2.8C	2.8D	2.8E	2.8F	2.8G
300	2.9A	2.9B	2.9C	2.9D	2.9E	2.9F	2.9G
400	2.10A	2.10B	2.10C	2.10D	2.10E	2.10F	2.10G
500	2.11A	2.11B	2.11C	2.11D	2.11E	2.11F	2.11G

3. Harmonic Phase Predistortion

3.1 Predistortion Function for Low Sidelobes

In this section we are interested in determining if and to what extent phase predistortion can be used to correct for amplitude or phase distortion in the receiver. Specifically we wish to know if some phase predistortion function can be employed to synthesize the equivalent desirable effect of sidelobe weighting functions which are pure amplitude distortions.

The doppler shifted radar signal return is denoted by $s_r(t)$ having a spectrum $S_r(\omega)$. If this signal is passed through a compression network having a transfer function $H_c(\omega)$, the output is $s_o(t)$ with spectrum $S_o(\omega)$. A weighting function $H_d(\omega)$ is used to reduce the sidelobe amplitude resulting in the output $s_w(t)$ with spectrum $S_w(\omega)$. These operations are shown in Figure 3.1 below.

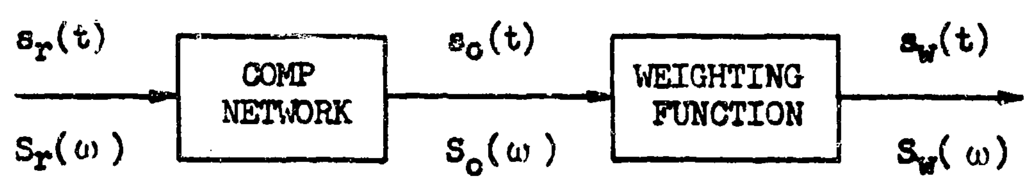


Figure 3.1 - OPERATIONS ON RECEIVED SIGNAL

The spectral weighting function is even in frequency and its effect on the input $s_o(t)$ is to produce replicas of $s_o(t)$ spaced symmetrically

in time about an undistorted output. To show this let the weighting function be generally given by an M term Fourier series about a center frequency ω_o,

$$H_d(\omega) = B_o + 2 \sum_{m=1}^{M} B_m \cos\left[m(\omega - \omega_o)/W \right] \tag{3.1}$$

where a flat phase is assumed for simplicity, and

W = frequency deviation of FM ramp,

B_m = constants of weighting function.

Equation (3.1) can be rewritten in the form

$$H_d(\omega) = \sum_{m=-M}^{M} B_m \exp\left[jm(\omega - \omega_o)/W \right] \tag{3.2}$$

The output spectrum for zero doppler is then

$$S_W(\omega) = S_o(\omega)\, H_d(\omega)$$

$$= \sum_{m=-M}^{M} B_m\, S_o(\omega) \exp\left[jm(\omega - \omega_o)/W \right] \tag{3.3}$$

and the time output by the inverse Fourier transform of equation (3.3) is

$$s_w(t) = \sum_{m=-M}^{M} B_m\, s_o(t + \frac{m}{W}) \exp(-jm\omega_o/W) \tag{3.4}$$

This shows that weighting produces both leading and lagging time replicas of the zero doppler, unweighted, compressed pulse. It is known that as

doppler shifts the weighting filter input spectrum, the sidelobes increase in amplitude due to the fact that the weighting is centered at ω_o. If the weighting function shifted in frequency, compensating for doppler, the sidelobes would remain at a low level. Such a weighting function represents the desired result of phase predistortion and can be described using equation (3.2) by

$$H_d(\omega) = \sum_{m=-M}^{M} B_m \exp\left\{ j\left[m(\omega - \omega_o - \omega_d)/W + \theta_m \operatorname{sgn} m \right] \right\} \qquad (3.5)$$

where sgn m means the sign of m and θ_m is an arbitrary phase angle. Using an input spectrum of

$$S_o(\omega) = \frac{1}{\beta} \operatorname{rect}\left[(\omega - \omega_o - \omega_d)/(2\pi W) \right] \qquad (3.6)$$

results in the time response

$$s_w(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_o(\omega)\, H_d(\omega) \exp(j\omega t)\, d\omega$$

$$= \sum_{m=-M}^{M} B_m \frac{\sin \pi W(t + m/W)}{\pi W(t + m/W)} \exp\left\{ j\left[(\omega_o + \omega_d)t + \theta_m \operatorname{sgn} m \right] \right\} \qquad (3.7)$$

Thus any phase predistortion function that is to synthesize the tracking weighting function given by equation (3.5) must produce a compressed pulse time response of the form given by equation (3.7).

3.2 Phase Predistortion Function

Since we are concerned with phase distortion occurring in time, the phase distortion components can be expressed as

$$\phi_d(t) = \sum_{n=1}^{N} c_n \sin(\omega_n t + \theta_n) \qquad (3.8)$$

where ω_n is the angular frequency of the nth component. For the cases of interest ω_n is related to the pulse length T of the expanded signal duration by

$$\omega_n = \frac{2\pi n}{T} \qquad (3.9)$$

The phase predistorted linear FM signal is then

$$s_d(t) = s(t) \exp[j\,\phi_d(t)]$$

$$= s(t) \prod_{n=1}^{N} \exp[\,j\,c_n \sin(\omega_n t + \theta_n)\,] \qquad (3.10)$$

where

$$s(t) = \text{rect}\,(t/T) \exp\, 2\pi j(f_o t + kt^2/2)$$

k = rate of FM = W/T

f_o = carrier frequency.

Using the relation[4]

$$\exp\left[j\, c_n \sin(\omega_n t + \theta_n)\right] = \sum_{k=-\infty}^{\infty} J_k(c_n) \exp\left[jk(\omega_n t + \theta_n)\right] \qquad (3.11)$$

we note that one predistortion component ($N = 1$) in equation (3.10) represents an infinite set of replicas of $s(t)$ that are weighted and shifted in frequency. Evaluation of equation (3.10) using the expression (3.11) is not very attractive; however, a useful approximation can be made if we assume c_n is small. In this case $J_k(c_n) \gg J_{k+1}(c_n)$ so we drop terms above $k=1$ in equation (3.11). Expanding equation (3.10), using only the three terms of equation (3.11) for several values of N, we note that all cross-product terms are smaller than primary terms of similar frequency by a factor of the form $J_1(c_n)/J_o(c_n)$. Neglecting the cross-product terms, the phase distorted signal can be written

$$s_d(t) = s(t)\left\{ A_o + \sum_{n=1}^{N} A_n \exp\left[j(\omega_n t + \theta_n)\right] - \sum_{n=1}^{N} A_n \exp\left[-j(\omega_n t + \theta_n)\right] \right\} \qquad (3.12)$$

where

$$A_o = \prod_{k=1}^{N} J_o(c_k)$$

$$A_n = \frac{J_1(c_n)}{J_o(c_n)} A_o$$

The spectrum of the phase distorted signal is given by the transform of equation (3.12),

$$S_d(\omega) = A_o\, S(\omega) + \sum_{n=1}^{N} A_n\, S(\omega - \omega_n) \exp(j\theta_n)$$
$$- \sum_{n=1}^{N} A_n\, S(\omega + \omega_n) \exp(-j\theta_n) \qquad (3.13)$$

The matched filter response to a linear FM signal spectrum that is doppler shifted is expressed as[5]

$$s(t) = \exp\left[j(\omega_o + \frac{\omega_d}{2})t\right] \cdot \frac{\sin\left[(\omega_d + \mu t)(T - |t|\,)/2\right]}{(\omega_d + \mu t)T/2}\,, \qquad (3.14)$$
$$-T < t < T$$

Applying this result to each of the spectral components in equation (3.13) gives the response

$$s_w(t) = A_o \exp(j\omega_o t)\, \frac{\sin[\pi W t(1 - |t/T|)]}{\pi W t}$$
$$+ \sum_{n=1}^{N} A_n \exp\left\{j\left[(\omega_o + \frac{n\pi}{T})t + \theta_n\right]\right\} \frac{\sin[\pi W(t + \frac{n}{W})(1 - |t/T|)]}{\pi W(t + \frac{n}{W})}$$
$$- \sum_{n=1}^{N} A_n \exp\left\{j\left[(\omega_o - \frac{n\pi}{T})t - \theta_n\right]\right\} \frac{\sin[\pi W(t - \frac{n}{W})(1 - |t/T|)]}{\pi W(t - \frac{n}{W})} \qquad (3.15)$$

This result is to be compared with equation (3.7) previously derived for the tracking weighting function. For this comparison, the doppler frequency can be assumed to be zero. The most significant difference in the two results is that the lagging echoes are negative in equation (3.15) and positive in equation (3.7). This means that only leading (or lagging) echoes can be synthesized as desired. Except for this

sign difference, the desired weighting function could be approximated. Thus phase predistortion cannot in general synthesize or correct for the effects of pure amplitude distortions in the receiver.

4. Taylor Weighted Nonlinear FM

The method given by Key, Fowle, and Haggarty[1] is used to determine the phase characteristic of the transmitted waveform such that the envelope of the signal is rectangular and the spectrum is shaped as desired. From the phase characteristic, the time delay is found as a function of frequency. This function is then reversed to determine the required nonlinear frequency in terms of time. Integration of the frequency yields the phase of the transmitted signal as a function of time. Knowledge of the phase function determines the matched receiver characteristic so that the overall system response can be calculated as a function of the compression ratio and doppler frequency.

4.1 Delay Function for Nonlinear FM

If the signal is denoted by f(t), then the signal spectrum is defined by the Fourier transform

$$\int_{-\infty}^{\infty} f(t)\ \exp(j\omega t)\ dt = \left|F(\omega)\right| \exp\left[j\,\psi(\omega)\right] \qquad (4.1)$$

where $F(\omega)$ = envelope of signal spectrum

$\psi(\omega)$ = phase of signal spectrum.

For a rectangular envelope, the relationship between the phase and the frequency spectrum of the signal is given by[1]

$$\left| F(\omega) \right|^2 = K \frac{d^2\psi}{d\omega^2} \qquad (4.2)$$

where K is a constant.

The group delay is defined as

$$t_d = -\frac{d\psi}{d\omega} = -\int \frac{d^2\psi}{d\omega^2}\, d\omega + \text{constant}$$

$$= -\frac{1}{K}\int_0^{\omega} \left|F(\omega)\right|^2 d\omega \qquad (4.3)$$

When the envelope of the spectrum is specified as 40 db Taylor weighted, then from equation (2.1)

$$F(\omega) = 1 + 2\sum_{m=1}^{5} F_m \cos m\omega/W \qquad (4.4)$$

Using this value of $F(\omega)$ in equation (4.3) and neglecting terms involving F_n^2 and F_nF_k for n and k greater than 2 results in the approximate delay function (see Appendix I).

$$-t_d(\omega)/T = \frac{\omega}{2\pi W} + \sum_{m=1}^{4} K_m \sin\frac{m\omega}{W}, \quad -\pi W < \omega < \pi W \qquad (4.5)$$

where

$$K_1 = \frac{2F_1(1 + F_2)}{\pi(1 + 2F_1^2)}, \qquad K_2 = \frac{2F_2 + F_1^2}{2\pi(1 + 2F_1^2)}$$

$$K_3 = \frac{2(F_3 + F_1F_2)}{3\pi(1 + 2F_1^2)}, \qquad K_4 = \frac{F_4}{2\pi(1 + 2F_1^2)}$$

T = signal time duration = $2\pi W(1 + 2F_1^2)/K$

F_m's as defined in equation (2.1).

Equation (4.5) can be written in the normalized form

$$-\underline{t_d} = \Delta + \sum_{m=1}^{4} K_m \sin 2\pi m \Delta, \quad -1/2 < \Delta < 1/2 \tag{4.6}$$

where

$$\underline{t_d} = t_d(\omega)/T$$

$$\Delta = \omega/(2\pi W) = f/W$$

This equation has been plotted in Figure 4.1 from which the degree of nonlinearity can be seen.

4.2 Phase as Function of Time

Since the transmitted signal is to be of the form

$$u(t) = \text{rect}(t/T) \exp\left[i\,\psi(t)\right], \tag{4.7}$$

the phase ψ must be determined as a function of time. If equation (4.6) is reversed so that ω is made a function of t, then

$$\psi(t) = \int_0^t \omega(t)\, dt \tag{4.8}$$

The most convenient method of reversing equation (4.6) is to make an N point harmonic analysis of t_d as shown in Figure 4.1, obtaining a finite series representation of $\omega(t)$,

$$\omega(t) = 2\pi W \left[\frac{t}{T} + \sum_{n=1}^{m} K_n \sin 2\pi nt/T \right] \qquad (4.9)$$

where

K_1	=	-.1648	K_5	=	-.0181
K_2	=	.0673	K_6	=	.0133
K_3	=	-.0380	K_7	=	-.0098
K_4	=	.0260			

Integration of equation (4.9) yields the phase function

$$\psi(t) = \pi TW \left[\left(\frac{t}{T}\right)^2 - \sum_{n=1}^{m} \frac{K_n}{\pi n} \cos 2\pi n \frac{t}{T} \right] \qquad (4.10)$$

Letting $TW = D = T/\tau$, where τ is the compressed pulse width, then $t/T = t'/D$, where t' is the time normalized to the compressed pulse. With this notation the phase becomes

$$\psi(t') = \pi D \left[(t'/D)^2 - \sum_{n=1}^{m} \frac{K_n}{\pi n} \cos 2\pi nt'/D \right] \qquad (4.11)$$

4.3 Spectrum of Transmitted Signal

Since the inverse Fourier transform of $f(t)$ can be expressed as

$$F(f) = \int_{-\infty}^{\infty} f(t) \exp(-j2\pi ft)\, dt \qquad (4.12)$$

the spectrum in terms of the normalized time $t' = Wt$ becomes

$$F(f/W) = \frac{1}{W} \int_{-\infty}^{\infty} f(t'/W) \exp(-j2\pi ft'/W)\, dt' \qquad (4.13)$$

For the transmitted signal $u(t')$ defined by equation (4.7), the spectrum is

$$U(\Delta) = \frac{1}{W} \int_{-\infty}^{\infty} u(t') \exp(-j\, 2\pi \Delta t')\, dt' \qquad (4.14)$$

or

$$U(\Delta) = \frac{1}{W}\int_{-\infty}^{\infty} \text{rect}(t'/D)\ \exp[\psi(t') - 2\pi\Delta t']\ dt'$$

$$= \frac{1}{W}\int_{-D/2}^{D/2} \exp\left\{j\pi D\left[(t'/D)^2 - \sum_{n=1}^{m}\frac{K_n}{\pi n}\cos 2\pi n t'/D - 2\Delta t'/D\right]\right\} dt' \tag{4.15}$$

where Δ = normalized frequency variable f/W.

4.4 Receiver Processing of the Return Signal

The receiver phase function $\exp(j\psi_R)$, is the conjugate of that for the transmitted signal so that $\psi_R = -\psi_T$. The transmitted phase angle is determined from the group delay by

$$\psi_T(\omega) = -\int_0^{\omega} t_d(\lambda)\ d\lambda \tag{4.16}$$

where $t_d(\lambda)$ is given by equation (4.5). Thus the phase is *

$$\psi_T(\omega) = -T\int_0^{\omega}\left[\lambda/(2\pi W) + \sum_{m=1}^{4} K_m \sin(m\lambda/W)\right] d\lambda\ , \tag{4.17}$$

which can be integrated and written in the form

$$\psi_T(\Delta) = -D\left[\pi\Delta^2 - \sum_{m=1}^{4}\frac{K_m}{m}\cos 2\pi m\Delta\right] \tag{4.18}$$

where the constant phase shift term in the integration has been omitted. The receiver phase is then

$$\psi_R(\Delta) = D\left[\pi\Delta^2 - \sum_{m=1}^{4}\frac{K_m}{m}\cos 2\pi m\Delta\right] \tag{4.19}$$

---

* To correspond with the Fourier series representation given by Equation (4.9), the sign of t_d has been changed.

If the receiver has an amplitude characteristic $A(\Delta)$, the overall transfer function for the receiver becomes

$$H(\Delta) = A(\Delta) \exp\left[j\,\psi_R(\Delta)\right] \qquad (4.20)$$

The doppler shifted received signal spectrum is obtained from equation (4.15) as

$$U(\Delta - \delta) = \frac{1}{W}\int_{-D/2}^{D/2} \exp\left[j\pi D\left\{\left[\frac{t'}{T}\right]^2 - \sum_{n=1}^{m} \frac{K_n}{\pi n} \cos 2\pi\frac{t'}{T} - \frac{2(\Delta - \delta)t'}{D} \right\}\right] dt' \qquad (4.21)$$

where δ = normalized doppler shift ϕ/W.

The amplitude of the spectrum as given by equation (4.21) is plotted in Figure 4.2 for D = 50 and zero doppler.

Using the receiver transfer characteristic as given by equation (4.20) and the input spectrum as defined by equation (4.21), the output time response is the Fourier transform of the product spectra.

$$s(Wt) = W\int_{-\Delta_o/2}^{\Delta_o/2} H(\Delta)\, U(\Delta - \delta) \exp(j2\pi\Delta Wt) \quad d\Delta \qquad (4.22)$$

where Δ_o = normalized receiver bandwidth

$= f_o/W$

4.5 Calculated Results

The time response given by equation (4.22) has been computed with a time-bandwidth product D = 50 for different doppler offsets and receiver bandwidths. In this case a rectangular receiver amplitude

characteristic has been assumed so that $A(\Delta)$ in equation (4.20) is made unity. The calculated results have been plotted in Figures 4.3A, 4.3B, 4.4A and 4.4B, and these show that the compressed pulse becomes severely distorted with high sidelobes even for small relative doppler frequencies. This means that the receiver compression filter which is matched to the zero doppler signal is extremely sensitive to the phase mismatch produced by the doppler shift.

5. Square Root of Taylor Nonlinear FM

In order to reduce the degree of nonlinearity in the transmitted signal and also to provide a match system to improve the signal-to-noise ratio, square root of Taylor spectrum weighting in both the transmitter and receiver is considered next.

5.1 Delay Function for Transmitted Signal

The spectrum amplitude for square root of Taylor weighting is given by

$$F(\omega) = \left[1 + 2 \sum_{m=1}^{5} F_m \cos \frac{m\omega}{W} \right]^{1/2} \tag{5.1}$$

where the constants F_m are those for 40 db weighting defined in equation (2.1). Substitution of this function into equation (4.3) results in the time delay of the signal

$$t_d = - \frac{1}{K} \int_0^{\omega} \left| F(\lambda) \right|^2 d\lambda$$

$$= - \frac{1}{K} \left[\omega + 2 \sum_{m=1}^{5} \frac{W F_m}{m} \sin \frac{m\omega}{W} \right] \tag{5.2}$$

$$-\pi W \leq \omega \leq \pi W$$

Dividing equation (5.2) by the signal time duration T, we obtain the normalized delay

$$t_d/T = \underline{t_d} = - \frac{2\pi W}{T} \left[\frac{\omega}{2\pi W} + \sum_{m=1}^{5} \frac{F_m}{\pi m} \sin \frac{m\omega}{W} \right]$$

$$\underline{t_d} = -\left[\Delta + \sum_{m=1}^{5} \frac{F_m}{\pi m} \sin 2\pi m \Delta \right] \qquad (5.3)$$

where the relation $\Delta = f/W = \omega/(2\pi W)$ has been used and K has been chosen to be $2\pi W/T$ to arrive at the last expression. When the constants F_m as given in equation (2.1) are substituted the delay as a function of frequency becomes

$$\underline{t_d}(\Delta) = -\Big[\Delta + .12386 \sin 2\pi\Delta - .001504 \sin 4\pi\Delta + .000518 \sin 6\pi\Delta - .000128 \sin 8\pi\Delta \Big] \qquad (5.4)$$

This equation has been plotted in Figure 5.1 and, when compared with Figure 4.1, the corresponding curve for Taylor weighting, it is noted that the deviation from linearity has been reduced.

5.2 Spectrum of Transmitted Signal

From a 24 point harmonic analysis of the frequency versus delay curve shown in Figure 5.1, the frequency is found as a function of time. The Fourier approximation is expressed as

$$\Delta(\underline{t}) = \underline{t} + \sum_{n=1}^{m} K_n \sin 2\pi n\underline{t} \qquad (5.5)$$

and since $\Delta = \frac{\omega}{2\pi W}$, $\underline{t} = \frac{t}{T}$, the series can be written

$$\omega(t) = 2\pi W \left[\frac{t}{T} + \sum_{n=1}^{m} K_n \sin \frac{2\pi n t}{T} \right] \tag{5.6}$$

where

K_1	=	$-.1145$	K_5	=	$-.0082$
K_2	=	$.0396$	K_6	=	$.0055$
K_3	=	$-.0202$	K_7	=	$-.0040$
K_4	=	$.0118$			

The Fourier series approximation of the frequency function is also shown in Figure 5.1.

Since equation (5.6) for the frequency function is the same as that given previously for the Taylor weighted case (equation 4.9), the form of the transmitted phase and spectrum is that given by equations (4.11) and (4.15) respectively.

5.3 Received Signal Waveform

The overall receiver transfer function is again given by

$$H(\Delta) = A(\Delta) \exp \left[j\,\psi_R(\Delta) \right] \tag{5.7}$$

where $\psi_R(\Delta)$ is defined by equation (4.19) and $A(\Delta)$ is the square root of Taylor amplitude characteristic

$$A(\Delta) = \left[1 + 2 \sum_{m=1}^{5} F_m \cos 2\pi m \Delta \right]^{1/2} \tag{5.8}$$

Using the relations given by (4.21) and (5.7) in equation (4.22), the receiver output is obtained as a function of time.

5.4 Calculated Results

The amplitude spectrum of the transmitted signal as computed by use of equation (4.21) for square root of Taylor weighting is shown in Figure 5.2 for the zero doppler case for a pulse compression ratio of 25. The corresponding square root of Taylor weighting function is also shown for comparison purposes. Using equation (4.22), the receiver signal output has been calculated for D = 25 and D = 50, for small doppler offsets in units of $\delta = \phi/W$. These results have been plotted in Figures 5.3A, B, C, D, E and Figures 5.4A, B, C, D, E. Comparing the matched and mismatched systems under zero doppler conditions for D = 50 a received signal bandwidth of W, the square root of Taylor weighted system is found to have 1.21 db more signal than the system with Taylor weighting only in the transmitter. It is noted from a comparison of Figures 5.3 and 5.4 that for a given doppler offset the time sidelobe amplitude increases in amplitude as the compression ratio is increased. Also, as the compression ratio is increased, evaluation of the integral in equation (4.22) requires more computation time. To decrease the computation time for large values of D, and also to check the results obtained here, another method of determining the system response is developed in the next section.

6. Alternative Method of Analyzing Nonlinear FM System Response

6.1 Analysis of System Matched Only in Phase

As a check on the previous method of finding the system response, an alternative method of analysis of the nonlinear FM system is used. This analysis assumes a weighted amplitude spectrum for the transmitted signal and an all pass receiver with a linear delay characteristic. The truncation in time of the transmitted signal and truncation in frequency of the received signal in the previous analysis is replaced by a transmitted spectrum of finite width. Using this approach, the Fourier series approximation of the delay-frequency characteristic is unnecessary since the analysis is carried out in the frequency domain instead of in the time domain.

Let $A(f) \exp\left[j\psi(f)\right]$ be the Fourier transform of the transmitted waveform and let the receiver be described by the all pass filter characteristic $\exp\left[-j\psi(f)\right]$. The response of the receiver to the doppler-shifted transmitted waveform is the inverse Fourier transform

$$s(t) = \mathcal{F}^{-1}\left\{A(f-\phi)\exp\left[j\psi(f-\phi) - j\psi(f)\right]\right\}, \quad (6.1)$$

Where $A(f)$ = amplitude of transmitted spectrum

$\psi(f)$ = phase of transmitted spectrum

ϕ = doppler shift in cycles per second.

The system in the frequency domain is as shown in Figure 6.1

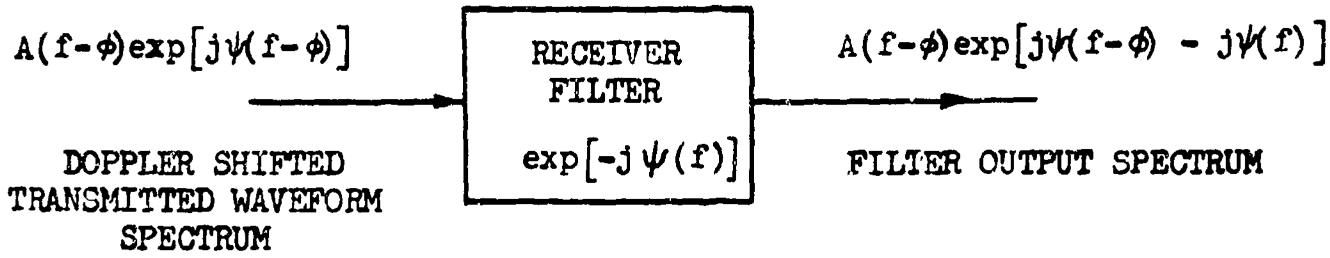


FIGURE 6.1

DIAGRAM OF SYSTEM IN FREQUENCY DOMAIN

6.2 Cosine on Pedestal Amplitude Spectrum

If the spectrum amplitude is described by

$$A(f) = \left[1 + 2\,F_1 \cos 2\pi \frac{f}{W} \right] \text{rect} \frac{f}{W} \qquad (6.2)$$

where

$F_1 = 0.42$

$W =$ width of spectrum in cycles per second,

the sidelobes of the response corresponding to $\phi = 0$, will all be suppressed to a level of 40 db or more below the peak response.

6.3 Required Phase Function

If the envelope of the transmitted waveform is to be approximately rectangular, then the phase function $\psi(f)$ must satisfy the condition[1]

$$\frac{d^2\psi}{df^2} = k\,A^2(f) \qquad (6.3)$$

where k is a constant to be determined later.

In this case

$$\frac{d^2\psi}{df^2} = k\left[1 + 2F_1 \cos 2\pi\frac{f}{W}\right]^2 , \quad |f| \le \frac{W}{2}$$

Integrating this expression with respect to frequency results in

$$\frac{d\psi}{df} = k\Big[(1 + 2F_1^2)f + 4F_1(W/2\pi)\sin 2\pi f/W$$

$$+ 2F_1^2 (W/4\pi)\sin 4\pi f/W\Big] \qquad (6.4)$$

If the delay $-\frac{1}{2\pi}\frac{d\psi}{df}$ is to change by $-T$ in the frequency interval $-\frac{W}{2}$ to $\frac{W}{2}$, then

$$-\frac{1}{2\pi}\frac{d\psi}{df}\Big|_{-W/2}^{W/2} = -T$$

which gives the value of

$$k = \frac{2\pi T}{(1 + 2F_1^2)W} \qquad (6.5)$$

The phase as a function of frequency on integration of (6.4) becomes

$$\psi(f) = \frac{\pi T}{W}f^2 - \frac{2TWF_1}{\pi(1 + 2F_1^2)}\cos 2\pi\frac{f}{W} - \frac{TWF_1^2}{4\pi(1 + 2F_1^2)}\cos 4\pi\frac{f}{W}$$

(6.6)

Equation (6.6) can be rewritten as

$$\psi(f) = \pi D \left[(f/W)^2 - C_1 \cos 2\pi \frac{f}{W} - C_2 \cos 4\pi \frac{f}{W} \right] \tag{6.7}$$

where

$$D = TW$$

$$C_1 = 2 F_1 / \pi^2 (1 + 2 F_1^2)$$

$$C_2 = F_1^2 / 4\pi^2 (1 + 2 F_1^2)$$

6.4 Output Phase Function and Spectrum

The phase difference, $\psi(f - \phi) - \psi(f)$, can be expressed as

$$\psi(f - \phi) - \psi(f) = \pi D \left[(\phi/W)^2 - 2f\phi/W - 2C_1 \sin \frac{\pi\phi}{W} \sin 2\pi \frac{f - \phi/2}{W} - 2 C_2 \sin \frac{2\pi\phi}{W} \sin 4\pi \frac{f - \phi/2}{W} \right] \tag{6.8}$$

The term $\pi D\left[\frac{\phi}{W}\right]^2$ in equation (6.8) represents a constant phase shift and $2\pi Df\phi/W$ is a linear phase shift with frequency so that these terms do not contribute to the distortion of the output waveform.

Considering only the distortion terms, the output spectrum can be written in the form

$$S(f) = A(f - \phi) \exp\left[- j(a \sin x + b \sin 2x)\right] \tag{6.9}$$

where $a = 2\pi C_1 D \sin \frac{\pi\phi}{W}$

$b = 2\pi C_2 D \sin \frac{2\pi\phi}{W}$

$x = \frac{2\pi}{W}(f - \frac{1}{2}\phi)$

Using the Bessel function expansion of the exponential

$$\begin{aligned}\exp(-j\alpha \sin\theta) = \; & J_0(\alpha) - J_1(\alpha)\left[\exp(j\theta) - \exp(-j\theta)\right] \\ & + J_2(\alpha)\left[\exp(j2\theta) + \exp(-j2\theta)\right] \\ & - J_3(\alpha)\left[\exp(j3\theta) - \exp(-j3\theta)\right] \\ & + \ldots\ldots ,\end{aligned} \tag{6.10}$$

the output spectrum becomes

$$\begin{aligned}S(f) = A(f - \phi)\Big\{ & J_0(a) - J_1(a)\left[\exp(jx) - \exp(-jx)\right] \\ & + J_2(a)\left[\exp(j2x) + \exp(-j2x)\right] \\ & - J_3(a)\left[\exp(j3x) - \exp(-j3x)\right] \\ & + \ldots \Big\}\Big\{ J_0(b) - J_1(b)\left[\exp(j2x) - \exp(-j2x)\right] \\ & + J_2(b)\left[\exp(j4x) + \exp(-j4x)\right] - \ldots \Big\}\end{aligned} \tag{6.11}$$

6.5 Result for Restricted Case

Consider the case where $TW = 50$ and $\phi = 0.1W$, the values of a and b are then $a = 6.15$, $b = 0.61$, and the corresponding Bessel functions are

$J_0(b) = 0.91, \quad J_1(b) = 0.29, \quad J_2(b) = .04$

In this case only the first two terms of the second Bessel function expansion will be considered. Under this condition the output spectrum can be written as

$$
\begin{aligned}
S(f) = A(f-\phi)\Big\{ & J_o(a)J_o(b) - \left[J_1(a)J_o(b) + J_1(a)J_1(b) + J_3(a)J_1(b)\right] \exp(jx) \\
& + \left[J_1(a)J_o(b) - J_1(a)J_o(b) - J_3(a)J_1(b)\right] \exp(-jx) \\
& + \left[J_2(a)J_0(b) - J_0(a)J_1(b) + J_4(a)J_1(b)\right] \exp(j2x) \\
& + \left[J_2(a)J_0(b) + J_0(a)J_1(b) - J_4(a)J_1(b)\right] \exp(-j2x) \\
& + \left[-J_3(a)J_0(b) + J_1(a)J_1(b) - J_5(a)J_1(b)\right] \exp(j3x) \\
& + \left[J_3(a)J_0(b) + J_1(a)J_1(b) - J_5(a)J_1(b)\right] \exp(-j3x) \\
& + \left[J_4(a)J_0(b) - J_2(a)J_1(b) + J_6(a)J_1(b)\right] \exp(j4x) \\
& + \left[J_4(a)J_0(b) + J_2(a)J_1(b) - J_6(a)J_1(b)\right] \exp(-j4x) \\
& + \left[-J_5(a)J_0(b) + J_3(a)J_1(b) - J_7(a)J_1(b)\right] \exp(j5x) \\
& + \left[J_5(a)J_0(b) + J_3(a)J_1(b) - J_7(a)J_1(b)\right] \exp(-j5x) \\
& + \ldots \Big\} \qquad (6.12)
\end{aligned}
$$

Equation (6.12) is of the form

$$S(f) = A(f-\phi) \sum_{n=-\infty}^{\infty} q_n \exp(jnx) \qquad (6.13)$$

where q_n are the respective coefficients of equation (6.12). The terms in equation (6.13) are of the type

$$S_n(f) = A(f-\phi)\, q_n \exp\left[j\, 2\pi n (f - \phi/2)/W \right]$$

$$= A(f-\phi)\, q_n \exp\left[j\, 2\pi n (f-\phi)/W \right] \exp(j\pi n \phi/W) \qquad (6.14)$$

6.6 Output Time Response

The time function corresponding to equation (6.14) is

$$s_n(t) = \int_{-\infty}^{\infty} S_n(f) \exp(j2\pi f t)\, df$$

$$= q_n \exp(j\pi n \phi/W) \int_{-\infty}^{\infty} A(f-\phi) \exp[j2\pi n(f-\phi)/W + j2\pi f t]\, df$$

$$= q_n\, a(t + \tfrac{n}{W}) \exp\left[j2\pi \phi (t + \tfrac{n}{2W}) \right] \qquad (6.15)$$

where

$$a(t) = \int_{-\infty}^{\infty} A(f) \exp(j2\pi f t)\, df$$

$$= \int_{-\infty}^{\infty} (1 + 2 F_1 \cos 2\pi \tfrac{f}{W}) \operatorname{rect} \tfrac{f}{W} \exp(j2\pi f t)\, df$$

$$= W \left[\operatorname{sinc} Wt + F_1 \operatorname{sinc} W (t + \tfrac{1}{W}) + F_1 \operatorname{sinc} W (t - \tfrac{1}{W}) \right] \qquad (6.16)$$

and

$$\operatorname{sinc} Wt = \frac{\sin \pi Wt}{\pi Wt}$$

The output time response is then expressed as

$$s(t) = \sum_{n=-\infty}^{\infty} s_n(t)$$

$$= \sum_{n=-\infty}^{\infty} q_n\, a(t + \tfrac{n}{W}) \exp\left[j2\pi \phi (t + \tfrac{n}{2W}) \right] \qquad (6.17)$$

In terms of the sinc functions the time response is

$$s(t) = W \exp(j2\pi \phi t) \sum_{n=-\infty}^{\infty} q_n \left\{ \text{sinc } W(t + \frac{n}{W}) + F_1 \text{ sinc } W(t + \frac{n+1}{W}) + F_1 \text{ sinc } W(t + \frac{n-1}{W}) \right\} \exp(j \frac{\pi\phi n}{W}) \quad (6.18)$$

When the time is taken at integer multiples of 1/W, only three terms are required for each point in time since $a(0) = W$, $a(\frac{1}{W}) = a(-\frac{1}{W}) = F_1W$ and $a(\frac{n}{W}) = 0$ for integer values of n other than 0 or $\pm$ 1.

The time response has been calculated for TW = D = 50 and $\phi = 0.1W$ using equation (6.18). The values of q_n used are given in Table 6.1 and the calculated values of s(t) are plotted in Figure 6.2. The solid curve shown in Figure 6.2 is the response as calculated by equation (4.22) of the first method of analysis. The agreement in the two methods of analysis is quite reasonable. For values of b greater than unity, more terms of the second Bessel function expansion should be taken. This leads to a different equation for the q_n's; however, the calculation of s(t) can be made in the same manner.

TABLE 6.1

q_0 = .173		q_6 = .160	q_{-6} = .318
q_1 = .273	q_{-1} = -.171	q_7 = -.0313	q_{-7} = .231
q_2 = -.202	q_{-2} = -.288	q_8 = -.014	q_{-8} = .134
q_3 = -.241	q_{-3} = -.115	q_9 = .018	q_{-9} = .064
q_4 = .461	q_{-4} = .153	q_{10} = .011	q_{-10} = .027
q_5 = -.358	q_{-5} = .314	q_{11} = .005	q_{-11} = .010

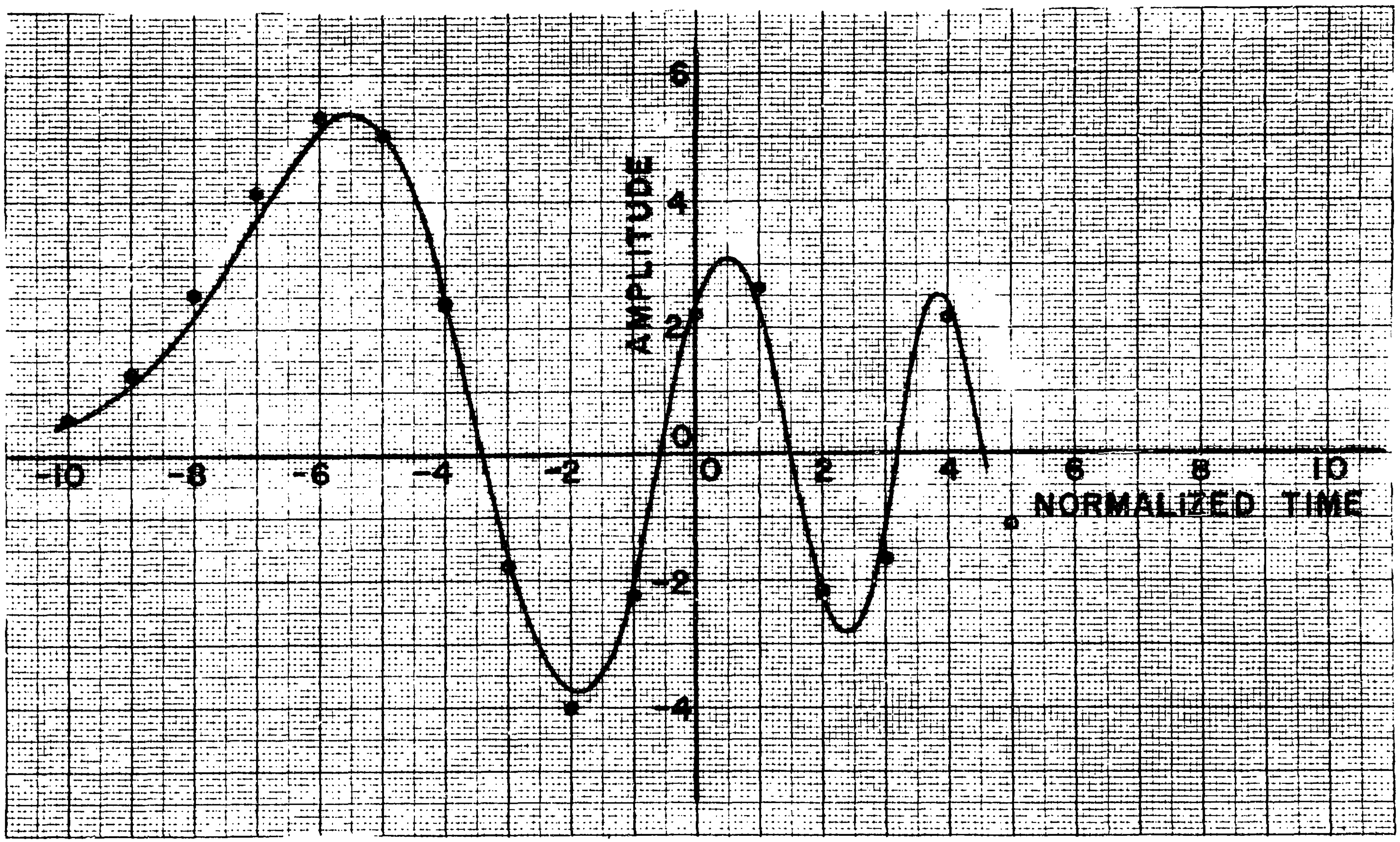


FIG. 6.2 NORMALIZED TIME RESPONSE BY TWO METHODS OF ANALYSIS

6.7 Analysis of Matched System

If the Fourier transform of the transmitted waveform is $A(f) \exp\left[j\,\psi(f)\right]$, then the matched receiver is given by $A(f) \exp\left[-j\,\psi(f)\right]$ where $A(f)$ is real. The response of the receiver to the doppler shifted transmitted waveform is then

$$s(t) = \mathcal{F}^{-1}\left[A(f)A(f-\phi) \exp\left\{ j\left[\psi(f-\phi) - \psi(f) \right] \right\} \right] \qquad (6.19)$$

6.8 Square Root of Cosine on Pedestal Amplitude Characteristic

To simplify the analysis a cosine on pedestal will be used instead of a 40 db Taylor weighted amplitude function. For the matched system, the transmitted spectrum will then be

$$A(f) = \left[1 + 2F_1 \cos 2\pi \frac{f}{W} \right]^{\frac{1}{2}} \operatorname{rect} \frac{f}{W} \qquad (6.20)$$

where $F_1 = 0.42$ for 40 db Taylor equivalence.

6.9 Spectral Phase Function

In order to satisfy the condition for a rectangular time envelope, use of equation (6.3) gives

$$\frac{d^2\psi}{df^2} = K\left[1 + 2F_1 \cos 2\pi \frac{f}{W} \right] , \quad |f| \le \frac{W}{2} \qquad (6.21)$$

Integration of (6.21) gives the delay

$$-\frac{1}{2\pi} \frac{d\psi}{df} = -\frac{K}{2\pi}\left[f + \frac{F_1 W}{\pi} \sin \frac{2\pi f}{W} \right] \qquad (6.22)$$

Since the delay is to change by $-T$ in the interval $-W/2$ to $W/2$, the value of K is $2\pi\, T/W$. This results in the spectral phase.

$$\psi(f) = TW \left[\pi \left(\frac{f}{W}\right)^2 - \frac{F_1}{\pi} \cos \frac{2\pi f}{W} \right] \tag{6.23}$$

and the phase difference

$$\psi(f-\phi) - \psi(f) = \pi D \left[\left(\frac{\phi}{W}\right)^2 - \frac{2f\phi}{W^2} - \frac{F_1}{\pi^2} \left\{ \cos 2\pi \frac{f-\phi}{W} - \cos 2\pi \frac{f}{W} \right\} \right]$$

$$= \pi D \left[\left(\frac{\phi}{W}\right)^2 - \frac{2f\phi}{W^2} - \frac{2F_1}{\pi^2} \sin \frac{\pi\phi}{W} \sin 2\pi \frac{f - \frac{1}{2}\phi}{W} \right] \tag{6.24}$$

where $D = TW$. The first two terms in equation (6.24) cause a constant phase shift and a linear phase shift, respectively, so that only the last term will be considered. In this case the phase distortion term is

$$\psi_d(f) = -\frac{2DF_1}{\pi} \sin \frac{\pi\phi}{W} \sin 2\pi \frac{f - \frac{1}{2}\phi}{W}$$

$$= -a \sin x \tag{6.25}$$

where
$$a = \frac{2DF_1}{\pi} \sin \frac{\pi\phi}{W}$$
$$x = 2\pi \frac{f - \frac{1}{2}\phi}{W}$$

6.10 Output Time Function

If it is assumed that doppler shifts are very small ($\phi/W \leq .04$), then

$$A(f)\,A(f-\phi) \;\cong\; \left[1 + 2F_1 \cos 2\pi \frac{f}{W}\right] \mathrm{rect}\,\frac{f}{W} \tag{6.26}$$

and the output time response will be

$$s(t) = \int_{-\infty}^{\infty} \mathrm{rect}\,\frac{f}{W} \cdot \left[1 + 2F_1 \cos 2\pi \frac{f}{W}\right] \exp\left[\, j\,\psi_d(f) + j2\pi\, ft\,\right] df$$

$$= \int_{-\infty}^{\infty} G(f) \exp\left[-ja \sin x\right] \cdot \exp\left[j2\pi\, ft\right] df \tag{6.27}$$

where

$$G(f) = \left[1 + 2F_1 \cos 2\pi \frac{f}{W}\right] \mathrm{rect}\,\frac{f}{W}$$

Using the Bessel function expansion given by equation (6.10), equation (6.27) can be expressed as the series of integrals

$$\begin{aligned} s(t) = \; & J_0(a) \int_{-\infty}^{\infty} G(f) \exp\,(j2\pi\, ft)\, df \\ & - J_1(a) \int_{-\infty}^{\infty} G(f) \left[\exp(jx) - \exp(-jx)\right] \exp(j2\pi\, ft)\, df \\ & + J_2(a) \int_{-\infty}^{\infty} G(f) \left[\exp(j2x) + \exp(-j2x)\right] \exp(j2\pi\, ft)\, df \\ & - J_3(a) \int_{-\infty}^{\infty} G(f) \left[\exp(j3x) - \exp(-j3x)\right] \exp(2\pi\, ft)\, df + \cdots \end{aligned} \tag{6.28}$$

The integrals in equation (6.28) are of the form

$$\begin{aligned} S_n(t) &= J_n(a) \int_{-\infty}^{\infty} G(f) \exp\left[j\,\frac{2\pi n}{W}\left(f - \tfrac{1}{2}\phi\right) + j2nft)\right] df \\ &= J_n(a) \exp\left(-j\,\frac{\pi n \phi}{W}\right) \int_{-\infty}^{\infty} G(f) \exp\left[\, j2\pi\, f\left(t + \frac{n}{W}\right)\right] df \\ &= J_n(a) \exp\left(-j\,\frac{\pi n \phi}{W}\right) g\left(t + \frac{n}{W}\right) \end{aligned} \tag{6.29}$$

where

$$g(t) = \int_{-\infty}^{\infty} G(f) \exp(j2\pi ft)\, df$$

$$= \int_{-\infty}^{\infty} \left[1 + 2F_1 \cos 2\pi \frac{f}{W}\right] \text{rect} \frac{f}{W} \cdot \exp(j2\pi ft)\, df$$

$$= W\left[\text{sinc } Wt + F_1 \text{ sinc } W\left(t + \frac{1}{W}\right) + F_1 \text{ sinc } W\left(t - \frac{1}{W}\right)\right] \quad (6.30)$$

The output response becomes

$$s(t) = J_0(a)g(t) - J_1(a)g\left(t + \frac{1}{W}\right) \exp\left(-j\frac{\pi\phi}{W}\right) + J_1(a)g\left(t - \frac{1}{W}\right) \exp\left(j\frac{\pi\phi}{W}\right)$$
$$+ J_2(a)g\left(t + \frac{2}{W}\right) \exp\left(-j\frac{2\pi\phi}{W}\right) + J_2(a)g\left(t - \frac{2}{W}\right) \exp\left(j\frac{2\pi\phi}{W}\right)$$
$$- J_3(a)g\left(t + \frac{3}{W}\right) \exp\left(-j\frac{3\pi\phi}{W}\right) + J_3(a)g\left(t - \frac{3}{W}\right) \exp\left(j\frac{3\pi\phi}{W}\right)$$
$$+ \dots$$

$$s(t) = \sum_{n=-\infty}^{\infty} (-1)^n J_n(a)g\left(t + \frac{n}{W}\right) \exp\left(-j\frac{n\pi\phi}{W}\right) \quad (6.31)$$

Equation (6.31) can be used to calculate the system time response for various doppler offsets and TW products. This equation has been derived on the assumption that the doppler offsets are so small that the receiver filter tracks the transmitted signal in amplitude. The advantage of equation (6.31) over the corresponding equation in the first method of analysis is that the computation time required using equation (6.31) is very much smaller. When the computed results from the two methods of analysis are compared, it is found that the time sidelobes of the response are lower in amplitude when equation (6.31) is used so that the results using this equation are optimistic.

6.11 Calculated Results

Through the use of equation (6.31), the received signal time response has been calculated for a number of different compression ratios and doppler frequencies. The key to the parameter values for the various curves are given in Table 6.2. A curve showing the signal loss as a function of doppler and compression ratio is plotted in Figure 6.11. A similar curve for the time sidelobes of the signal response is shown in Figure 6.12. From these results it is seen that the system performance with respect to signal loss and time sidelobe amplitude deteriorate with increased doppler frequency or compression ratio.

6.12 Comparison of the Two Analysis Methods

In the first method of analysis given in section 4, a rectangular time function is assumed for the envelope of the transmitted signal and a given amplitude spectrum is approximated by choice of the proper spectral phase function. Bandwidth truncation of the signal occurs in the receiver. A weighted amplitude spectrum that is truncated in frequency is assumed for the signal in the second method of analysis and a rectangular time function is approximated for the envelope by choice of the suitable phase function. Since the phase term in equation (6.1) vanishes for zero doppler, the signal spectrum for zero doppler is independent of the TW product in the second method of analysis. In the first method of analysis a direct cancellation of the phase function does not occur since the analysis is first carried out in the time domain ahead of the phase subtraction process in the receiver.

TABLE 6.2

KEY TO CALCULATED CURVES*

Time Bandwidth Product D		Compressed Pulse Response				
		$\delta = 0$	$\delta = .01$	$\delta = .02$	$\delta = .03$	$\delta = .04$
50	Fig.	6.3	6.4A	6.4B	6.4C	6.4D
100		6.3	6.5A	6.5B	6.5C	6.5D
150		6.3	6.6A	6.6B	6.6C	6.6D
200		6.3	6.7A	6.7B	6.7C	6.7D
300		6.3	6.8A	6.8B	6.8C	6.8D
400		6.3	6.9A	6.9B	6.9C	6.9D
500		6.3	6.10A	6.10B	6.10C	6.10D

*For doppler shifts in the order of $\delta = .1$, the curves are essentially a function of $D\delta$ so that the curve for $D = 500$, $\delta = .01$ can be used for the case where $D = 50$, $\delta = .1$.

7. Linear FM with Stepped AM in Transmitter

It has been seen that phase modulation of the transmitted signal results in an overall system response that becomes distorted with doppler shifts due to the mismatch between the signal and the compression filter. It is known that a system using amplitude modulation of a linear FM transmitted signal realizes no signal distortion with doppler when an all-pass receiver is used. To this end, a simple step type of amplitude modulation will be studied in this section to determine the degree of improvement provided as to sidelobe amplitude reduction and also to find the optimum values for the width and height of the steps.

7.1 Waveform Parameter Notation

The type of step modulation to be considered is shown in Figure 7.1 where the index N determines the number of steps. In the

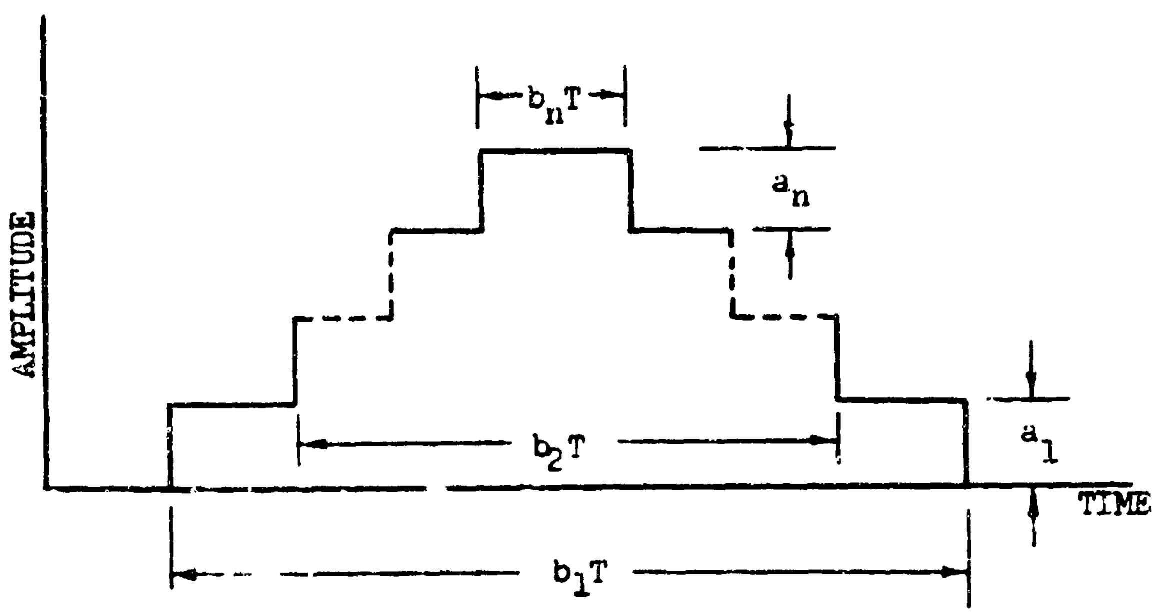


FIGURE 7.1 - STEPPED AMPLITUDE CARRIER ENVELOPE

notation used, the actual number of steps in the waveform is given by $2N - 1$ assuming a symmetrical time function. For the purposes of analysis it is convenient to assume that the waveform consists of N rectangular steps of amplitude a_n and width $b_n T$ as indicated in Figure 7.1. It will be assumed that $b_1 = 1$ and

$$\sum_{n=1}^{N} a_n = 1 \tag{7.1}$$

7.2 Response of Flat Receiver to Step Amplitude Waveform

We consider the case where the receiver is matched to the incoming signal only in phase, the receiver amplitude being flat. The transfer function of the receiver is then

$$H(f) = \exp\left[j\pi(f - fo)^2/k \right] \tag{7.2}$$

where

fo = carrier frequency

k = rate of linear FM.

The response of a receiver to the doppler shifted signal is evaluated as the convolution of the signal time function and the impulse response of the receiver transfer function. If a single rectangular pulse of amplitude a and width bT is considered, then the response can be written as

$$s(t) = e(t) * h(t) \tag{7.3}$$

where $e(t)$ = doppler shifted transmitted waveform

$$= a\ \mathrm{rect}\left(\frac{t}{bT}\right) \exp\left[2\pi j(f_o + \phi)t + kt^2/2 \right] \tag{7.4}$$

$*$ = convolution

$h(t)$ = receiver impulse response

$$= \int_{-\infty}^{\infty} H(f) \exp\left[j2\pi ft \right] df$$

$$= \sqrt{\frac{jW}{T}} \exp\left[j2\pi(f_o t - kt^2/2) \right] \tag{7.5}$$

ϕ = doppler frequency

W = net frequency sweep = kT

Using equations (7.4) and (7.5) in 7.3) gives the response

$$s(t) = a\sqrt{\frac{jW}{T}} \int_{-\infty}^{\infty} \text{rect}\left(\frac{\tau}{bT}\right) \exp\left\{ j2\pi\left[(f_o + \phi)\tau + k\tau^2/2 + f_o(t-\tau) - k(t-\tau)^2/2\right]\right\} d\tau$$

$$= a\sqrt{\frac{jW}{T}} \exp\left[j2\pi(f_o t - kt^2/2)\right] \int_{-bT/2}^{bT/2} \exp\left[j2\pi(\phi + kt)\tau\right] d\tau$$

$$= ab\sqrt{D} \exp\left[j2\pi(f_o t - kt^2/2) + j\pi/4\right] \frac{\sin \pi(\delta D + Wt) b}{\pi(\delta D + Wt) b} \tag{7.6}$$

where $D = TW$.

Equation (7.6) is the response of an all pass receiver to a linear FM signal with a rectangular time envelope. The envelope of the response is given by

$$|s(t)| = ab\sqrt{D} \frac{\sin \pi(\delta D + Wt) b}{\pi(\delta D + Wt) b} = ab\sqrt{D}\, \text{sinc}(\delta D + Wt)\, b \tag{7.7}$$

Since the transmitted signal envelope consists of a sum of rectangular pulses of amplitude a_n and width $b_n T$, the transmitted signal can be expressed as

$$e(t) = \sum_{n=1}^{N} a_n \,\text{rect}\left(\frac{t}{b_n T}\right) \exp\left[j2\pi(f_o + \phi)\, t + {kt^2}/{2} \right] \tag{7.8}$$

and the corresponding response envelope of the compressed pulse becomes

$$|s(t)| = \sqrt{D} \sum_{n=1}^{N} a_n b_n \operatorname{sinc}\left[(\delta D + Wt) b_n\right] \tag{7.9}$$

by superposition of individual responses as given by equation (7.7).

7.3 Optimization of Amplitude and Width of Subpulses

We wish to determine the optimum values of a_n and b_n which will yield the minimum time sidelobes for the response given by equation (7.9). The response function can be expressed in the form

$$s(\alpha) = \sum_{n=1}^{N} a_n b_n \frac{\sin b_n \alpha}{b_n \alpha} \tag{7.10}$$

where

$$s(\alpha) = \frac{|s(t)|}{\sqrt{D}}$$

$$\alpha = \pi(\delta D + Wt)$$

In general, the function given by equation (7.10) will be sin x/x in form, with a positive main lobe and positive and negative sidelobes. The problem of minimizing the time sidelobes that occurs here is the same as that of reducing the antenna pattern sidelobes for an antenna utilizing an amplitude distribution that varies in a stepwise manner.[6] We define the criterion of performance or efficiency η as the ratio of energy in the main lobe between the first zeros to the total energy in the pulse. Thus

$$\eta = \frac{\int_0^{\alpha_0} s(\alpha)^2 \, d\alpha}{\int_0^{\infty} s(\alpha)^2 \, d\alpha} \tag{7.11}$$

where

α_o = value of α at first zero.

The integral in the denominator of equation (7.11) can be integrated as

$$\int_0^{\infty} \left[\sum_{n=1}^{N} a_n b_n \frac{\sin b_n \alpha}{b_n \alpha} \right]^2 d\alpha = \frac{\pi}{2} \Big[a_1^2 (b_1 - b_2) + (a_1 + a_2)^2 (b_2 - b_3) + \ldots$$

$$\ldots + (a_1 + a_2 + \ldots + a_{n-1})^2 (b_{n-1} - b_n)$$

$$+ (a_1 + a_2 + \ldots + a_n)^2 \, b_n \Big] \qquad (7.12)$$

This equation represents the total energy in the pulse and can be assumed to be constant. If evaluated for $N = 1$, $a_1 = 1$, $b_1 = 1$, the value of the integral given in equation (7.12) becomes $\pi/2$. The value of is then

$$= \frac{2}{\pi} \int_0^{\alpha} \left[\sum_{n=1}^{N} a_n \frac{\sin b_n \alpha}{\alpha} \right]^2 d\alpha \qquad (7.13)$$

This integral has been maximized with respect to a_n and b_n by Nash[6] for values of N up to 4. His results are plotted in terms of the present notation in Figure 7.2. The efficiency is seen to be a rather slowly varying function of the pulse width b_N. It is also found to be a slowly varying function with respect to the amplitude of the steps. Nash found in his studies that the best results were obtained when the change in the pulse width from subpulse to subpulse was kept constant (e.g. $b_1 - b_2 = b_2 - b_3$, etc.)

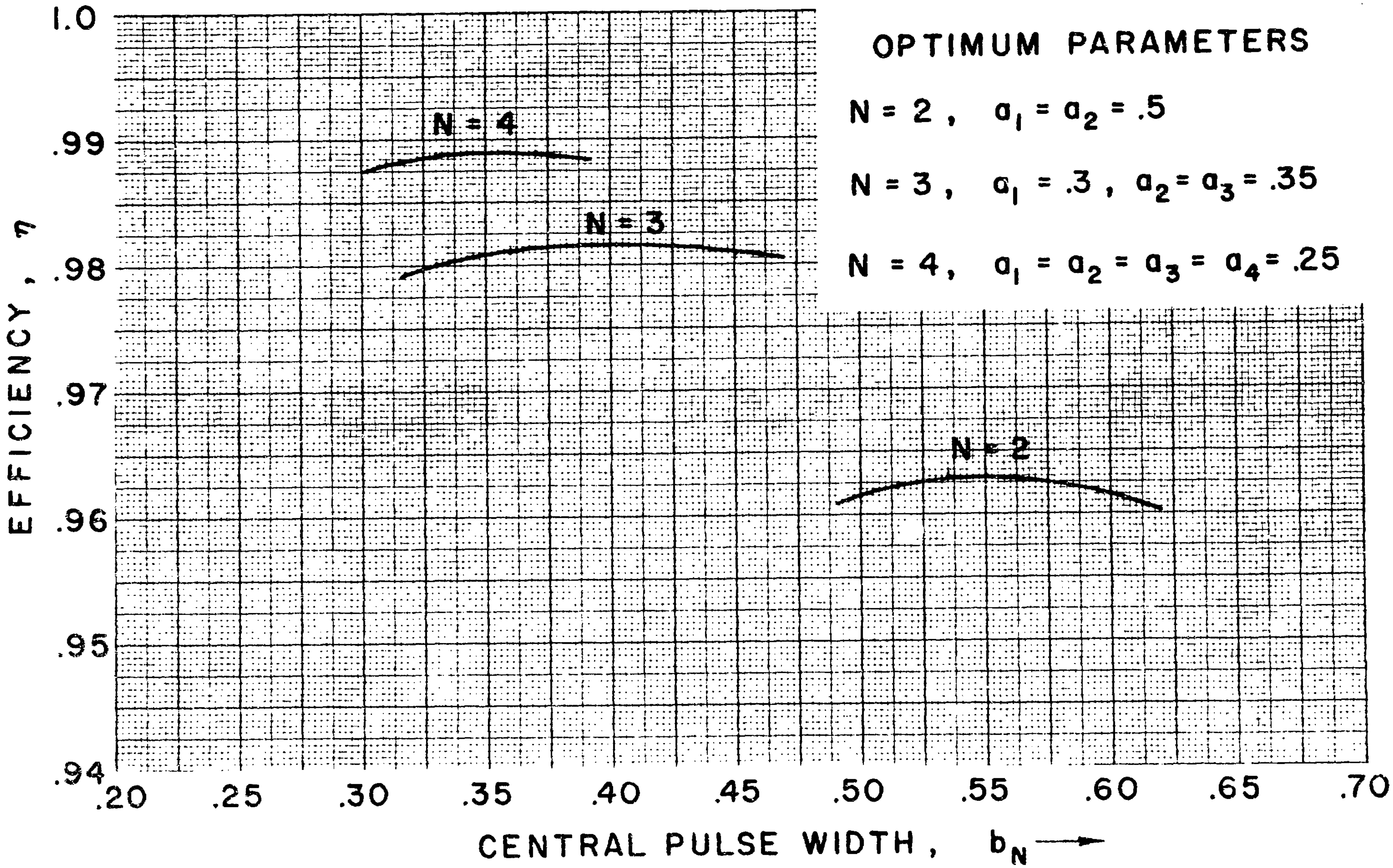


FIG. 7.2 EFFICIENCY AS FUNCTION OF N AND CENTRAL PULSE WIDTH

7.4 Calculated Results

Using a normalized form of equation (7.10) expressed by

$$\underline{s(\alpha)} = \frac{\sum_{n=1}^{N} a_n b_n \frac{\sin b_n \alpha}{b_n \alpha}}{\sum_{n=1}^{N} a_n b_n} \qquad (7.14)$$

the time response for a number of step-amplitude waveforms have been calculated. Table 7.1 lists the various Figures with their significant parameters and pertinent comments. Inspection of Figures 7.4A, 7.5A, 7.6A, 7.7A and 7.7C shows that the optimum values of the step width and amplitude are such that the inner corners of the steps lie very close to the 40 db Taylor weighting curve. The sidelobe characteristics of the curves are summarized in Figure 7.8 and shows that even with only two subpulses, there is approximately 7 db improvement in the time sidelobe response. As the number of subpulses is increased beyond 5, the reduction obtained in the sidelobe amplitude is small for each unit increment in N. The normalized pulse width as a function of the number of subpulses is shown in Figure 7.9 . The comparison in the widths is made at half the pulse amplitude and the reference pulse is the normal sin x/x pulse.

TABLE 7.1

WAVEFORM PARAMETERS AND PERFORMANCE

Fig. No.	N	a_1	a_2	a_3	a_4	a_5	b_1	b_2	b_3	b_4	b_5	Peak Sidelobe Amplitude (db)	Comment
7.3	1	1	-	-	-	-	1	-	-	-	-	-13.2	rectangular chirp
7.4A	2	.5	.5	-	-	-	1	.55	-	-	-		maximum efficiency transmitted envelope
7.4B	2	.5	.5	-	-	-	1	.55				-20.9	maximum efficiency
7.4C	2	.556	.444	-	-	-	1	.55	-	-	-	-20.8	near min. sidelobe
7.4D	2	.5	.5	-	-	-	1	.50	-	-	-	-19.1	non-optimum
7.5A	3	.30	.35	.35	-	-	1	.70	.40	-	-		maximum efficiency transmitted envelope
7.5B	3	.30	.35	.35	-	-	1	.70	.40	-	-	-23.	maximum efficiency
7.5C	3	.35	.35	.30	-	-	1	.625	.350	-	-	-23.7	min. sidelobe
7.5D	3	.35	.35	.30	-	-	1	.675	.40	-	-	-23.4	near min. sidelobe
7.5E	3	.40	.375	.225	-	-	1	.55	.35	-	-	-20.8	non-optimum
7.6A	4	.25	.25	.25	.25	-	1	.78	.56	.34	-		maximum efficiency transmitted waveform
7.6B	4	.25	.25	.25	.25	-	1	.78	.56	.34	-	-27.6	maximum efficiency
7.6C	4	.35	.30	.25	.10	-	1	.675	.445	.225	-	-27.1	near min. sidelobe
7.6D	4	.400	.275	.225	.10	-	1	.640	.430	.220	-	-25.6	non-optimum
7.6E	4	.265	.315	.295	.125	-	1	.750	.500	.250	-	-24.8	non-optimum
7.7A	5	.21	.24	.20	.23	.12	1	.80	.60	.40	.20		maximum efficiency transmitted envelope
7.7B	5	.21	.24	.20	.23	.12	1	.80	.60	.40	.20	-29.3	maximum efficiency
7.7C	5	.300	.225	.235	.170	.07	1	.72	.54	.36	.18		min. sidelobe transmitted envelope
7.7D	5	.300	.225	.235	.170	.07	1	.72	.54	.36	.18	-29.6	minimum sidelobe
7.7E	5	.210	.240	.220	.210	.120	1	.80	.60	.40	.20	-28	non-optimum
7.7F	5	.210	.240	.180	.240	.130	1	.80	.60	.40	.20	-29	non-optimum
7.7G	5	.210	.240	.260	.210	.080	1	.80	.60	.40	.20	-25.75	non-optimum
7.7H	5	.350	.225	.225	.150	.050	1	.67	.50	.32	.15	-26.5	non-optimum
7.8													sidelobe amplitude vs N
7.9													pulse width vs N

8. Summary, Conclusions and Recommendations

8.1 Summary

In this section a comparison is made of the different systems that were studied to aid in summarizing the results:

(a) The two pertinent quantities by which the performance of the different systems is judged are the time sidelobes and the signal-to-noise loss characteristic as a function of doppler shifts.

(b) A set of curves showing these quantities plotted against normalized doppler is shown in Figure 8.1 for a system with a time-bandwidth product of 50.

(c) To obtain an idea as to the range of the normalized doppler δ, a radar operating in C-band with a bandwidth of 4.5 Mc and a target velocity of 60,000 ft/sec, corresponds to a δ of 0.15.

(d) Curves a, A, b and B of Figure 8.1 show that both the sidelobe and signal-to-noise performance of the nonlinear FM systems deteriorate very rapidly with doppler offset.

(e) In the case of linear FM shown by curves C, D and d, the change in performance with respect to these quantities is relatively slow.

(f) Curve C of Figure 8.1 is for a stepped amplitude modulated linear FM transmitted signal feeding a receiver with a flat amplitude-frequency characteristic so does not show the sidelobe degradation with

CURVE

a, A NONLINEAR FM WITH TAYLOR SPECTRUM WEIGHTING IN TRANSMITTER, D = 50

b, B NONLINEAR FM WITH TAYLOR SPECTRUM WEIGHTING IN TRANSMITTER AND RECEIVER, D = 50

c, C LINEAR FM WITH STEP AMPLITUDE MODULATION IN TRANSMITTER, FLAT RESPONSE RECEIVER, N=

d, D LINEAR FM WITH TAYLOR FREQUENCY WEIGHTING IN RECEIVER, D = 50

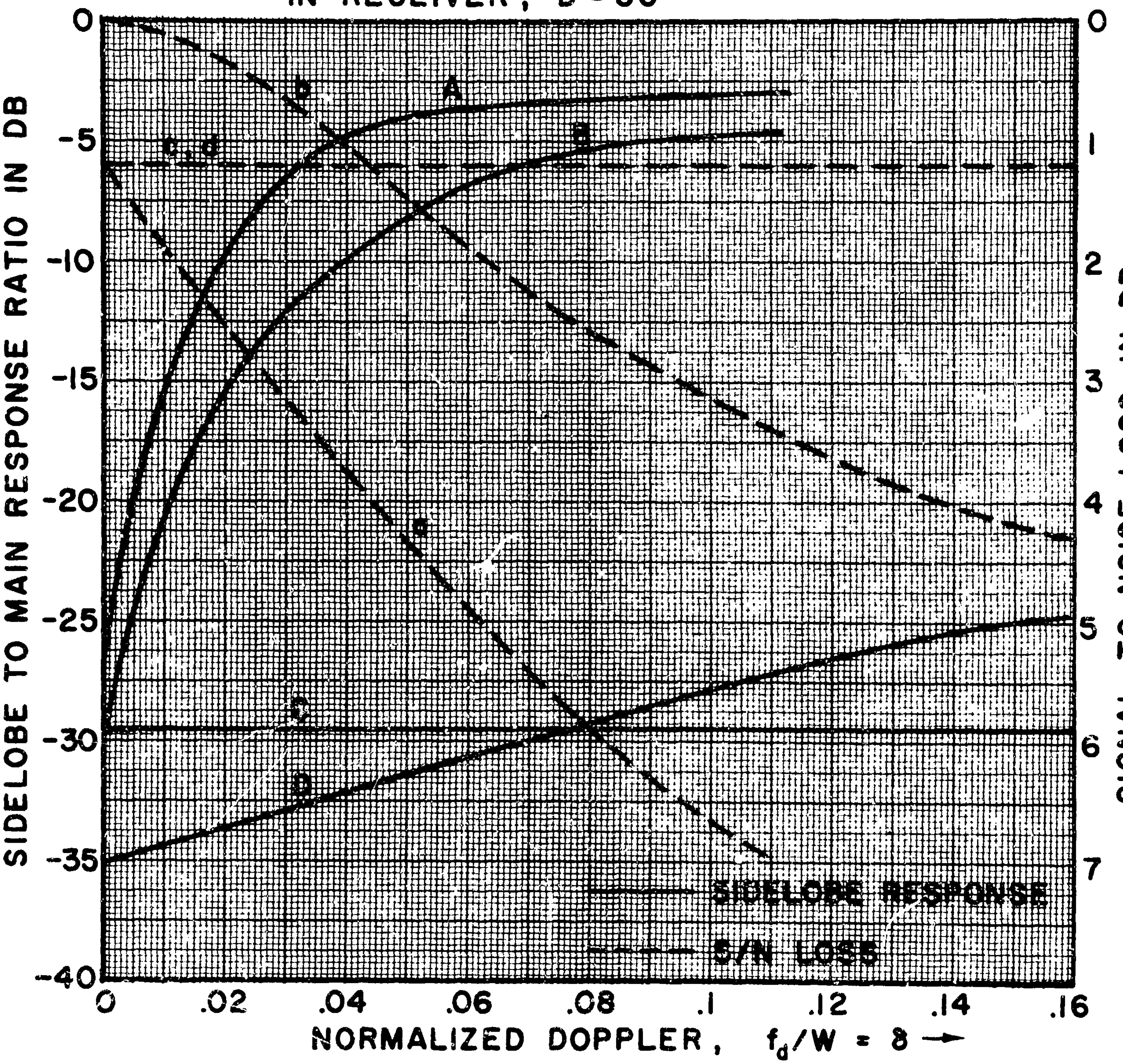


FIG. 8.1 TIME SIDELOBE AND SIGNAL-TO-NOISE PERFORMANCE

doppler shift.

(g) When the step-amplitude modulated type of transmitted signal is used in conjunction with a receiver of bandwidth W, the sidelobe versus doppler curve would have an upward slope slightly less than that for the rectangular envelope, linear FM case shown by Curve D.

8.2 Conclusions

Although nonlinear FM techniques result in relatively low time sidelobes for zero doppler, the compressed pulse becomes severely distorted with high sidelobes as the doppler frequency is increased. Since the time response for the nonlinear FM system is a function of the product of doppler and the time bandwidth product, the system performance deteriorates with increased doppler frequency or compression ratio. It is concluded that, when applied to pulse doppler radars operating in the search mode, nonlinear FM techniques are inferior to those using linear FM. In the track mode of operation of a pulse doppler radar or in a mode where most of the doppler shift has been removed, full advantage could be taken of the nonlinear FM signals with respect to sidelobe level and signal-to-noise ratio since a rectangular envelope signal is being transmitted and the system is matched for near zero dopplers.

It has been shown that substantial reduction of the time sidelobe level can be obtained by a small number of steps in the time envelope of the linear FM transmitted signal. By appropriately step-amplitude weighting the transmitted signal and spectrum weighting in the receiver, the

sidelobe and signal-to-noise performance could be improved over that obtained in this study.

8.3 Recommendations

Based upon the results of this study, it is recommended that experimental evaluation be performed involving amplitude-weighted frequency-coded waveforms, and that further experimental and theoretical study be performed in additional areas relating to linear FM pulse compression systems. These recommendations are summarized below.

a. Experimental simulation should be implemented for the case of amplitude-weighted, frequency coded transmitted signals and frequency weighting in the receiver.

b. The amplitude weighting in the transmitter and the frequency weighting in the receiver should approximate either square root of Taylor or square root of cosine-on-a-pedestal, and the frequency coding should be linear FM.

c. Initial experimental investigation of amplitude weighting for the transmitted signal should be of the discrete-step type following a square root of Taylor curve.

d. Experimental simulation should be implemented for the case of un-weighted linear FM transmitted signals with a Taylor weighted receiver, and with overall equivalent parameters (i.e. bandwidth, compression ratio, etc.) for use as a basis of comparison.

e. Experimental data should be taken on the two systems and evaluation made to demonstrate the signal-to-noise improvement and sidelobe behaviour in the presence of doppler shift when compared with the linear FM Taylor weighted receiver case.

f. Experimental and theoretical study are needed in other areas related to linear FM pulse compression systems:

(1) Accurate measurement of the linearity of the FM sweep is difficult using present techniques, and the development of more precise methods is necessary.

(2) In the design of a practical pulse compression system, the reduction of the time sidelobes is limited by the presence of the Fresnel wiggles in the spectrum, and studies to reduce this effect should be conducted once the actual spectrum and non-linearities are determined.

(3) Investigations should be conducted on methods for implementing and improving the efficiency of the transmitter for amplitude modulated waveforms.

When amplitude weighting is applied to the transmitted signal, doppler shifts do not disturb the relative amplitudes of the spectral components. Therefore, the compressed pulse output from a flat amplitude characteristic receiver remains unchanged with doppler shifts. Thus, the system is equivalent to a receiver that has a weighting filter that "tracks" a rectangular envelope linear FM signal as the signal is shifted

by the doppler frequency. In the case of a fixed weighting filter in the receiver, as recommended above, the system is partially "tracking" since the weighted incoming signal spectrum is shifted by the doppler frequency with respect to this fixed weighting filter characteristic. The sidelobe behaviour with doppler shifts should be better for this system than for the rectangular envelope linear FM system with a Taylor weighted receiver since there is no "tracking" at all in the latter system. With weighting in the transmitter and in the receiver, the system recommended for the investigation is closer to a matched system so that the signal-to-noise loss should be less than in the Taylor receiver case.

APPENDIX I

Delay Function for Nonlinear FM

The delay function for a system that is constrained to a rectangular time envelope and a specified spectrum amplitude is defined by

$$t_d(\omega) = -\frac{1}{K}\int_0^{\omega} \left|F(\lambda)\right|^2 d\lambda \qquad (I.1)$$

For a 40 db Taylor weighted spectrum

$$F(\omega) = 1 + 2\sum_{m=1}^{5} F_m \cos m\omega/W \qquad (I.2)$$

so that

$$\left|F(\omega)\right|^2 = 1 + 4\sum_{m=1}^{5} F_m \cos\frac{m\omega}{W} + 4\sum_{m=1}^{5}\sum_{k=1}^{5} F_m F_k \cos\frac{m\omega}{W}\cos\frac{k\omega}{W} \qquad (I.3)$$

Substituting this value of $\left|F(\omega)\right|^2$ in equation (I.1) results in the delay

$$-t_d(\omega) = \frac{1}{K}\Big[\omega + 4\sum_{m=1}^{5} F_m \int_0^{\omega} \cos\frac{m\lambda}{W}\, d\lambda$$

$$+ 4\sum_{m=1}^{5}\sum_{k=1}^{5} F_m F_k \int_0^{\omega} \cos\frac{m\lambda}{W}\cos\frac{k\lambda}{W}\, d\lambda\Big]$$

$$= \frac{1}{K}\Big[\omega + 4\sum_{m=1}^{5} F_m \frac{W}{m}\sin\frac{m\omega}{W} + 2\sum_{m=1}^{5}\sum_{k=1}^{5} F_m F_k \Big\{\frac{\sin(m-k)\omega/W}{(m-k)/W}$$

$$+ \frac{\sin(m+k)\omega/W}{(m+k)/W}\Big\}\Big] \qquad (I.4)$$

For 40 db Taylor weighting, terms involving $F_m F_k$, where m and k are greater than 2 are negligible, so that cross product terms other than those

for k=1 or m=1 can be dropped. Under these conditions the delay will be

$$- t_d(\omega) \cong \frac{1}{K}\left\{ \omega + 4\sum_{m=1}^{5} F_m \frac{W}{m} \sin\frac{m\omega}{W} + F_1^2\left[2\omega + W\sin(2\omega/W)\right] \right.$$

$$\left. + 4\sum_{m=2}^{5} F_m F_1 \left[\frac{\sin(m-1)\omega/W}{(m-1)/W} + \frac{\sin(m+1)\omega/W}{(m+1)/W} \right] \right\} \qquad (I.5)$$

Considering only those terms for $F_m F_1$ where m=2, the delay after factoring the quantity $2\pi W(1 + 2F_1^2)$ can be expressed as

$$- t_d(\omega) = \frac{2\pi W(1 + 2F_1^2)}{K} \left\{ \frac{\omega}{2\pi\omega} + \frac{2\,F_1(1 + F_2)}{\pi(1 + 2F_1^2)} \sin(\omega/W) \right.$$

$$+ \frac{F_1^2 + 2F_2}{2\pi(1 + 2F_1^2)} \sin\frac{2\omega}{W} + \frac{2(F_3 + F_1F_2)}{3\pi(1 + 2F_1^2)} \sin\frac{3\omega}{W}$$

$$\left. \frac{F_4}{2\pi(1 + 2F_1^2)} \sin\frac{4\omega}{W} + \frac{F_5}{2\pi(1 + 2F_1^2)} \sin\frac{5\omega}{W} \right\} \qquad (I.6)$$

If the total time duration of the pulse is denoted by $T = 2\pi W(1 + 2F_1^2)/K$, then

$$- t_d(\omega) = T\left[\frac{\omega}{2\pi W} + \sum_{m=1}^{5} K_m \sin(m\omega/W) \right] \qquad (I.7)$$

where K_m are the coefficients as indicated in equation (I.6).

REFERENCES

1. E. L. Key, E. N. Fowle, R. D. Haggarty,
"A Method of Designing Signals of Large Time-Bandwidth Product",
I.R.E. International Convention Record, Part 4, 1961, pp 146-154.

2. T. T. Taylor,
"Design of Line-Source Antennas for Narrow Beamwidth and Low Sidelobes",
I.R.E. Trans. AP-3, No. 1, January 1955, pp 16-28.

3. J. R. Klauder, A. C. Price, S. Darlington, W. J. Albersheim,
"The Theory and Design of Chirp Radars",
B. S. T. J., Vol. 39, July 1960, pp 745-808.

4. C. L. Cuccia,
"Harmonics, Sidebands, and Transients in Communication Engineering",
McGraw-Hill Book Co., 1952, p. 449.

5. C. E. Cook,
"General Matched-Filter Analysis of Linear FM Pulse Compression",
Proc. I.R.E., Vol. 49, April 1961. (Correspondence)

6. Robert T. Nash,
"Stepped Amplitude Distributions",
I.E.E.E. Transactions on Antennas and Propagation, Vol. AP-12, No. 4.
July 1964. pp 515-516.

LIST OF SYMBOLS

a, a_n	=	amplitude constant, $n = 1, 2, \ldots N$
$a(t)$	=	amplitude response
$A(\ )$	=	spectrum amplitude
A_n	=	amplitude constant, $n = 0, 1, 2, \ldots$
b, b_n	=	constants, $n = 1, 2, \ldots N$
B_m	=	constants, $m = 0, 1, 2 \ldots$
C_1, C_2	=	constants
D	=	time-bandwidth product, TW
$e(t)$	=	doppler shifted transmitted waveform
f	=	frequency in cycles per second
f_o	=	carrier frequency, cycles per second
$f(t)$	=	signal function
$F(\ )$	=	spectrum amplitude
F_m	=	constant $m = 1, 2, \ldots$
FM	=	frequency modulation
$g(t)$	=	time function corresponding to the spectrum $G(f)$

$h(t)$ = receiver impulse response

$H(f)$ = receiver transfer function

$H_c(\omega)$ = transfer function of compression network

$H_d(\omega)$ = frequency weighting function

j = $\sqrt{-1}$

$J_n(\ )$ = Bessel functions of the first kind $n = 0, 1, 2, \ldots$

k = rate of linear FM

K, K_n = constants, $n = 1, 2, \ldots$

m, M = integers

n, N = integers

o = subscript for output function

r = subscript for received signal

$$\text{rect}(t/T) = \begin{cases} 1 & |t| \leq T/2 \\ 0 & |t| > T/2 \end{cases}$$

R = subscript for receiver

$s(t)$ = time response amplitude

$S(f)$ = frequency spectrum amplitude

$\text{sinc}\ t$ = $(\sin \pi t)/(\pi t)$

t	=	time
t'	=	normalized time t/T
t_d	=	time delay
$\underline{t_d}$	=	normalized time delay
T	=	time duration of pulse
$u(t)$	=	transmitted signal time function
u, u_1, u_2	=	normalized frequency variable
U	=	received signal spectrum amplitude
w	=	subscript for weighted function
W	=	bandwidth in cycles per second
x	=	normalized frequency variable
$Z(u)$	=	complex Fresnel integral
α	=	normalized time variable
α_o	=	value of α at first zero
δ	=	normalized doppler frequency, ϕ/W
Δ	=	normalized frequency variable, f/W
η	=	efficiency of pulse defined as ratio of main pulse energy to total energy of pulse

θ	=	angular variable
λ	=	dummy frequency variable
π	=	3.14159265...
τ	=	time variable
ϕ	=	doppler frequency in cycles per sec.
ψ	=	phase of signal spectrum
ω	=	angular frequency

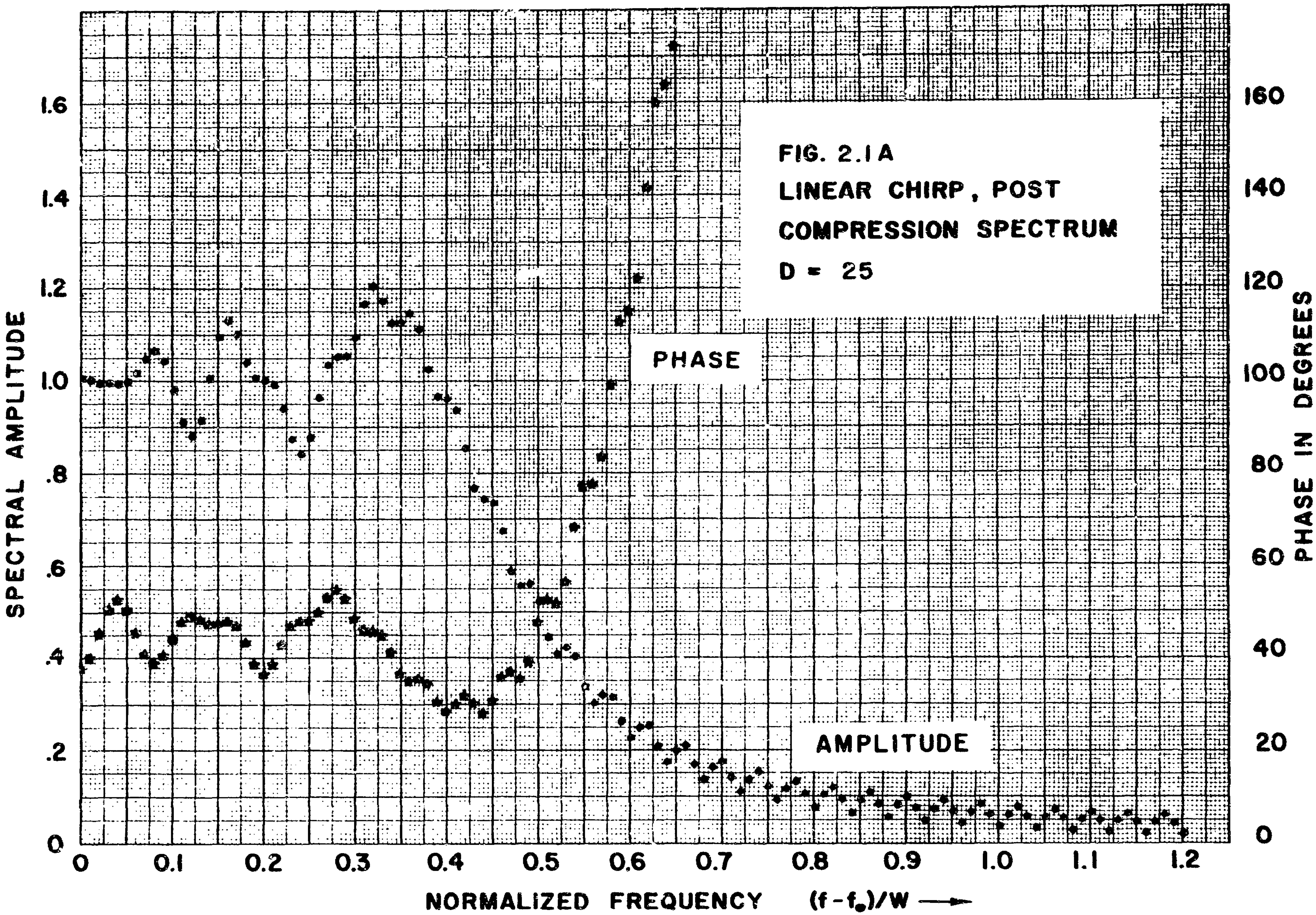


FIG. 2.1A
LINEAR CHIRP, POST COMPRESSION SPECTRUM
D = 25

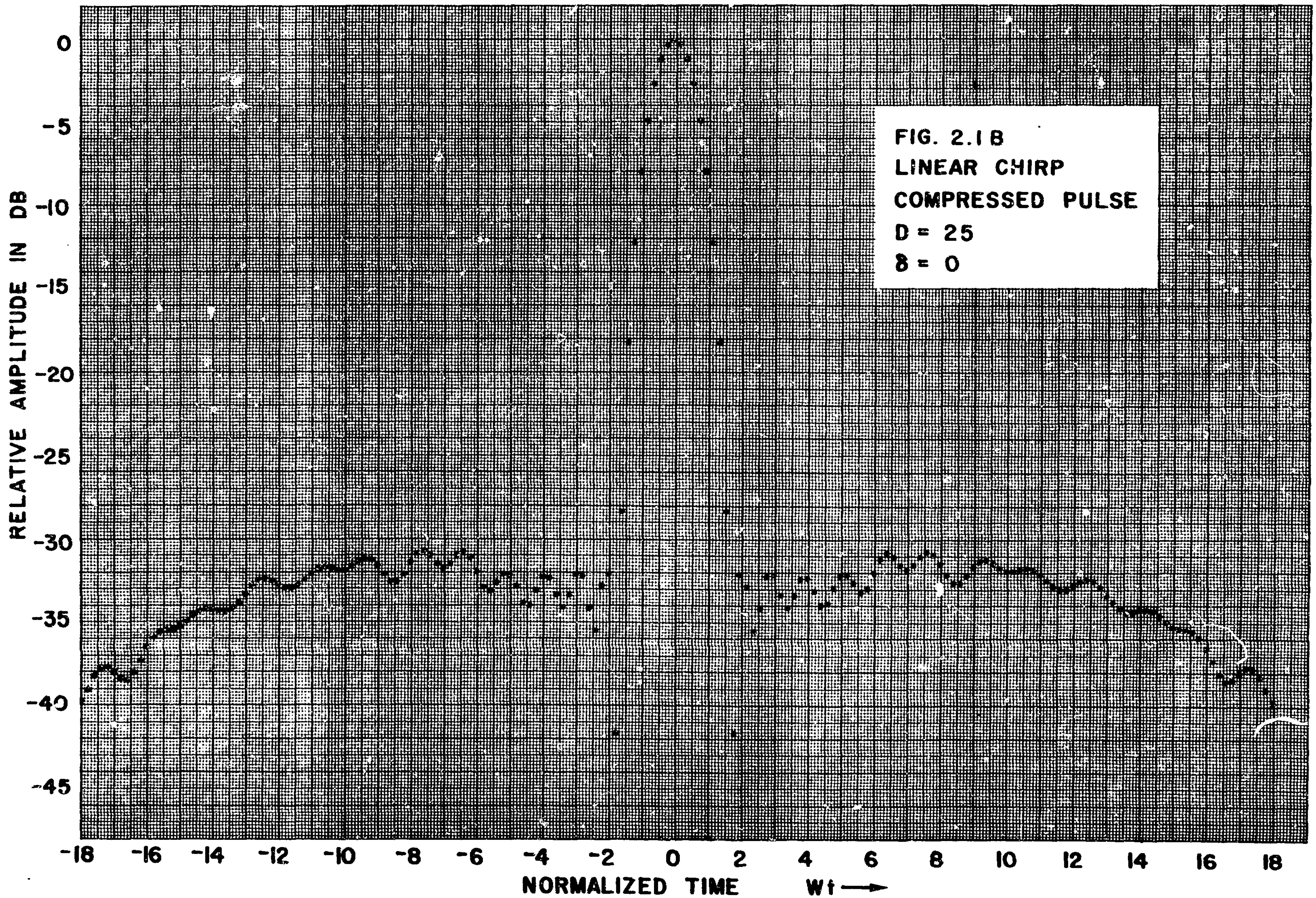


FIG. 2.1 B
LINEAR CHIRP
COMPRESSED PULSE
D = 25
δ = 0
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt ⟶

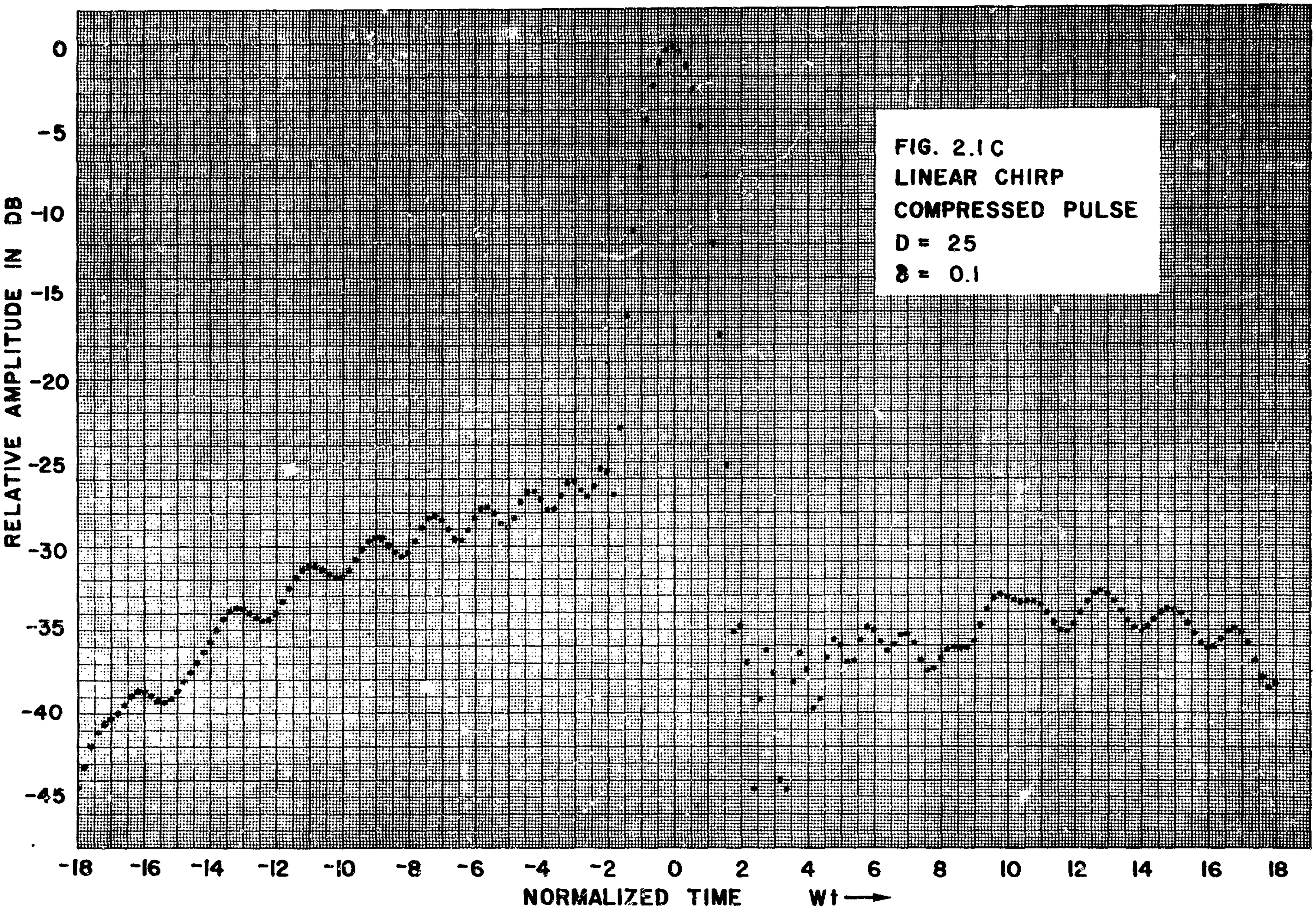


FIG. 2.1C
LINEAR CHIRP
COMPRESSED PULSE
D = 25
δ = 0.1
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

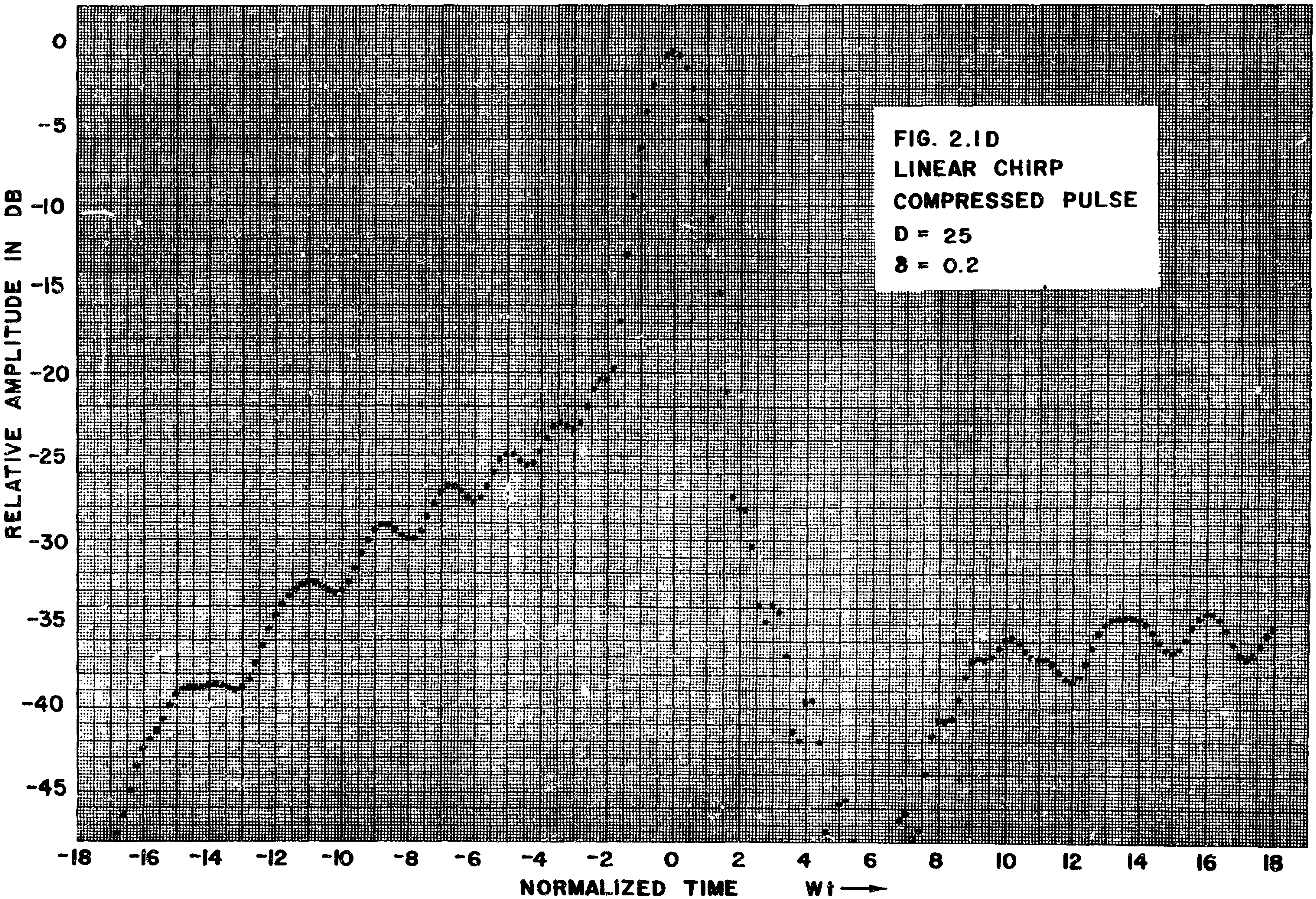


FIG. 2.1D
LINEAR CHIRP
COMPRESSED PULSE
D = 25
δ = 0.2

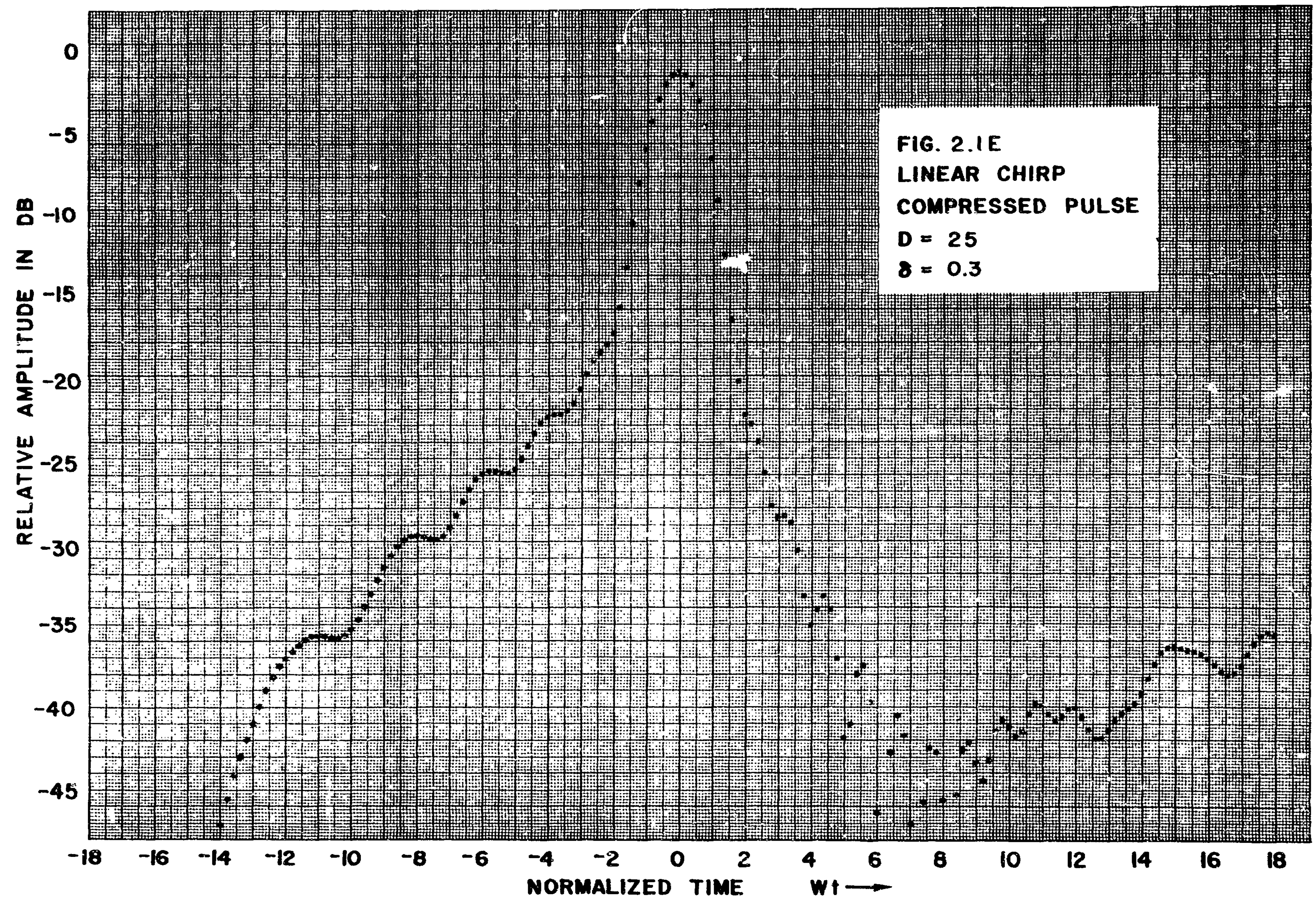


FIG. 2.1E
LINEAR CHIRP
COMPRESSED PULSE
D = 25
δ = 0.3
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt

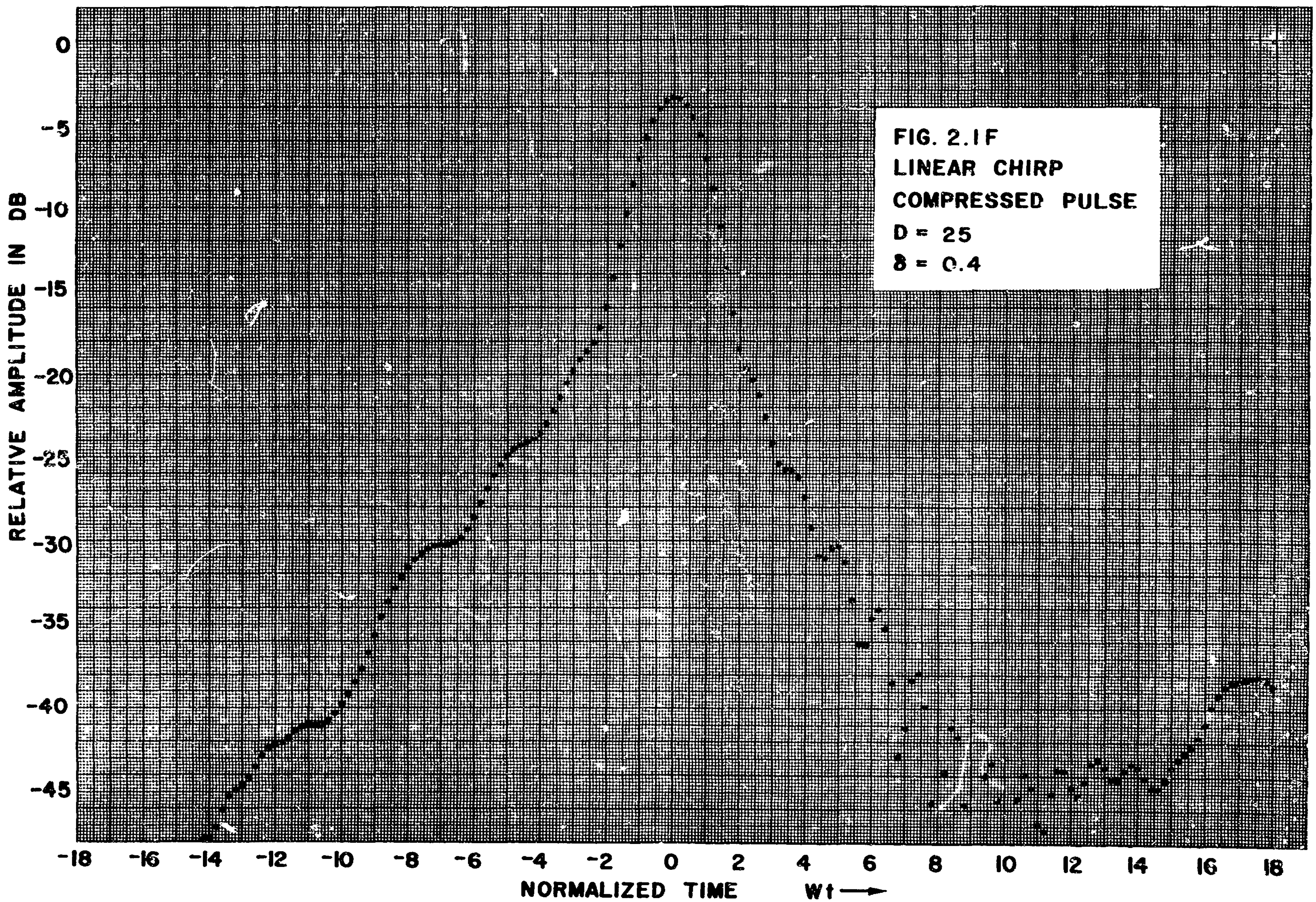


FIG. 2.1F
LINEAR CHIRP
COMPRESSED PULSE
D = 25
δ = 0.4
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

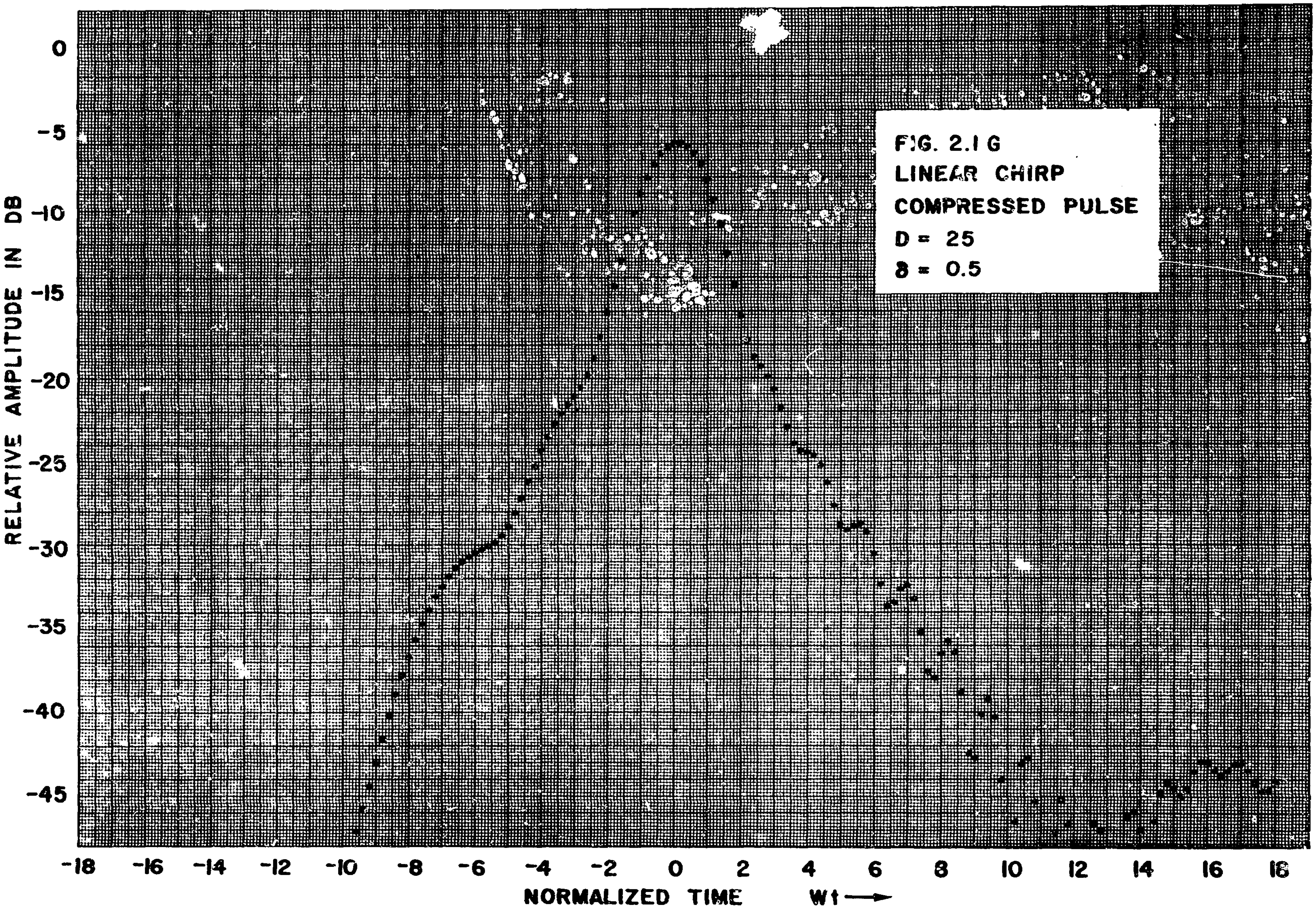
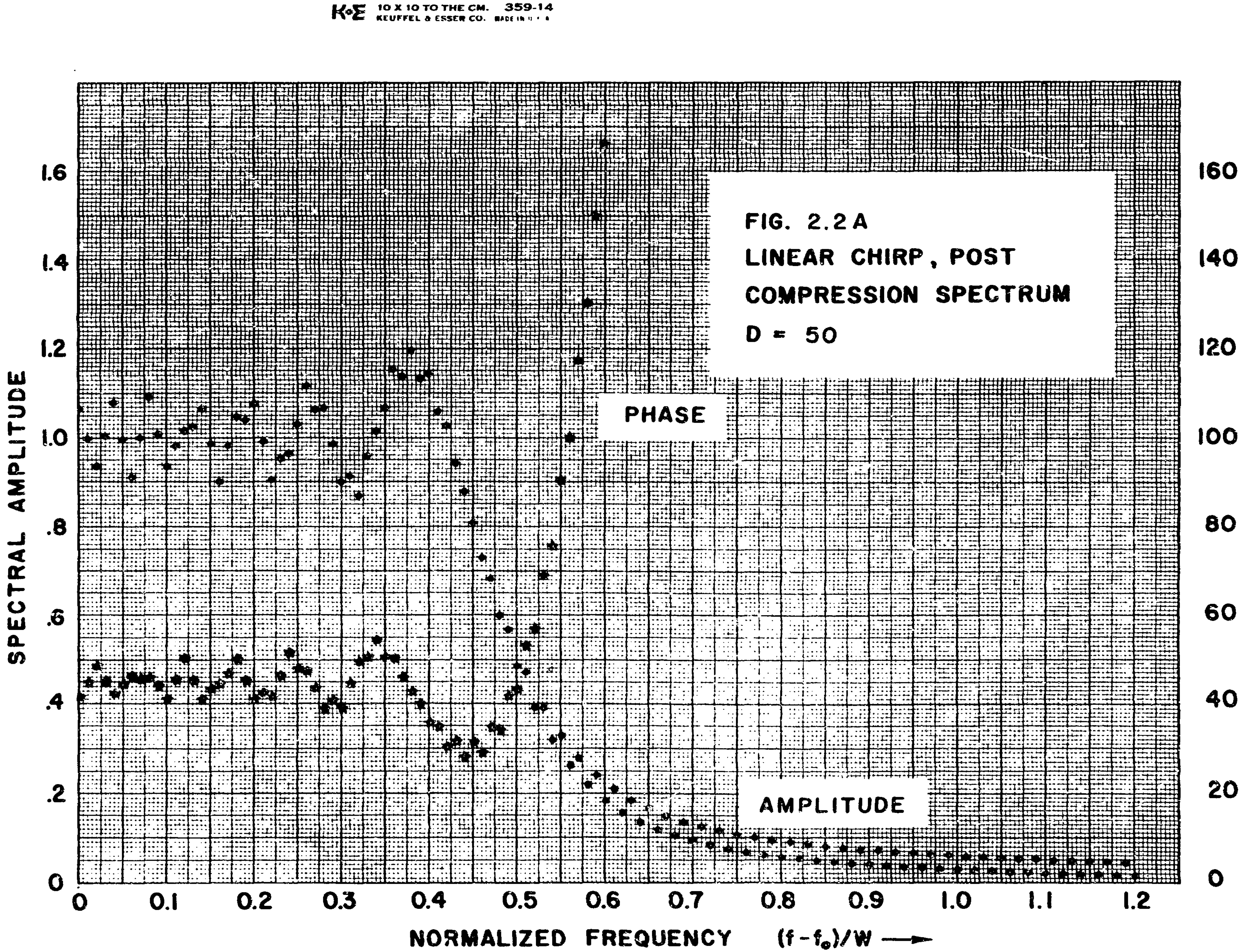


FIG. 2.1 G
LINEAR CHIRP
COMPRESSED PULSE
D = 25
δ = 0.5


K+E 10 X 10 TO THE CM. 359-14
KEUFFEL & ESSER CO. MADE IN U.S.A.
FIG. 2.2A
LINEAR CHIRP, POST
COMPRESSION SPECTRUM
D = 50
PHASE
AMPLITUDE
SPECTRAL AMPLITUDE
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
PHASE IN DEGREES
0
20
40
60
80
100
120
140
160
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
NORMALIZED FREQUENCY $(f-f_o)/W$ →

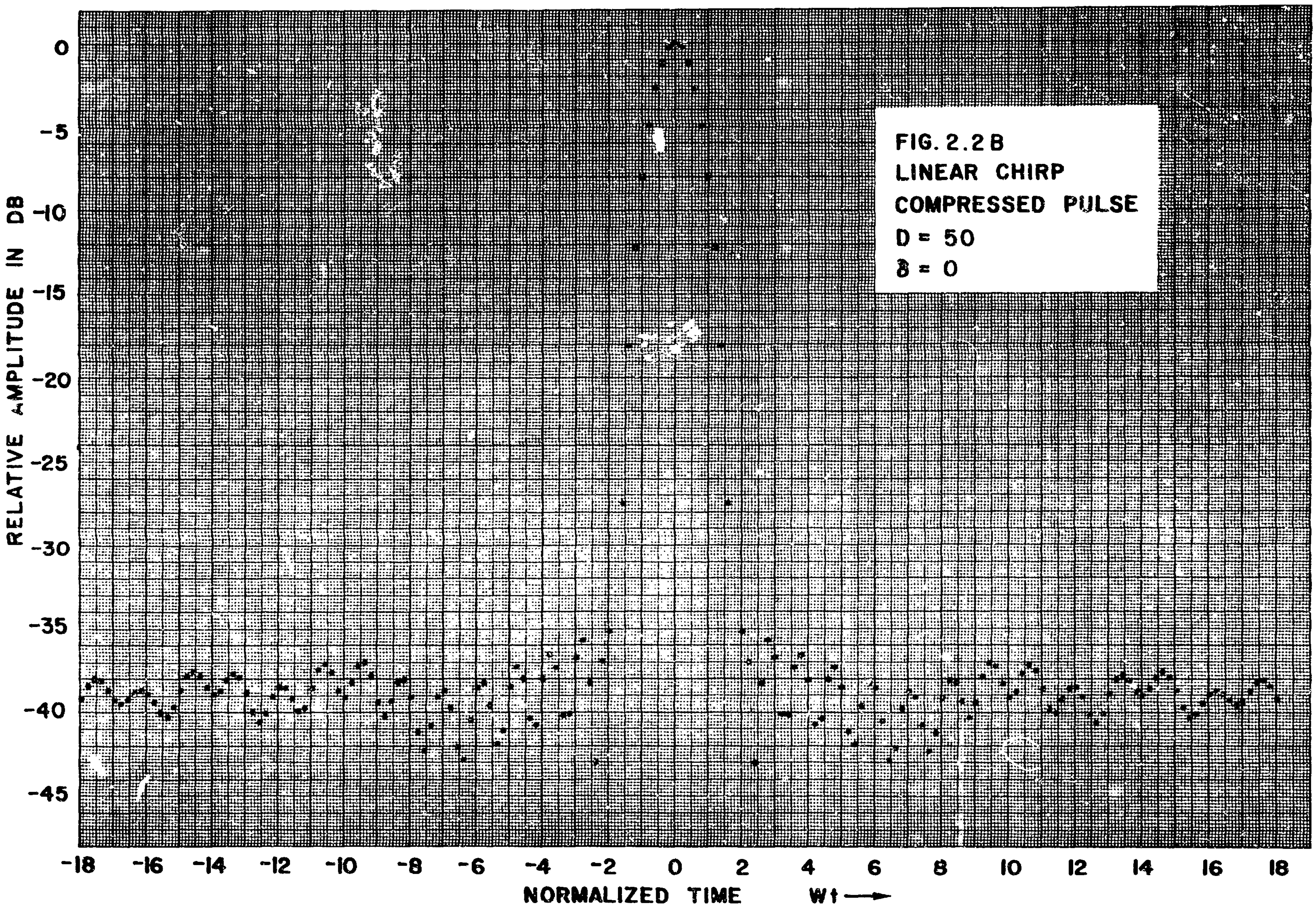


FIG. 2.2B
LINEAR CHIRP
COMPRESSED PULSE
D = 50
δ = 0

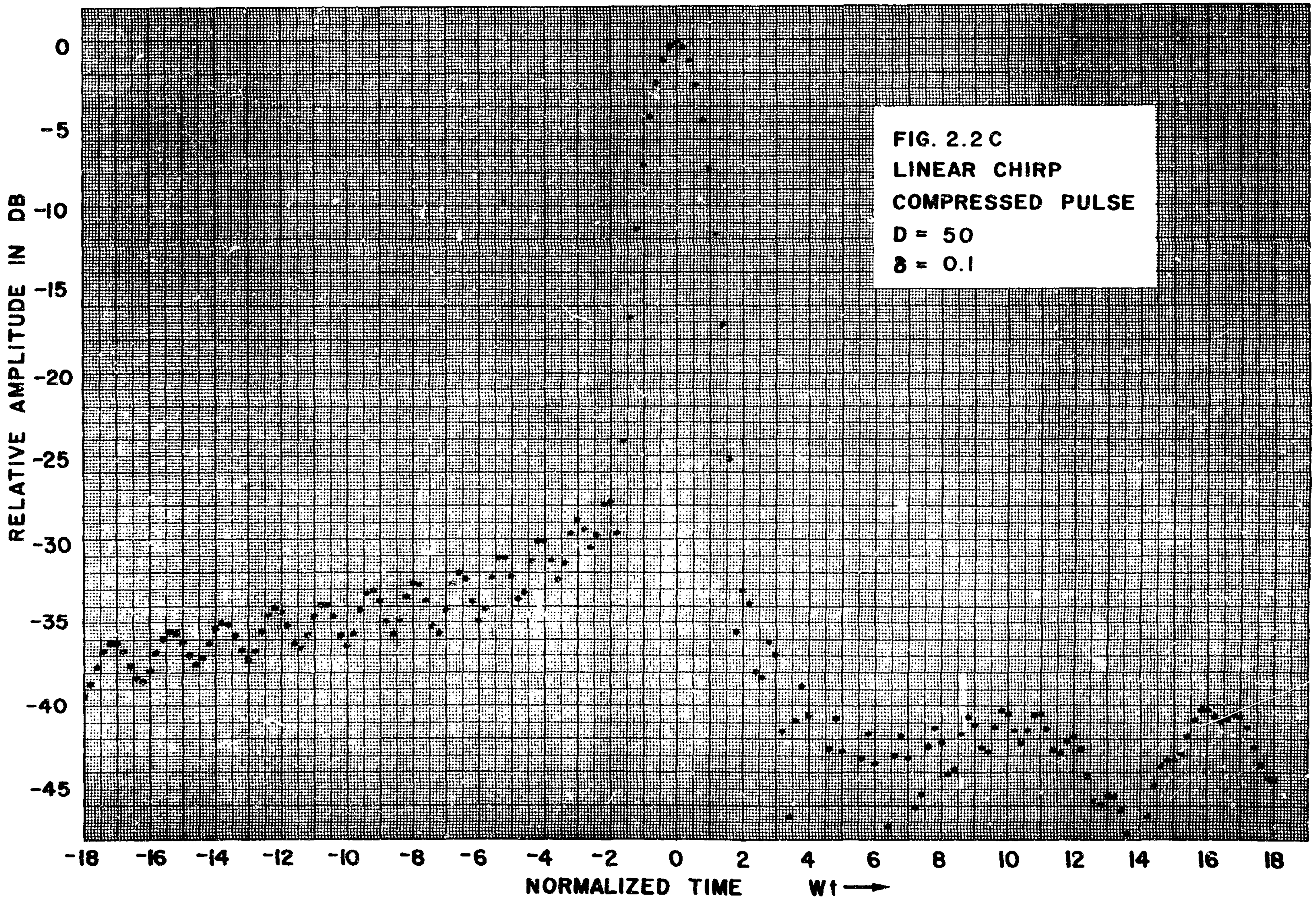


FIG. 2.2 C
LINEAR CHIRP
COMPRESSED PULSE
D = 50
δ = 0.1
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

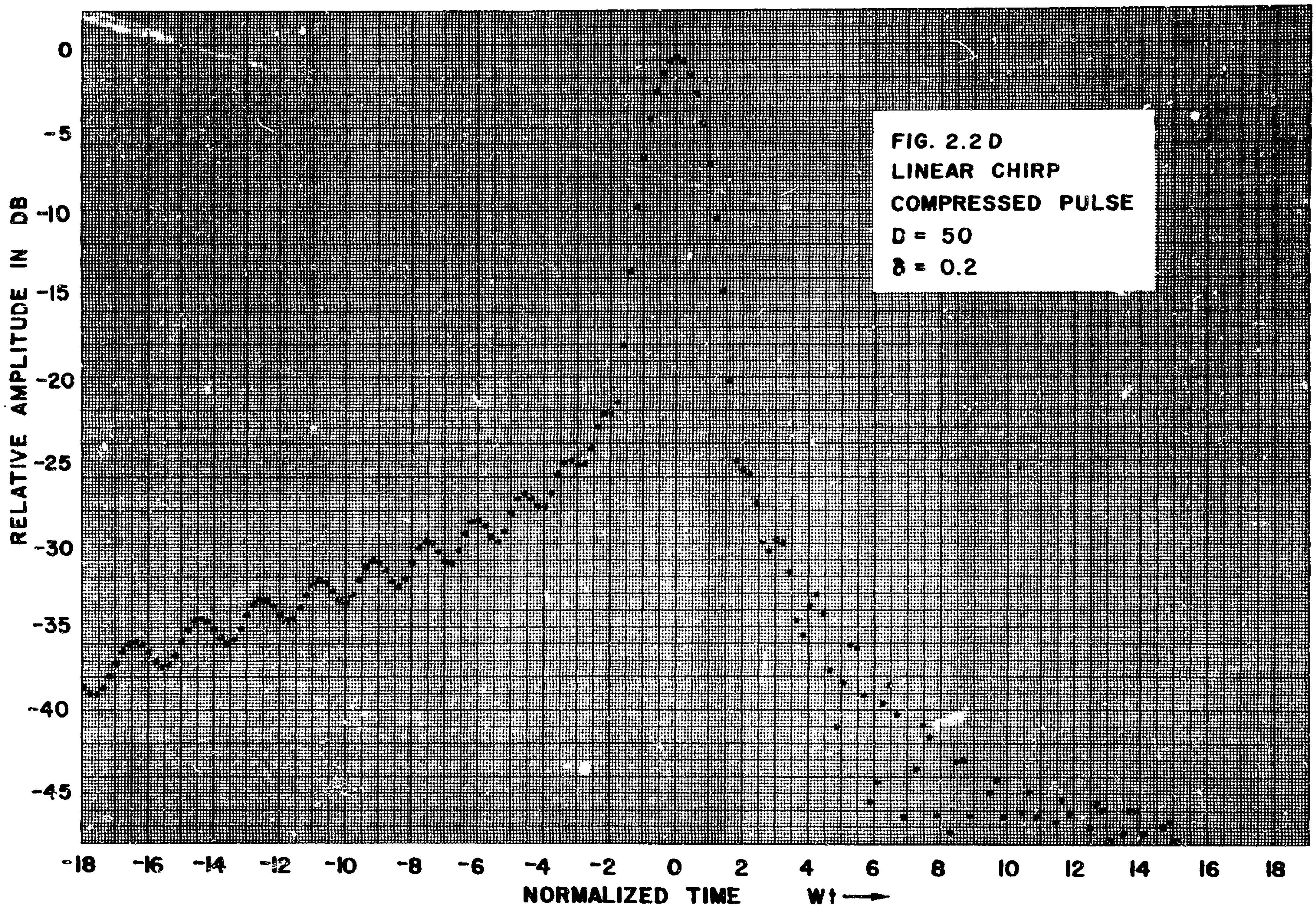


FIG. 2.2D
LINEAR CHIRP
COMPRESSED PULSE
D = 50
δ = 0.2

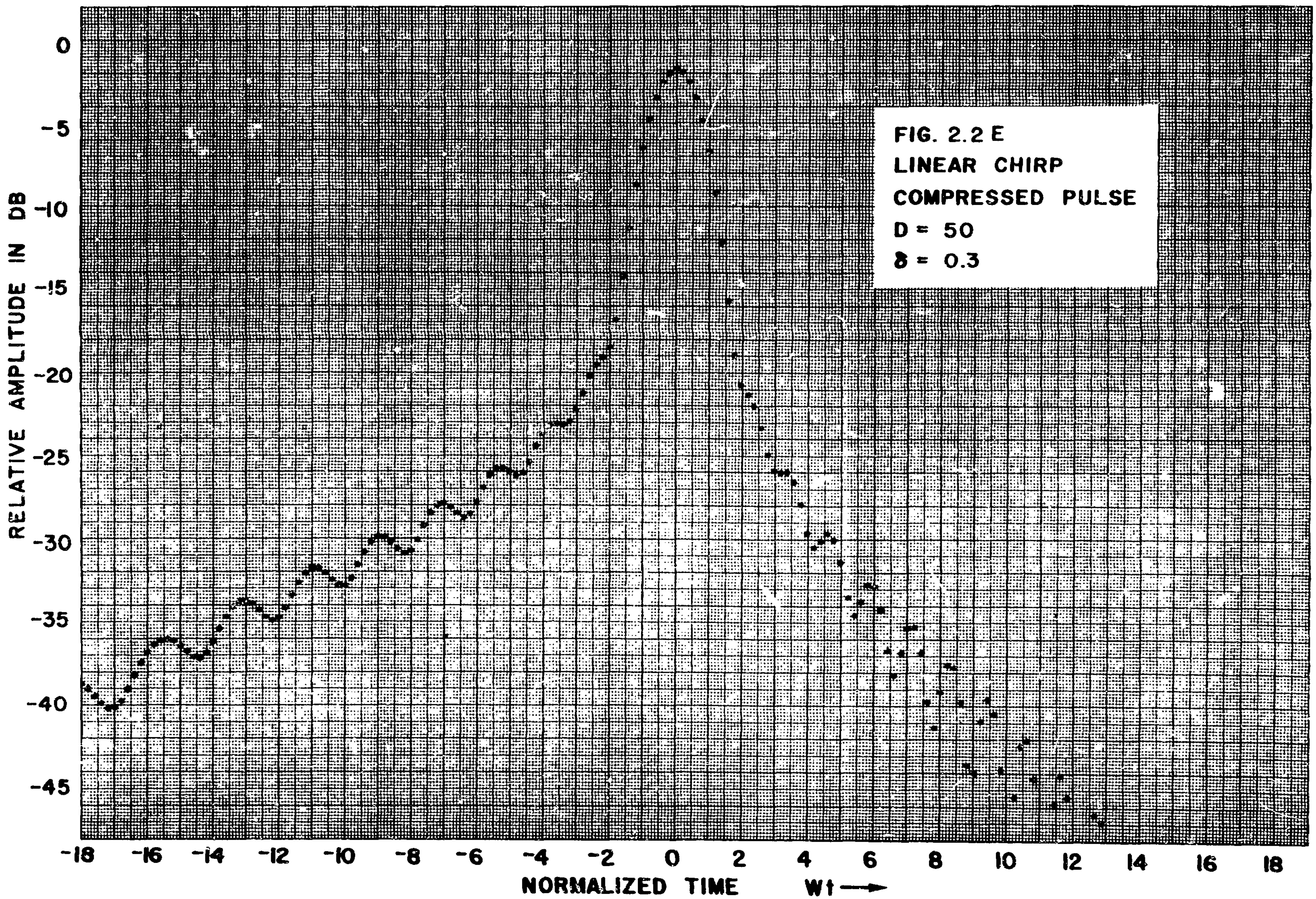


FIG. 2.2 E
LINEAR CHIRP
COMPRESSED PULSE
D = 50
δ = 0.3
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME Wt

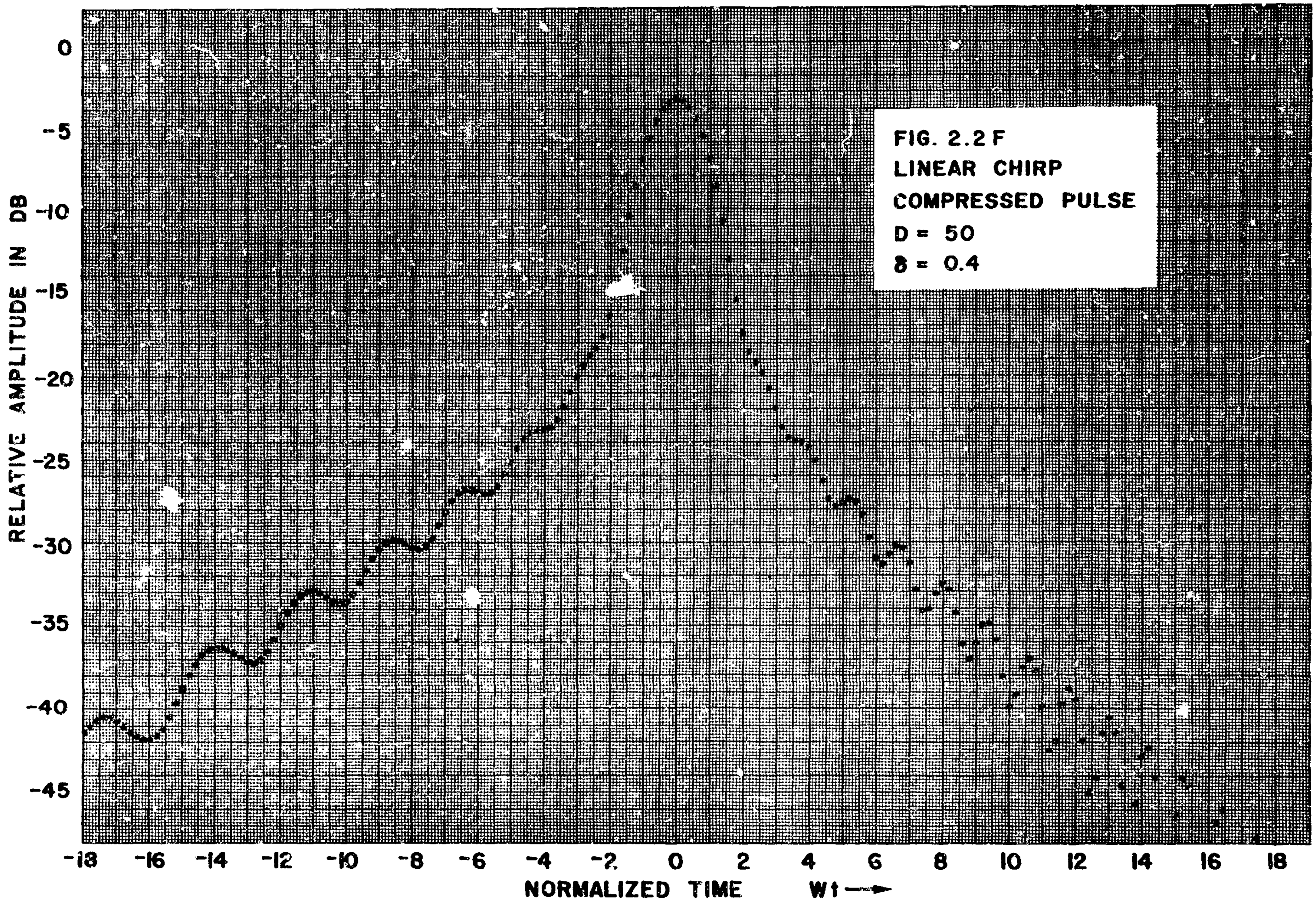


FIG. 2.2 F
LINEAR CHIRP
COMPRESSED PULSE
D = 50
δ = 0.4
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt

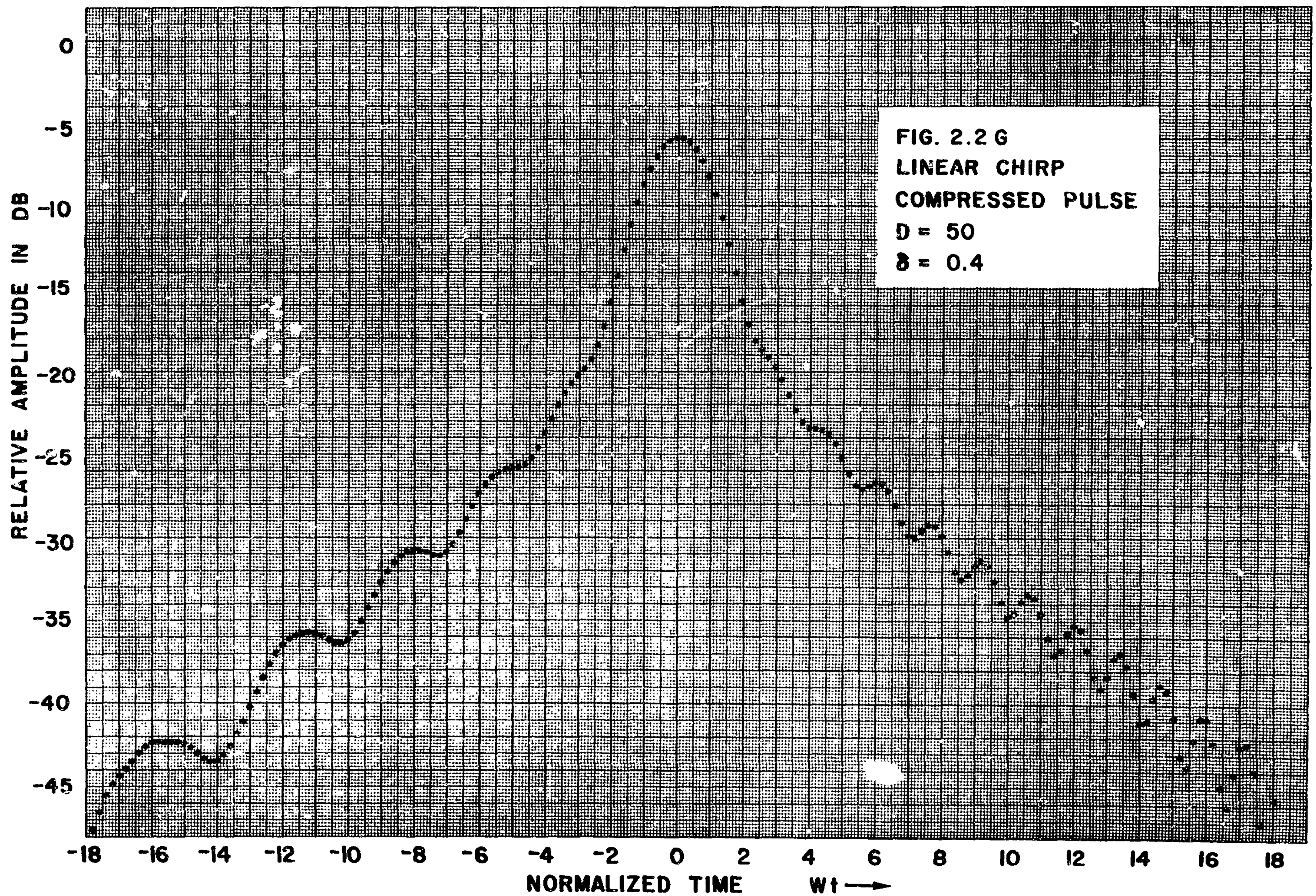


FIG. 2.2 G
LINEAR CHIRP
COMPRESSED PULSE
D = 50
δ = 0.4
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

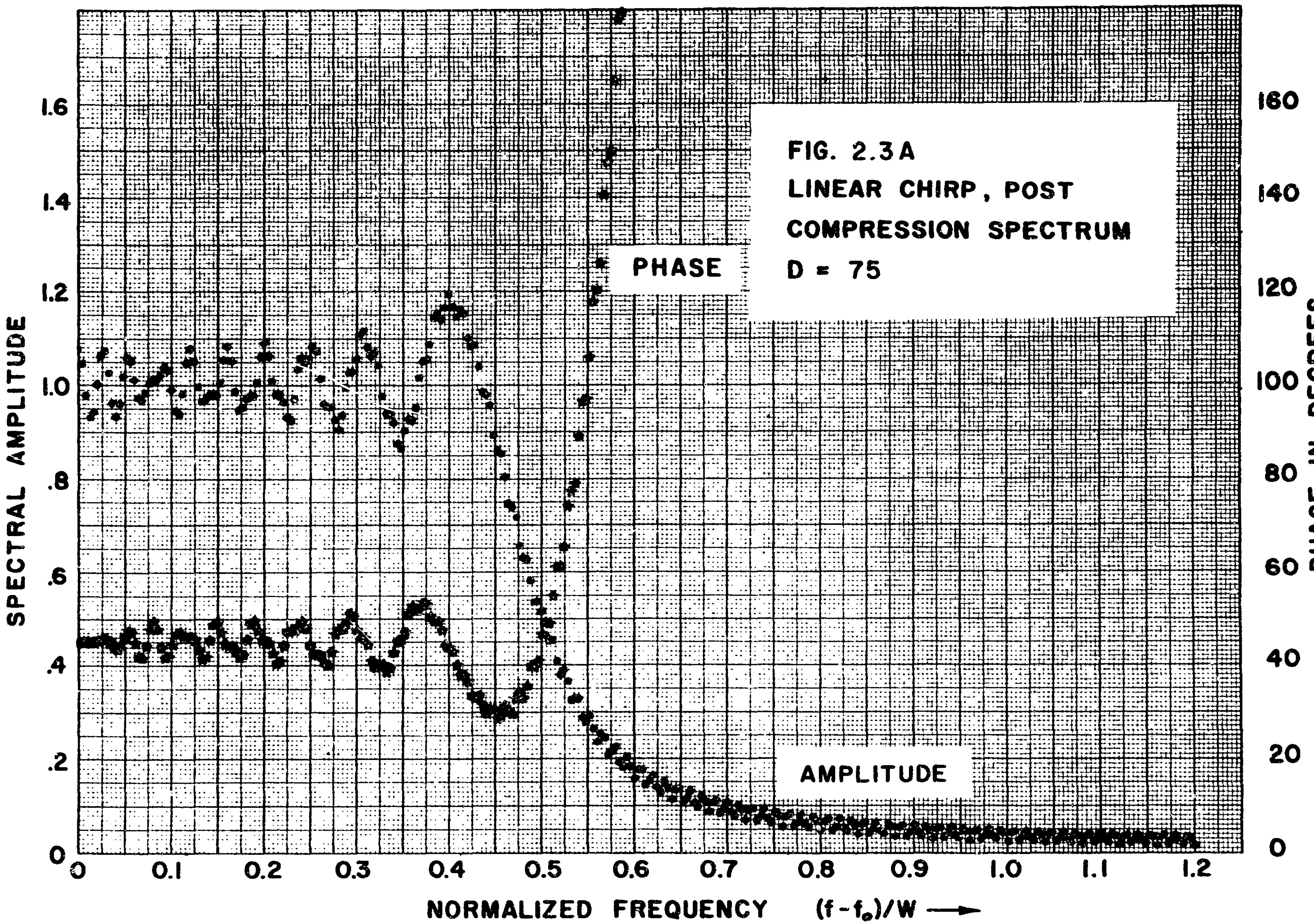


FIG. 2.3A
LINEAR CHIRP, POST COMPRESSION SPECTRUM
D = 75

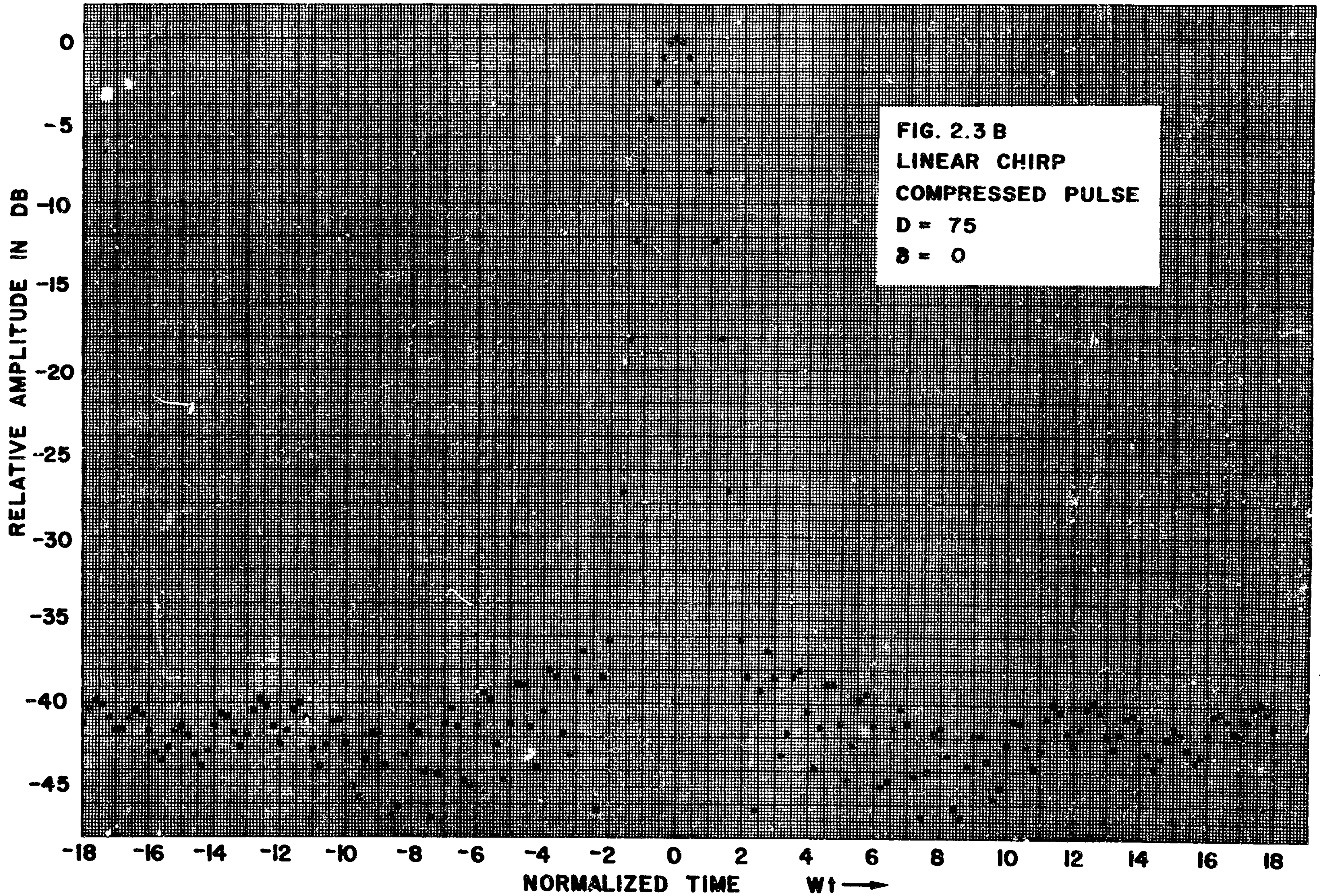


FIG. 2.3 B
LINEAR CHIRP
COMPRESSED PULSE
D = 75
δ = 0
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME Wt

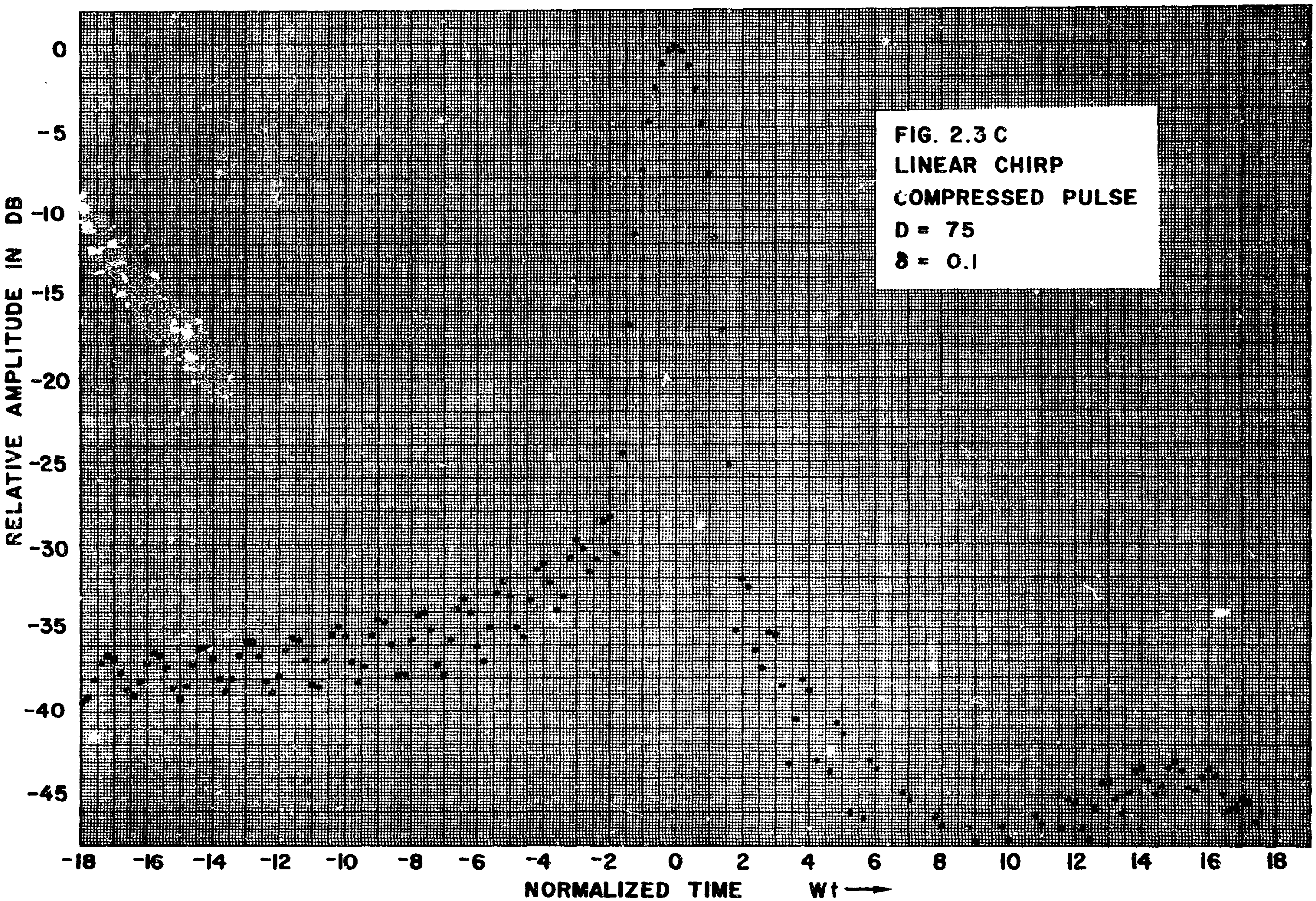


FIG. 2.3 C
LINEAR CHIRP
COMPRESSED PULSE
D = 75
δ = 0.1
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt

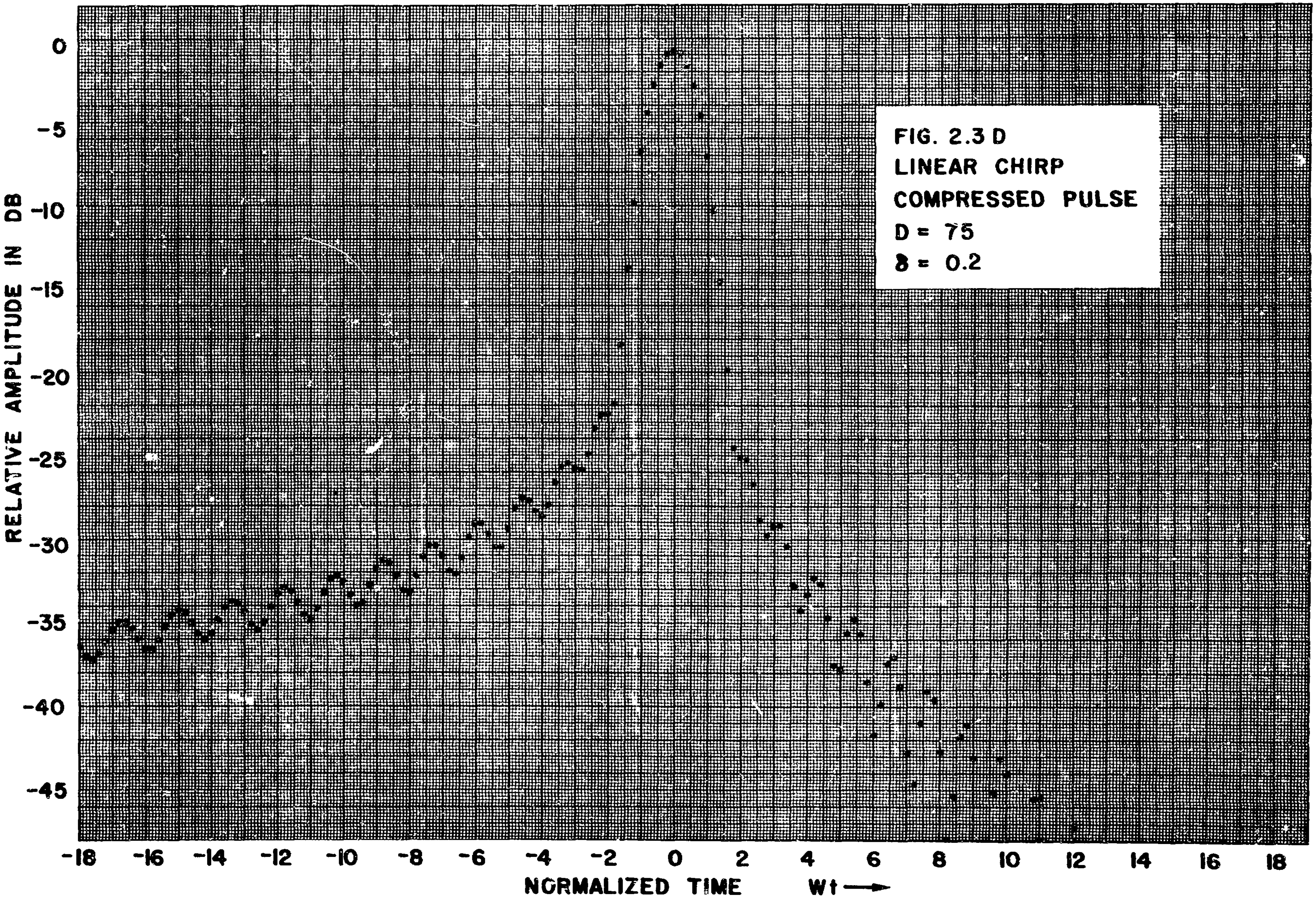


FIG. 2.3 D
LINEAR CHIRP
COMPRESSED PULSE
D = 75
δ = 0.2
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

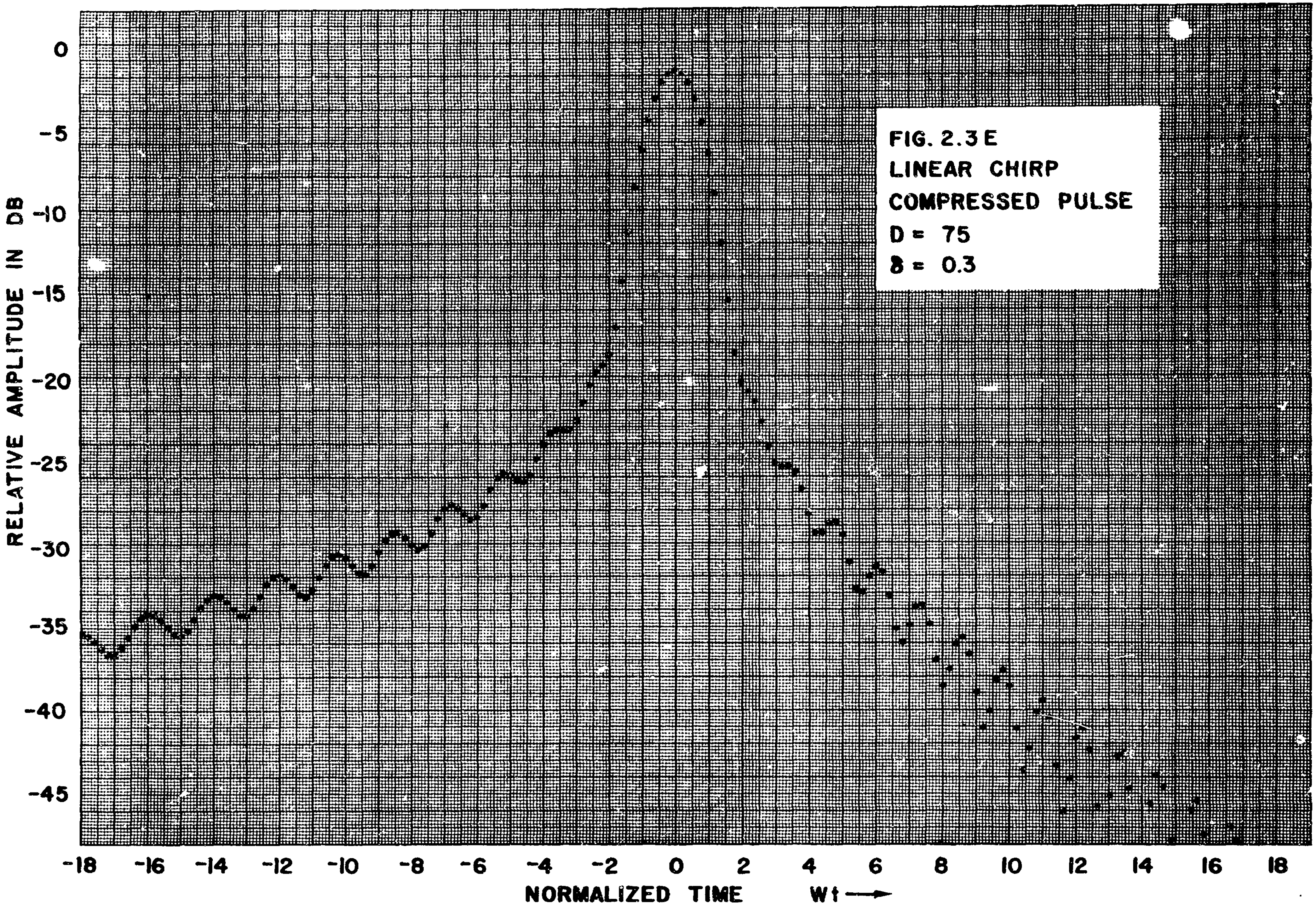


FIG. 2.3 E
LINEAR CHIRP
COMPRESSED PULSE
D = 75
δ = 0.3

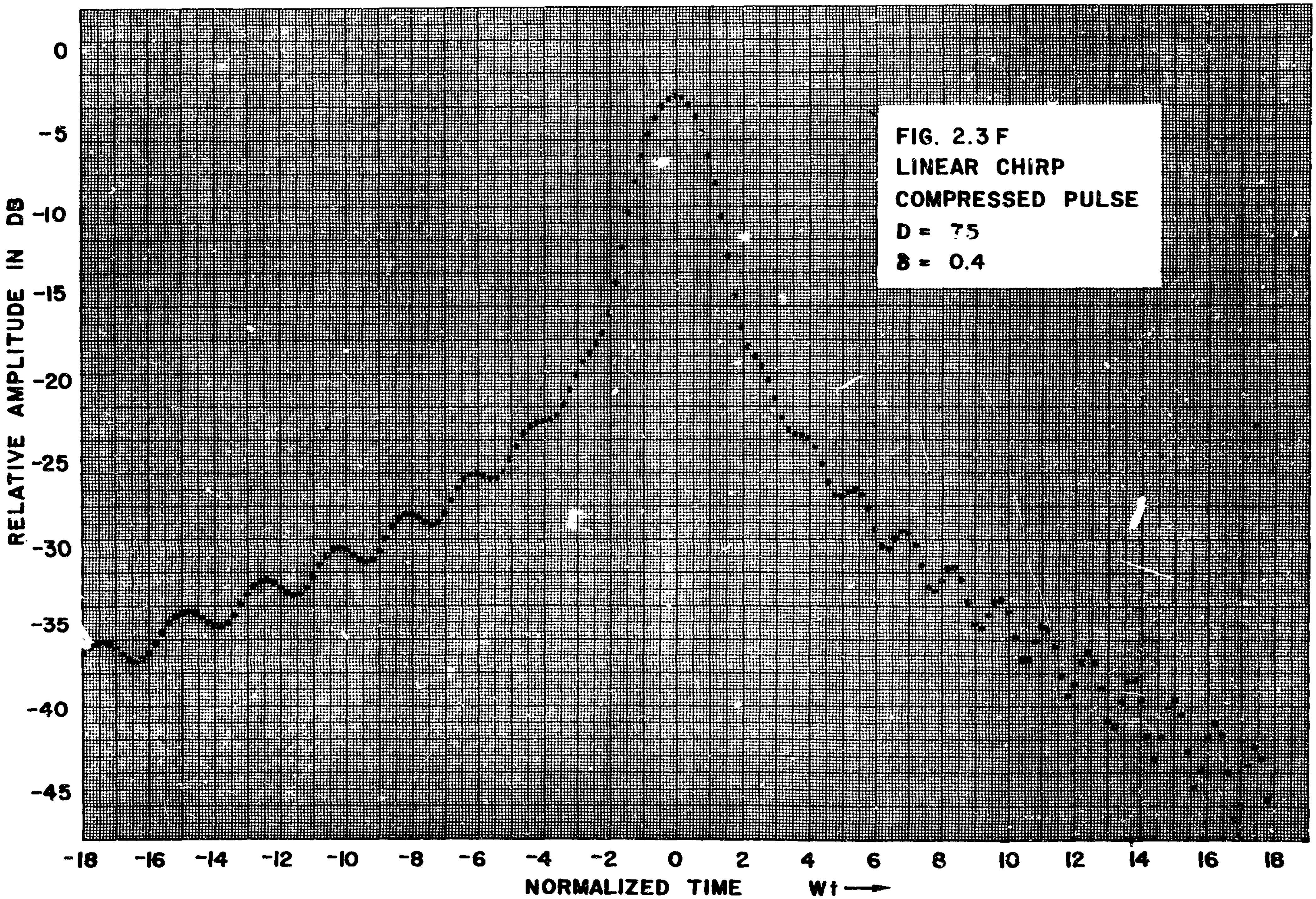


FIG. 2.3 F
LINEAR CHIRP
COMPRESSED PULSE
D = 75
δ = 0.4

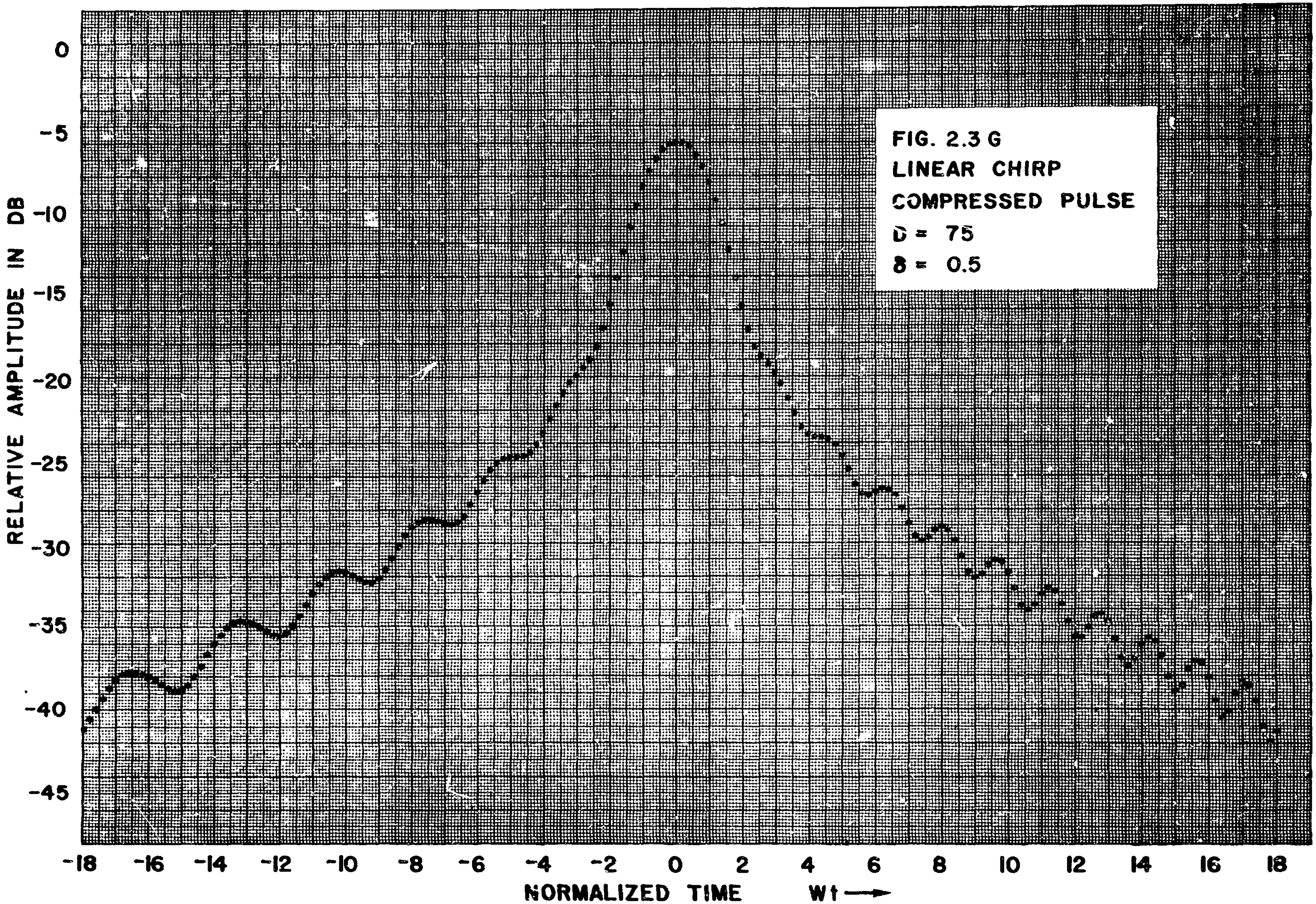


FIG. 2.3 G
LINEAR CHIRP
COMPRESSED PULSE
D = 75
δ = 0.5

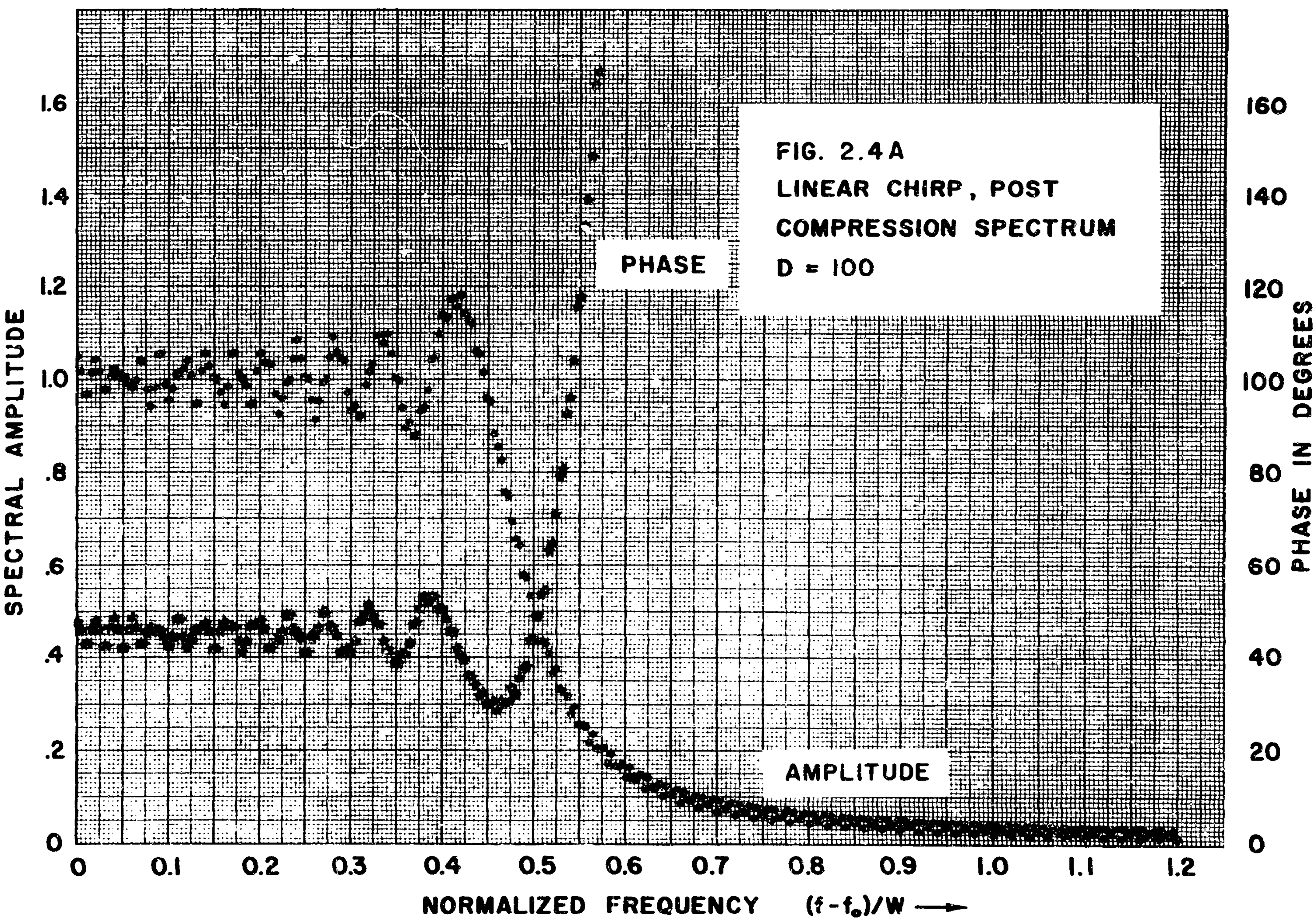


FIG. 2.4 A
LINEAR CHIRP, POST
COMPRESSION SPECTRUM
D = 100
PHASE
AMPLITUDE
SPECTRAL AMPLITUDE
PHASE IN DEGREES
NORMALIZED FREQUENCY (f - f₀)/W
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
20
40
60
80
100
120
140
160
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2

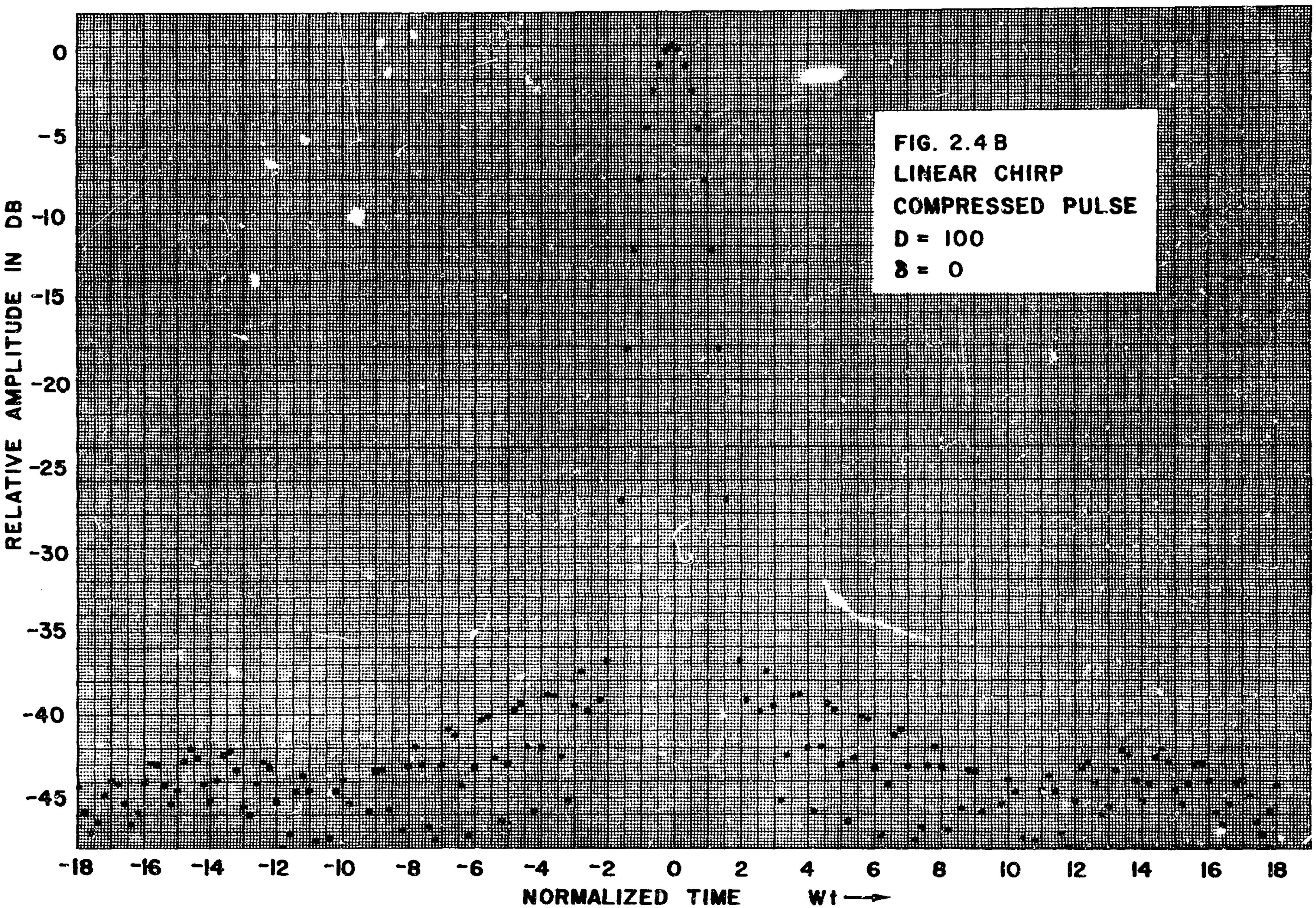


FIG. 2.4 B
LINEAR CHIRP
COMPRESSED PULSE
D = 100
δ = 0
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt →

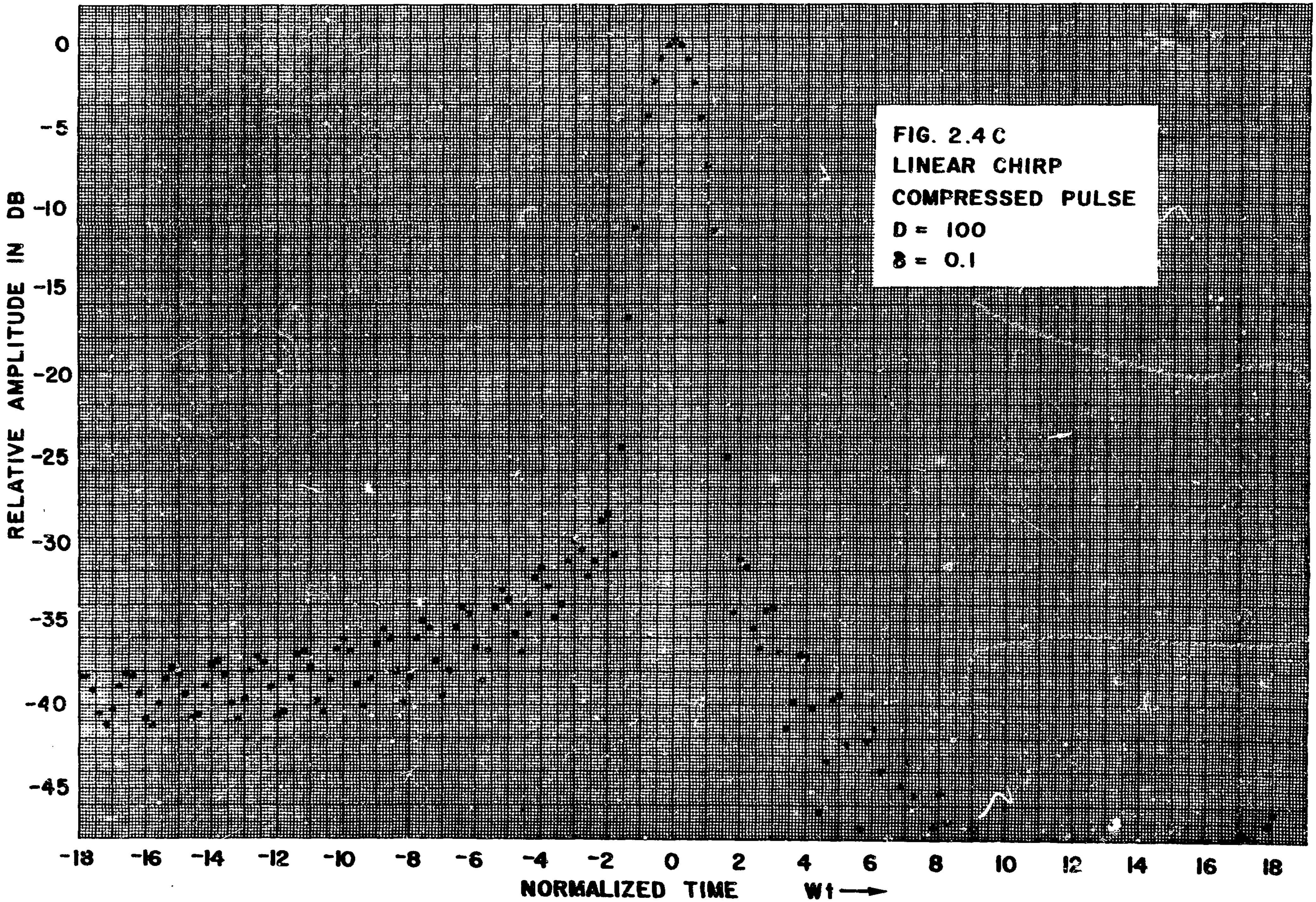


FIG. 2.4 C
LINEAR CHIRP
COMPRESSED PULSE
D = 100
δ = 0.1

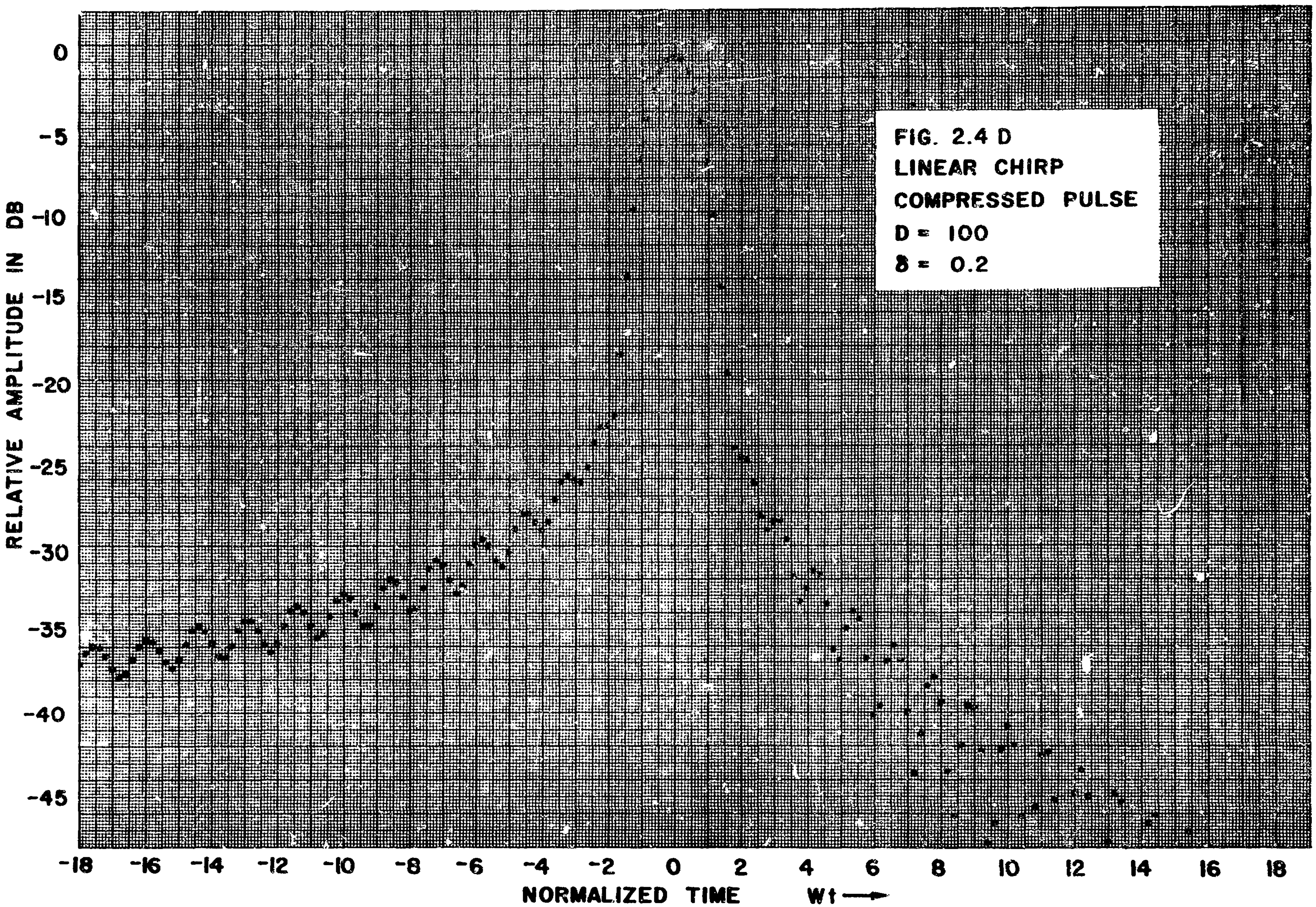


FIG. 2.4 D
LINEAR CHIRP
COMPRESSED PULSE
D = 100
δ = 0.2
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME Wt

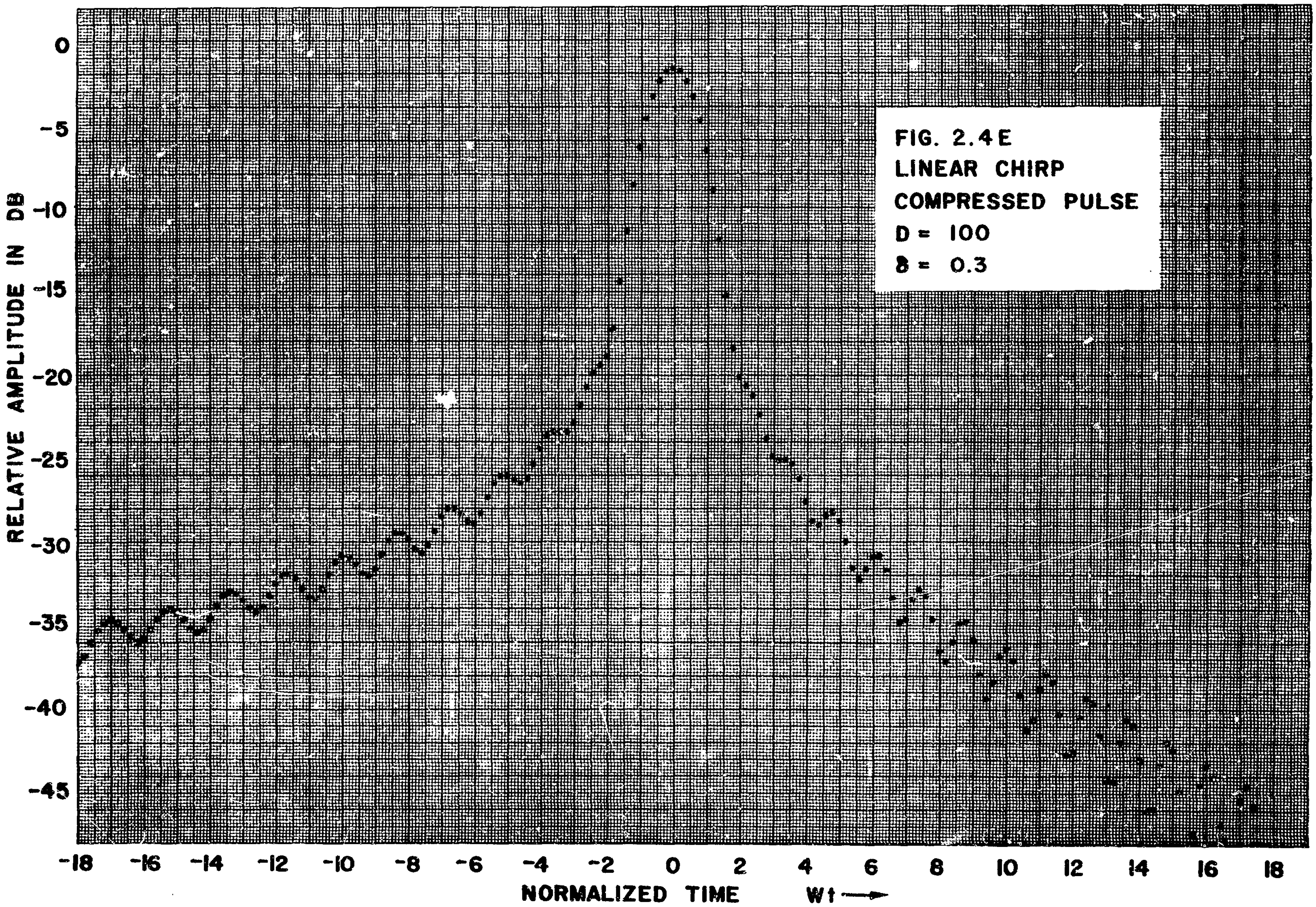


FIG. 2.4E
LINEAR CHIRP
COMPRESSED PULSE
D = 100
δ = 0.3

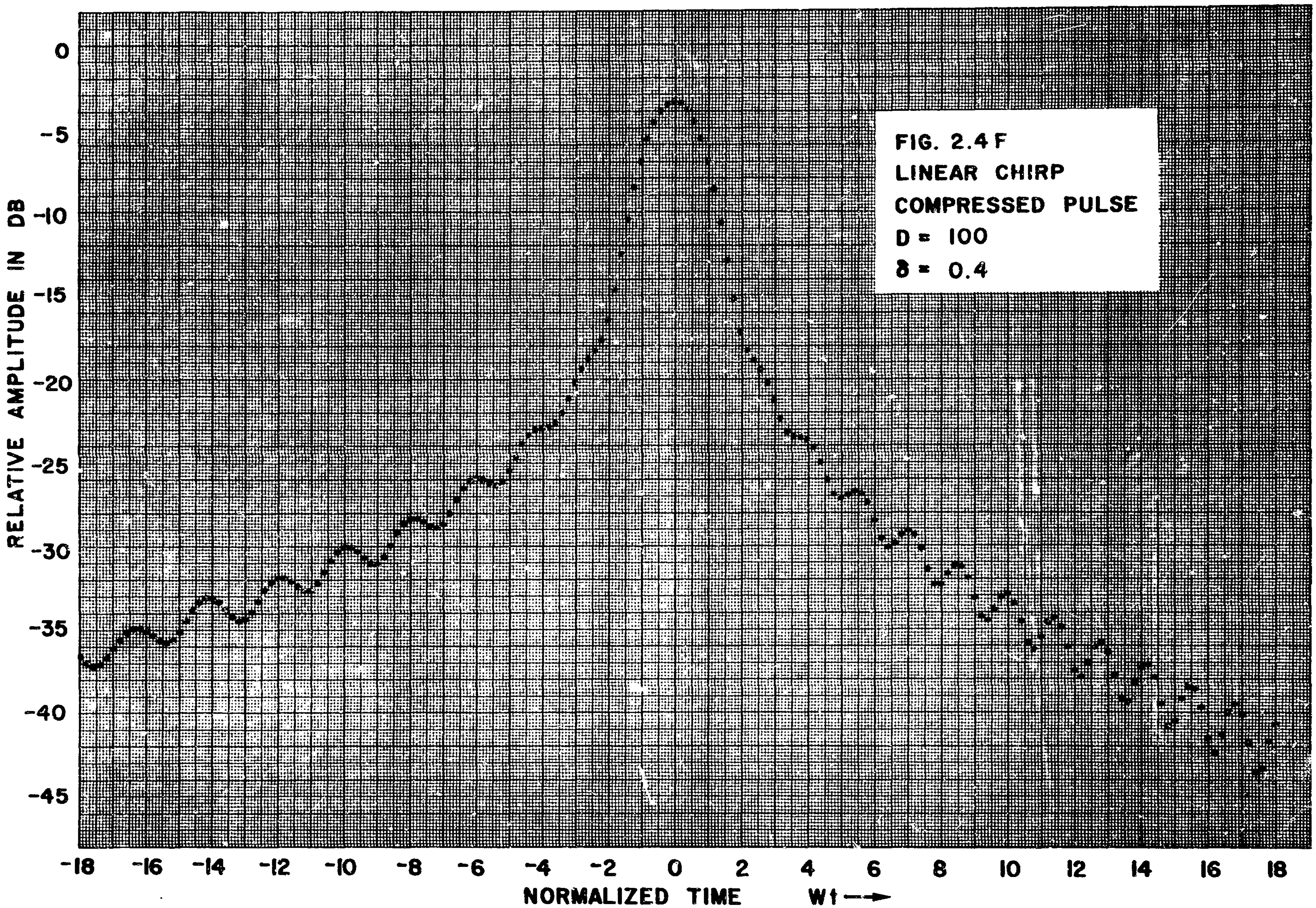


FIG. 2.4 F
LINEAR CHIRP
COMPRESSED PULSE
D = 100
δ = 0.4
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME Wt →

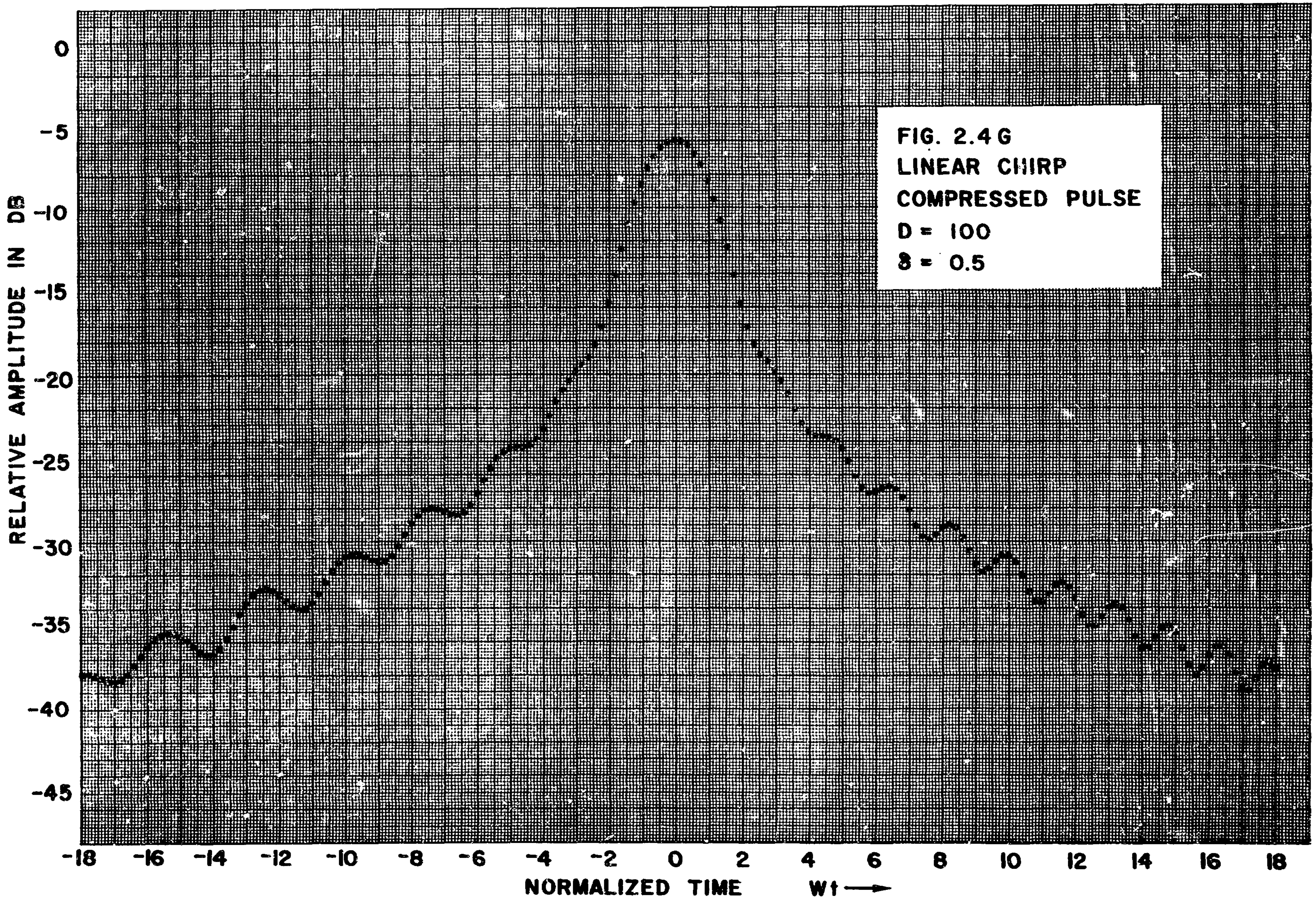


FIG. 2.4 G
LINEAR CHIRP
COMPRESSED PULSE
D = 100
δ = 0.5
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt

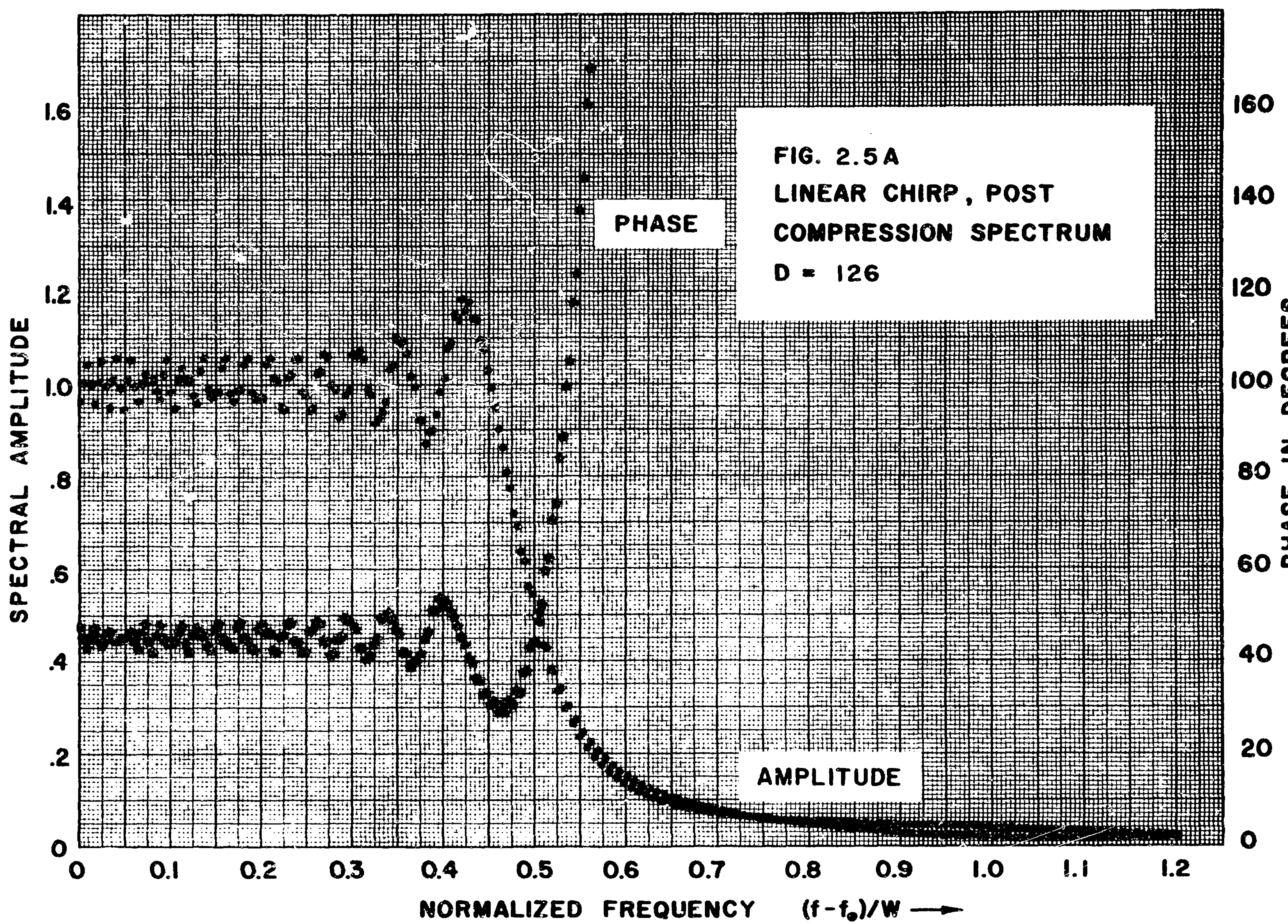


FIG. 2.5A
LINEAR CHIRP, POST COMPRESSION SPECTRUM
D = 126

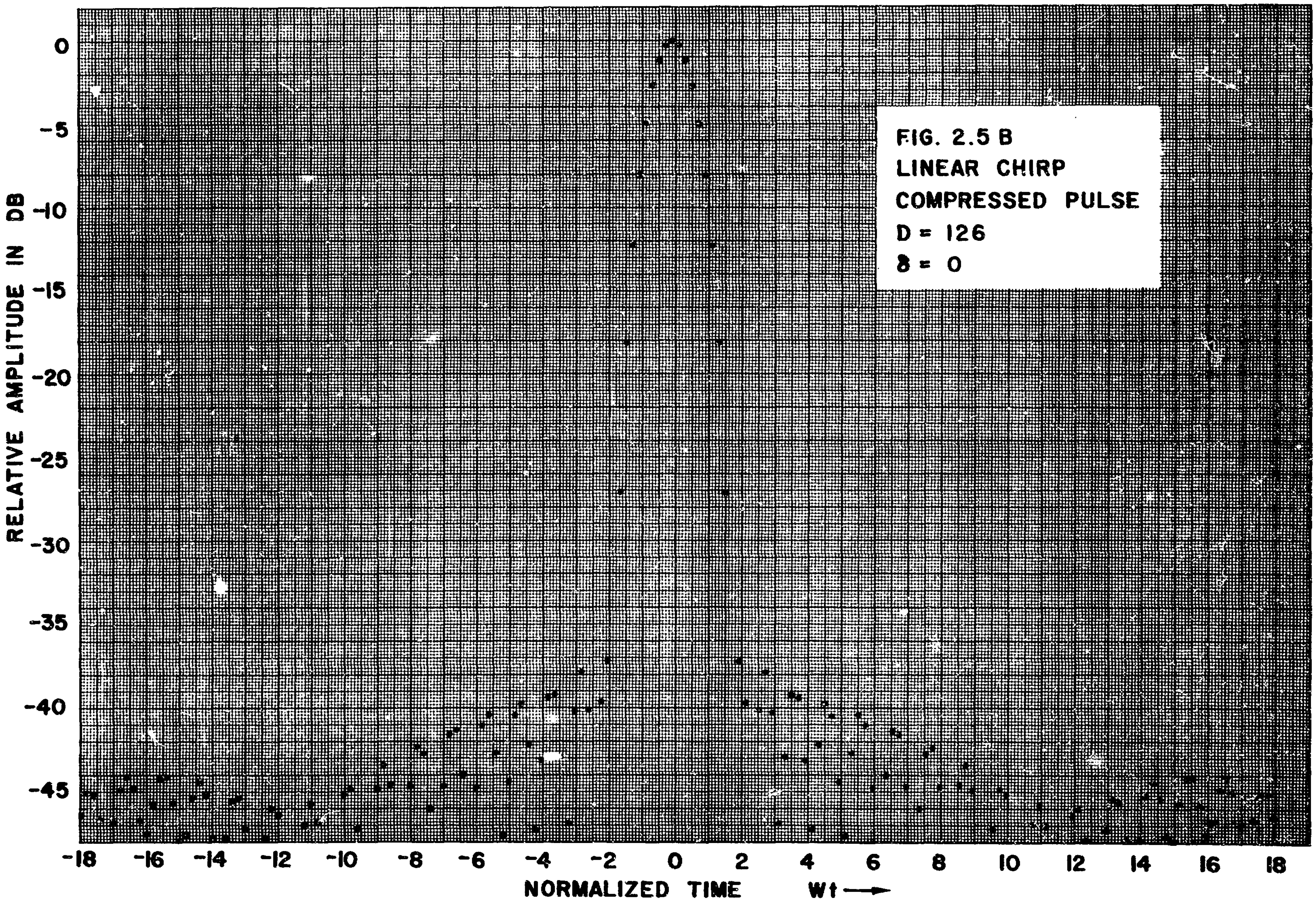


FIG. 2.5 B
LINEAR CHIRP
COMPRESSED PULSE
D = 126
δ = 0
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

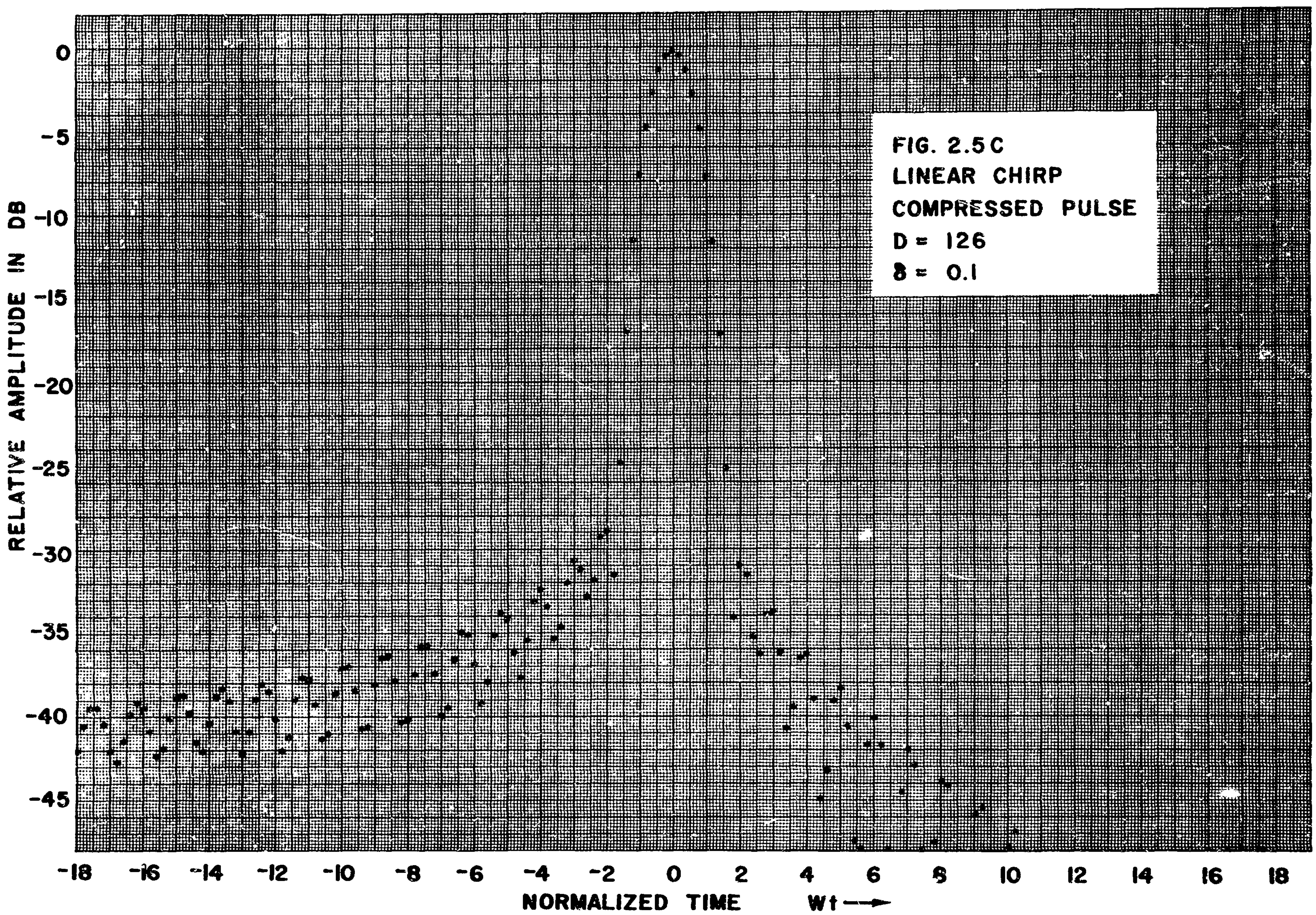


FIG. 2.5C
LINEAR CHIRP
COMPRESSED PULSE
D = 126
δ = 0.1

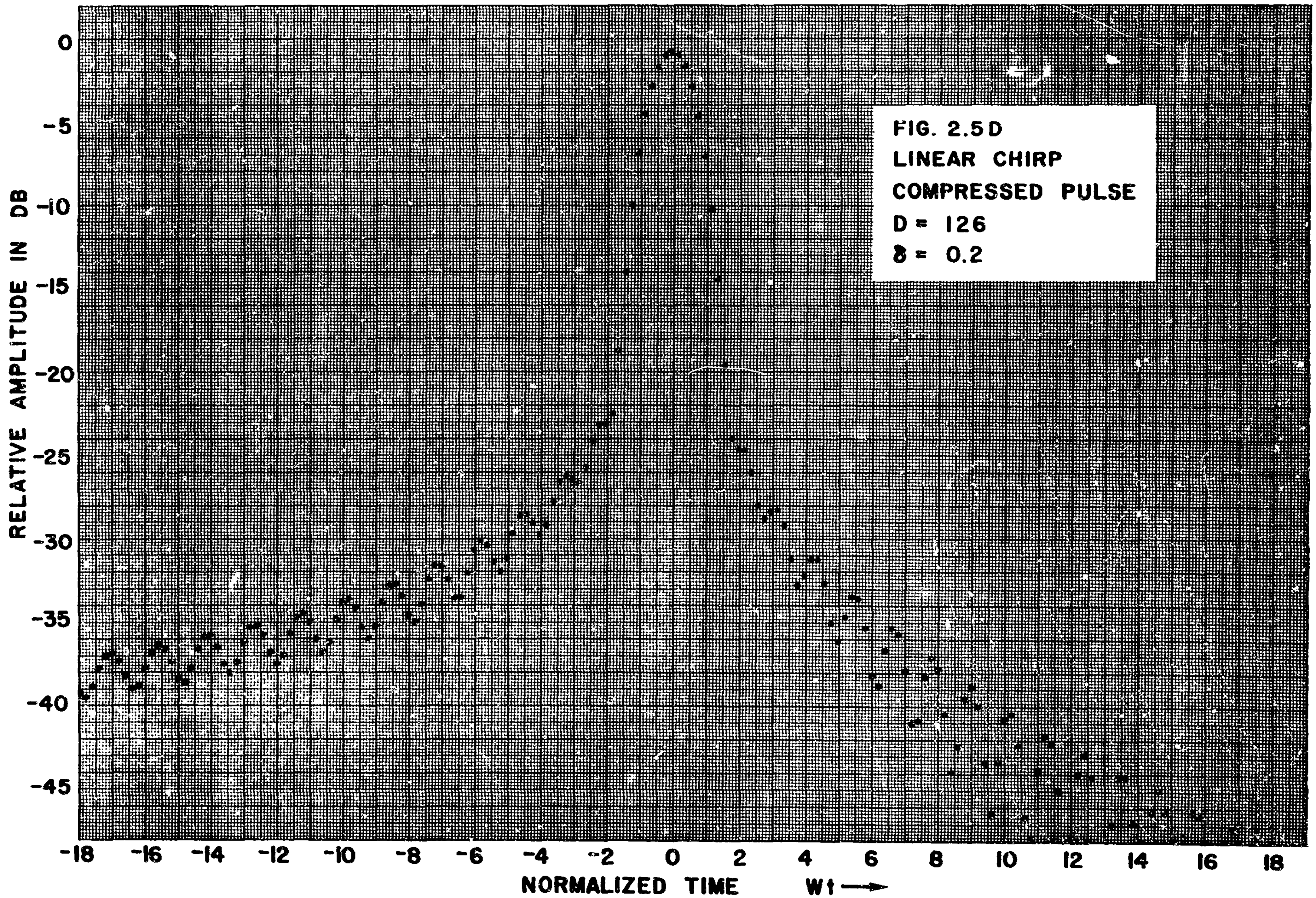


FIG. 2.5D
LINEAR CHIRP
COMPRESSED PULSE
D = 126
δ = 0.2
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

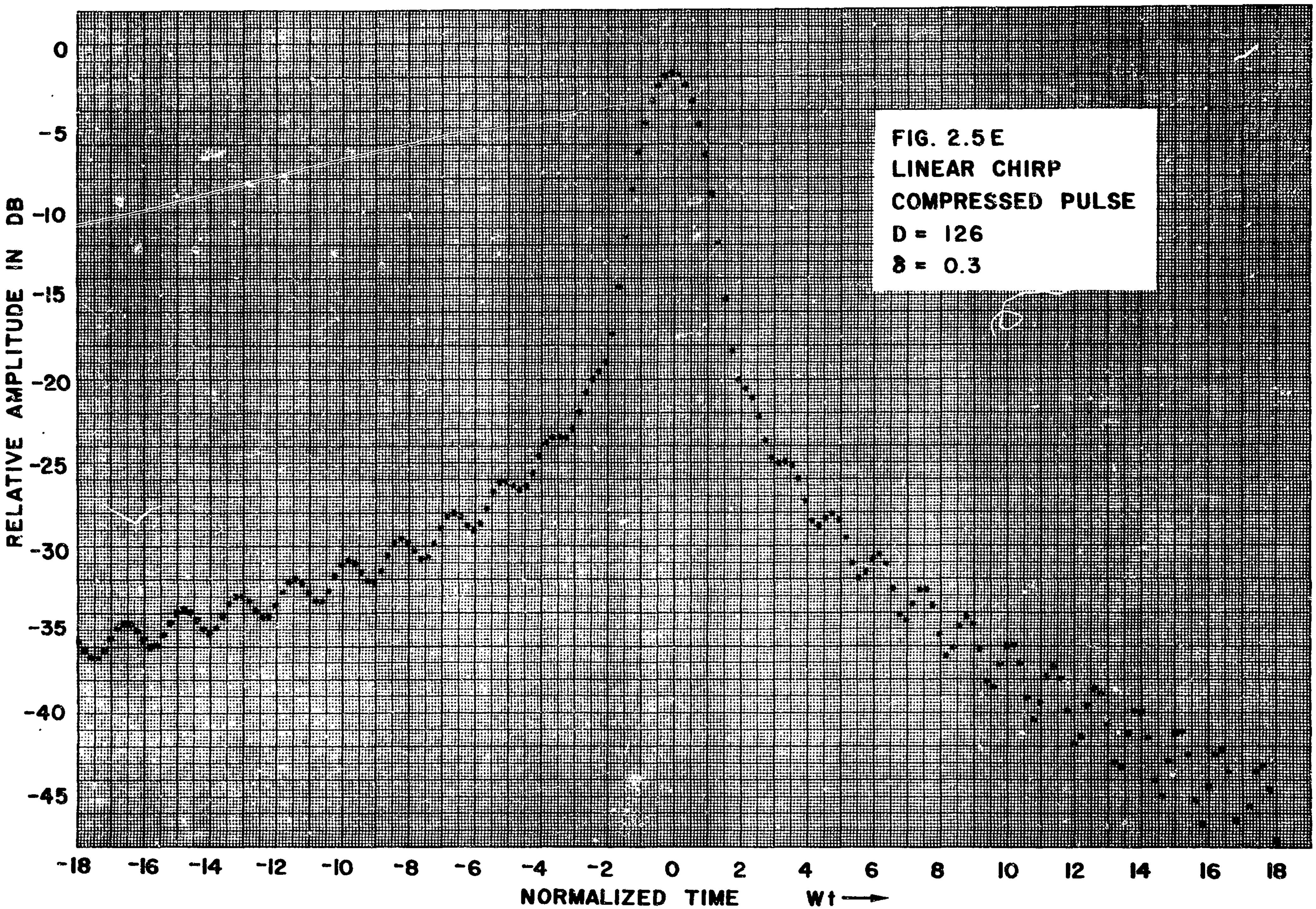


FIG. 2.5E
LINEAR CHIRP
COMPRESSED PULSE
D = 126
δ = 0.3

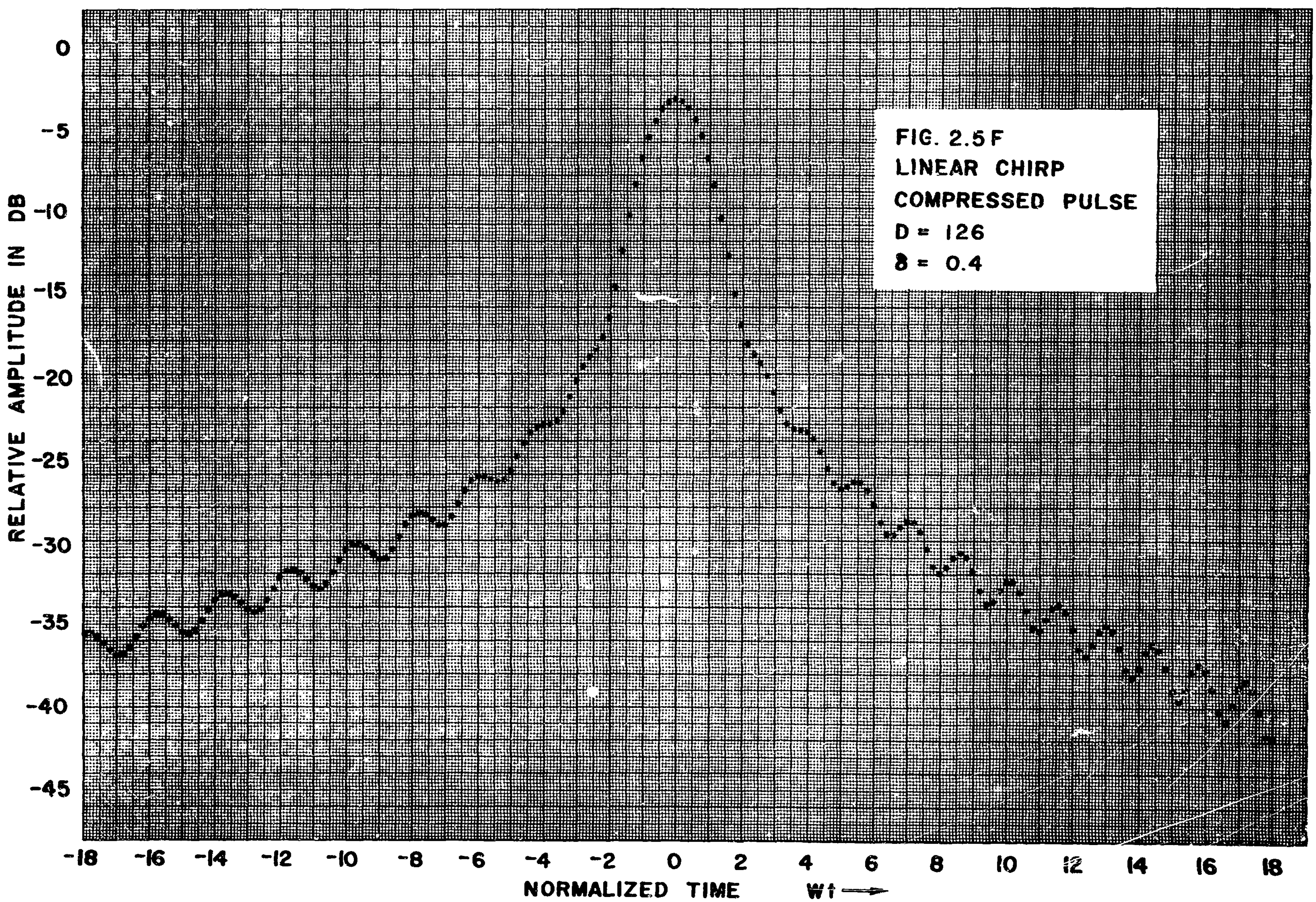


FIG. 2.5 F
LINEAR CHIRP
COMPRESSED PULSE
D = 126
δ = 0.4
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

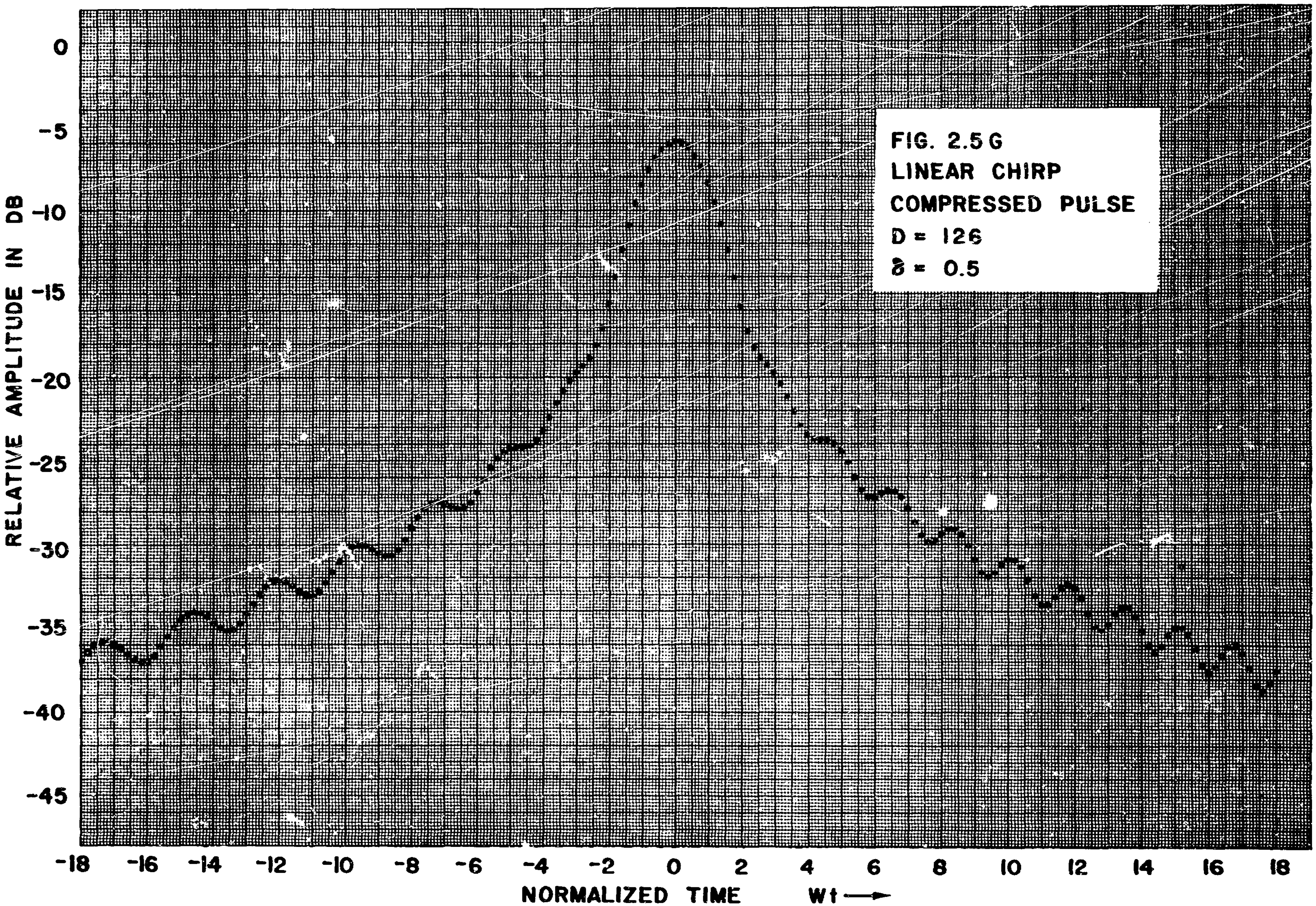


FIG. 2.5 G
LINEAR CHIRP
COMPRESSED PULSE
D = 126
δ = 0.5

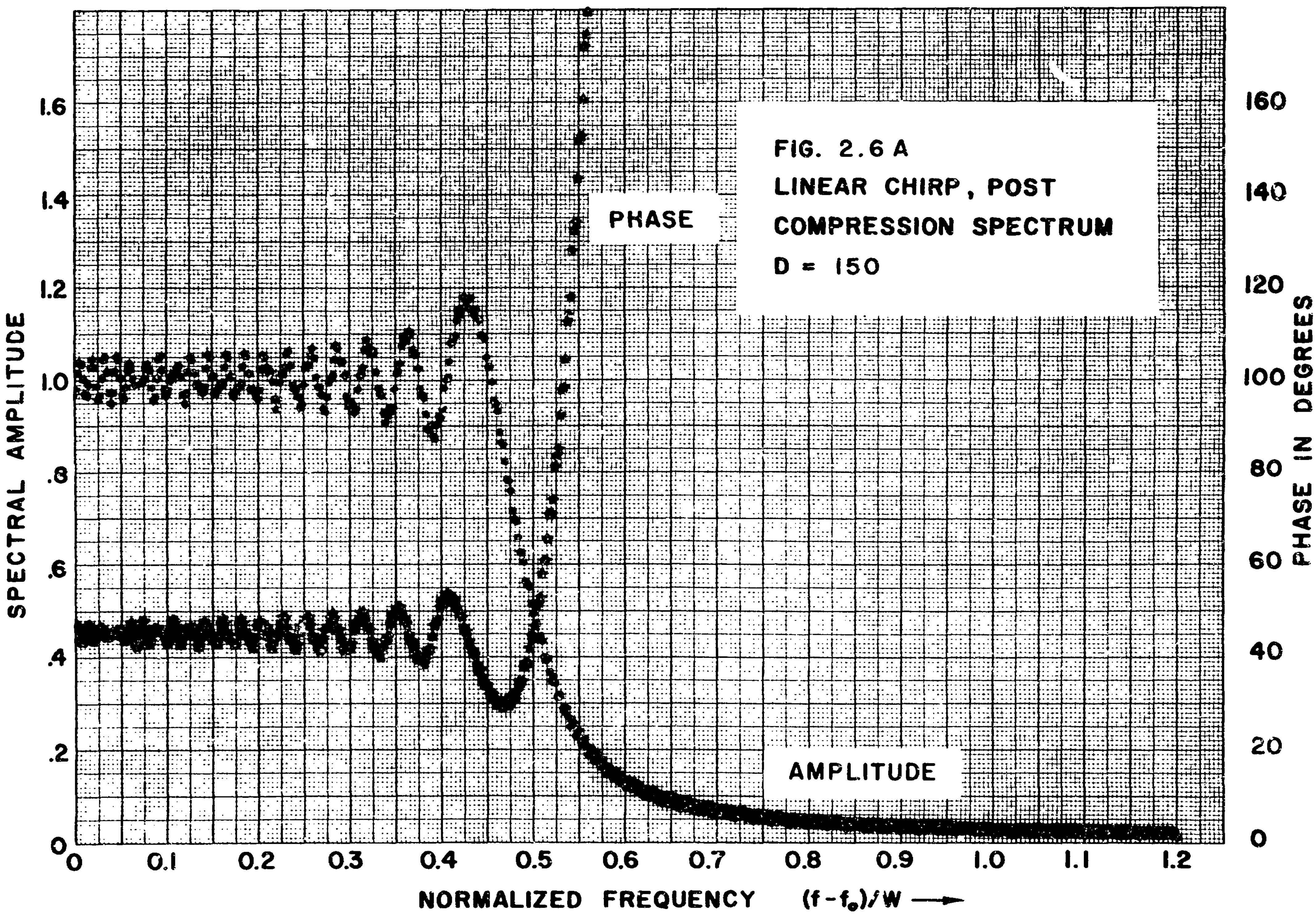


FIG. 2.6 A
LINEAR CHIRP, POST COMPRESSION SPECTRUM
D = 150

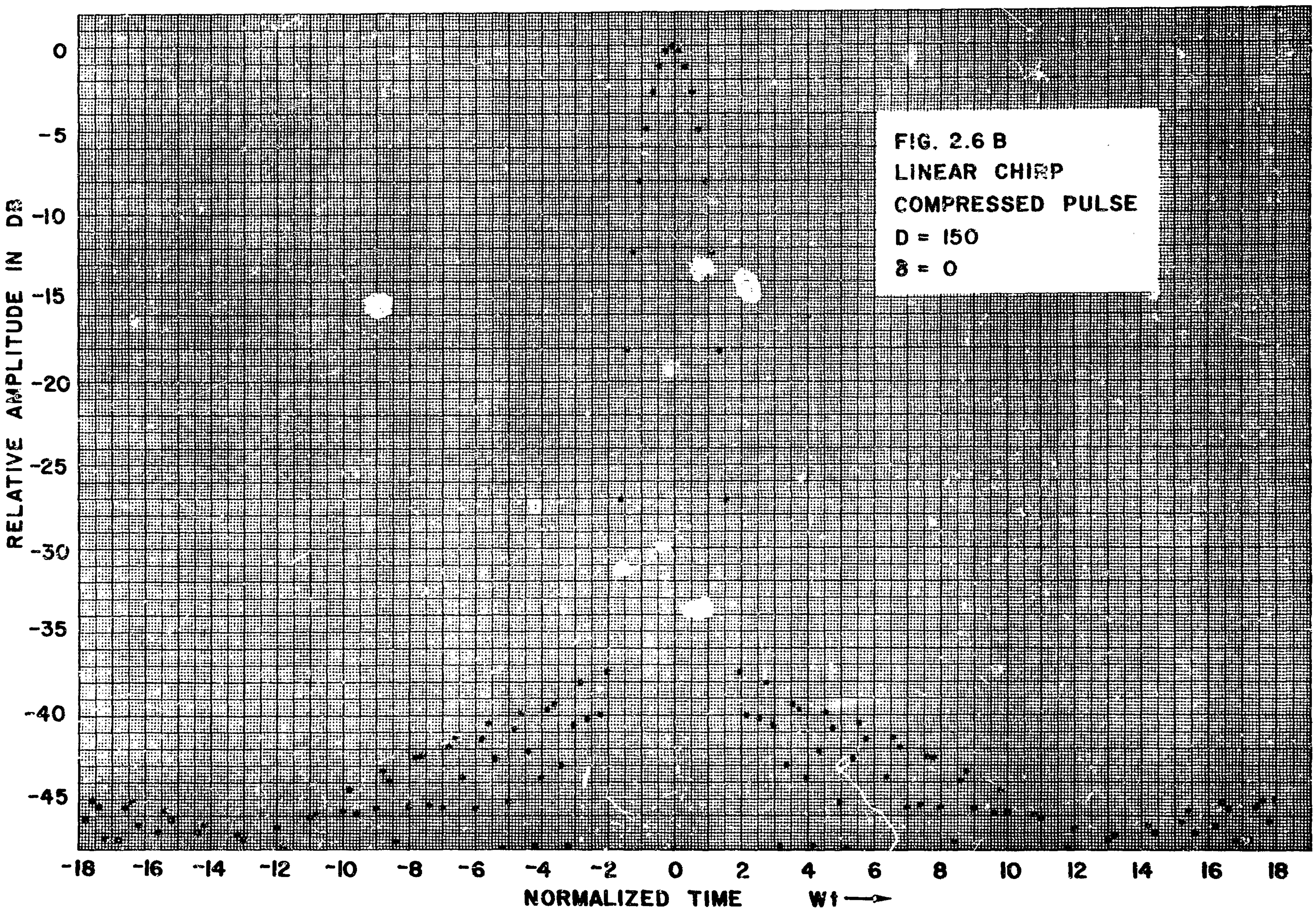


FIG. 2.6 B
LINEAR CHIRP
COMPRESSED PULSE
D = 150
δ = 0

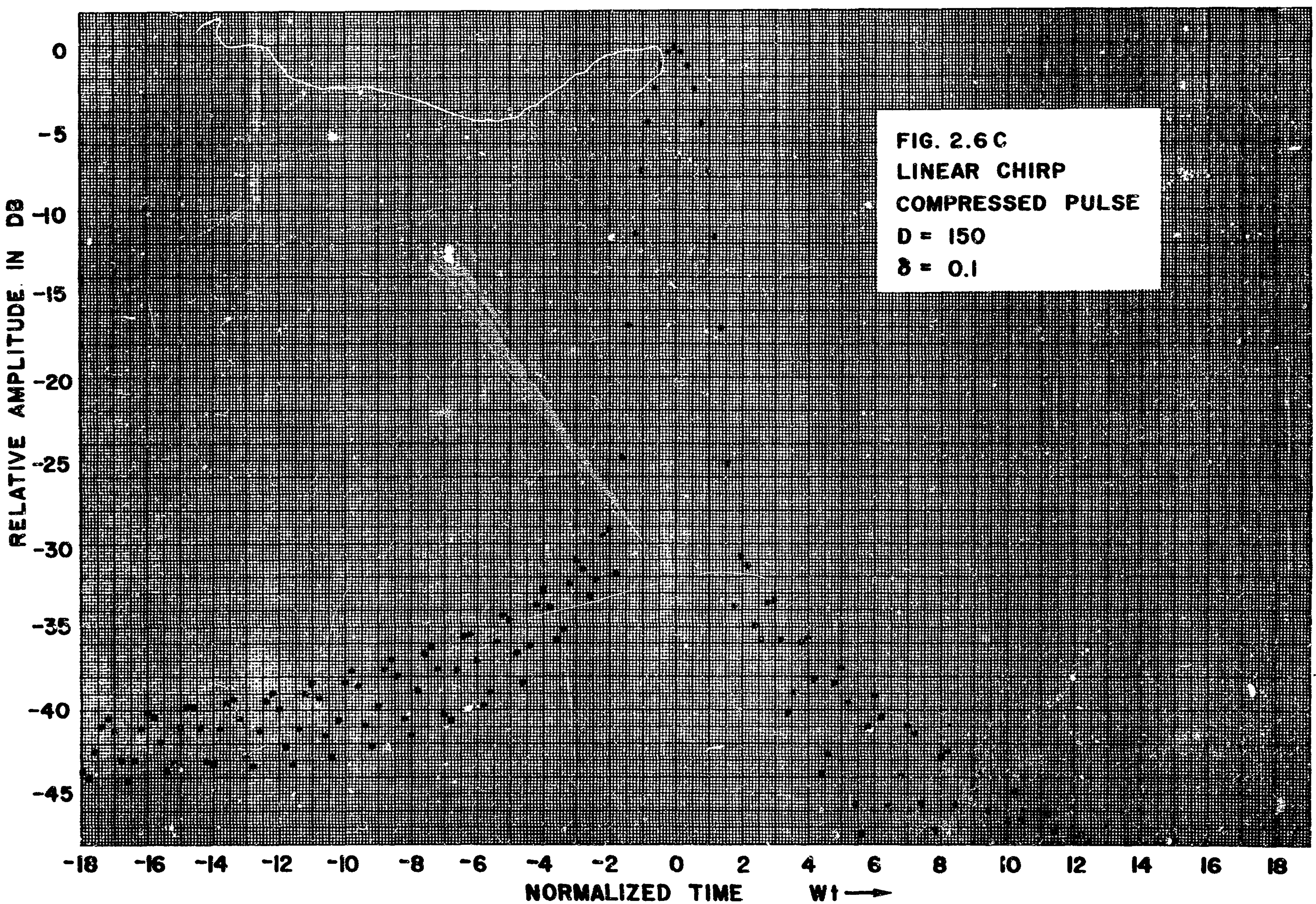


FIG. 2.6 C
LINEAR CHIRP
COMPRESSED PULSE
D = 150
δ = 0.1
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

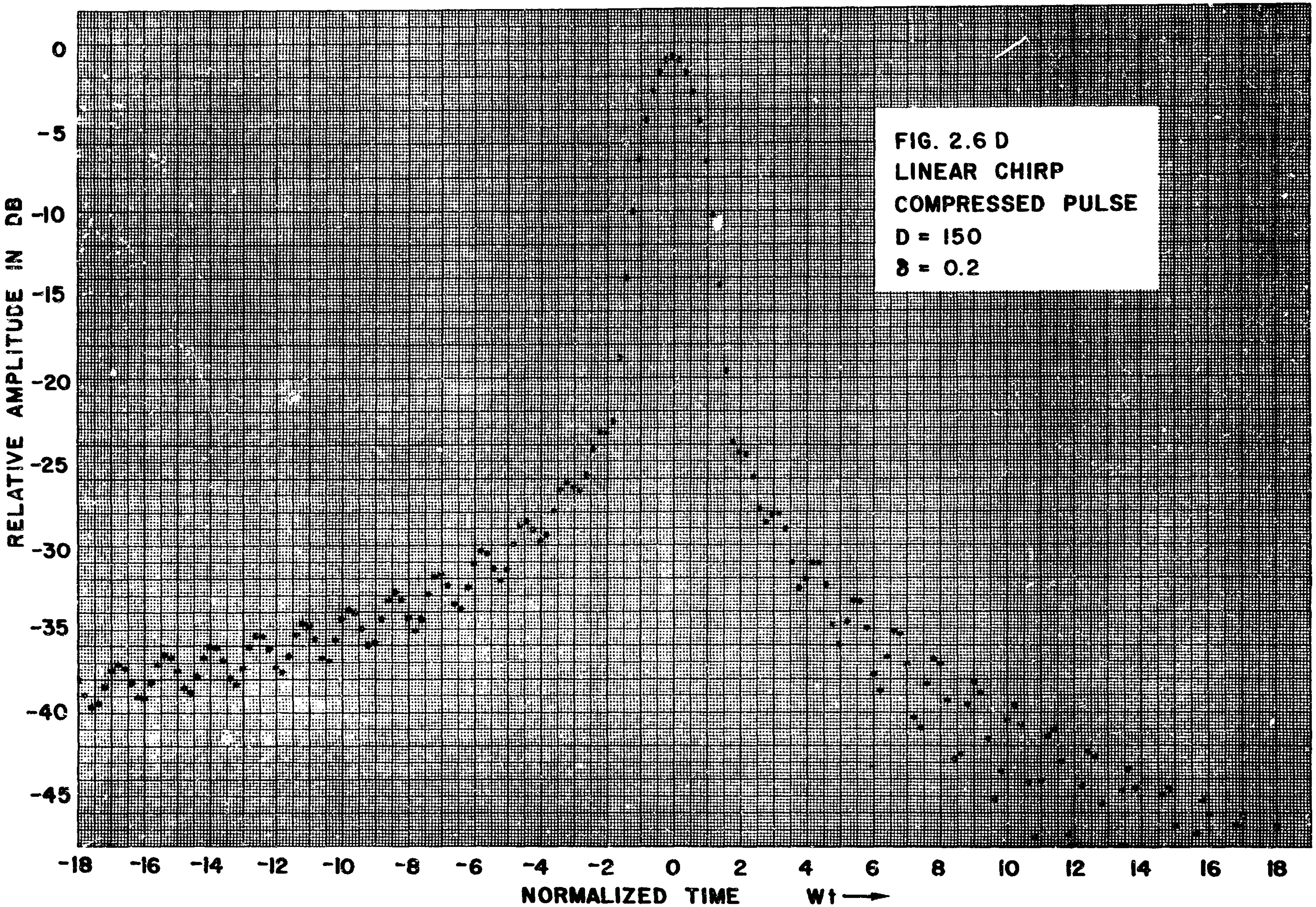


FIG. 2.6 D
LINEAR CHIRP
COMPRESSED PULSE
D = 150
δ = 0.2
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt →

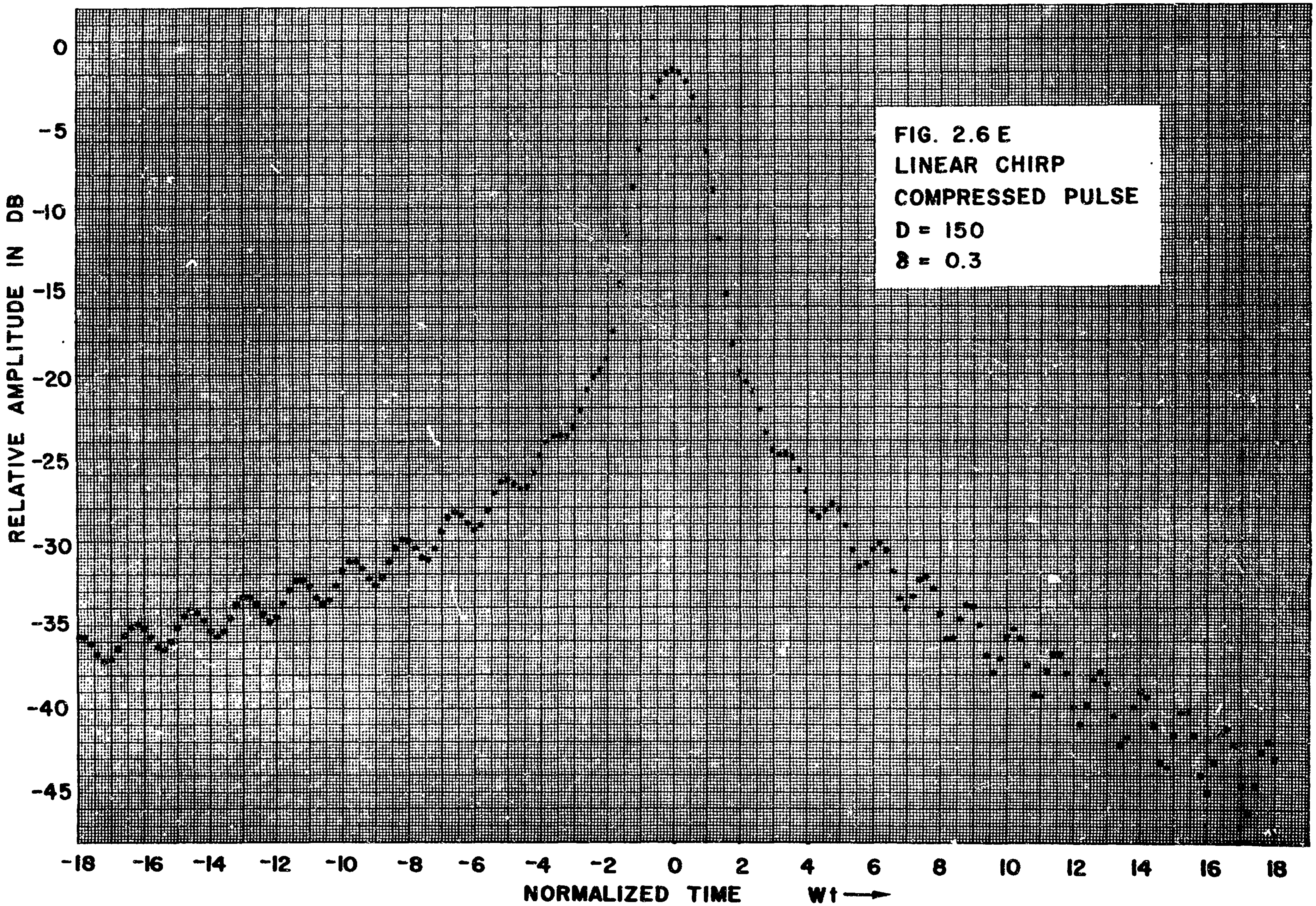


FIG. 2.6 E
LINEAR CHIRP
COMPRESSED PULSE
D = 150
δ = 0.3
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

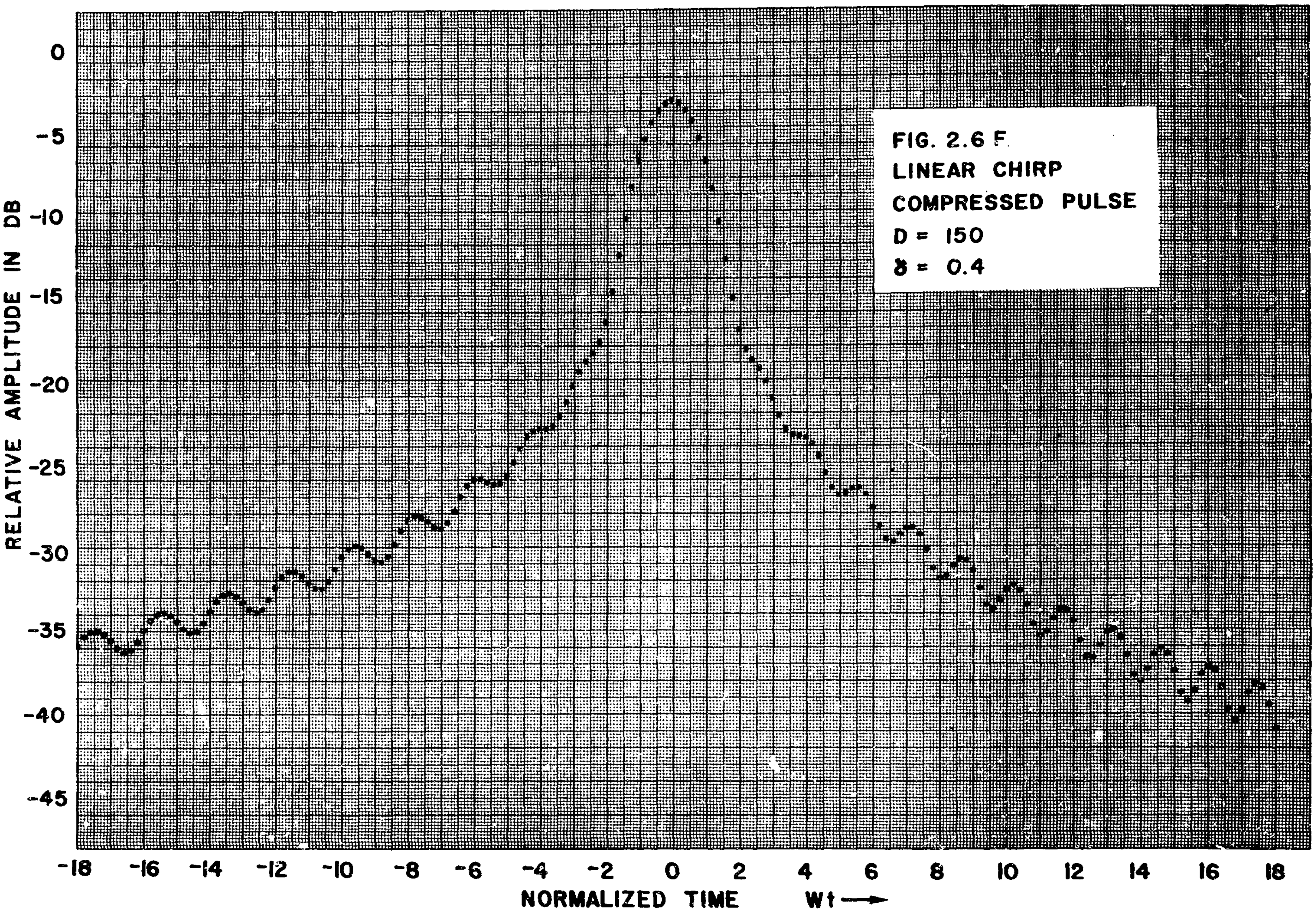


FIG. 2.6 F.
LINEAR CHIRP
COMPRESSED PULSE
D = 150
δ = 0.4
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt

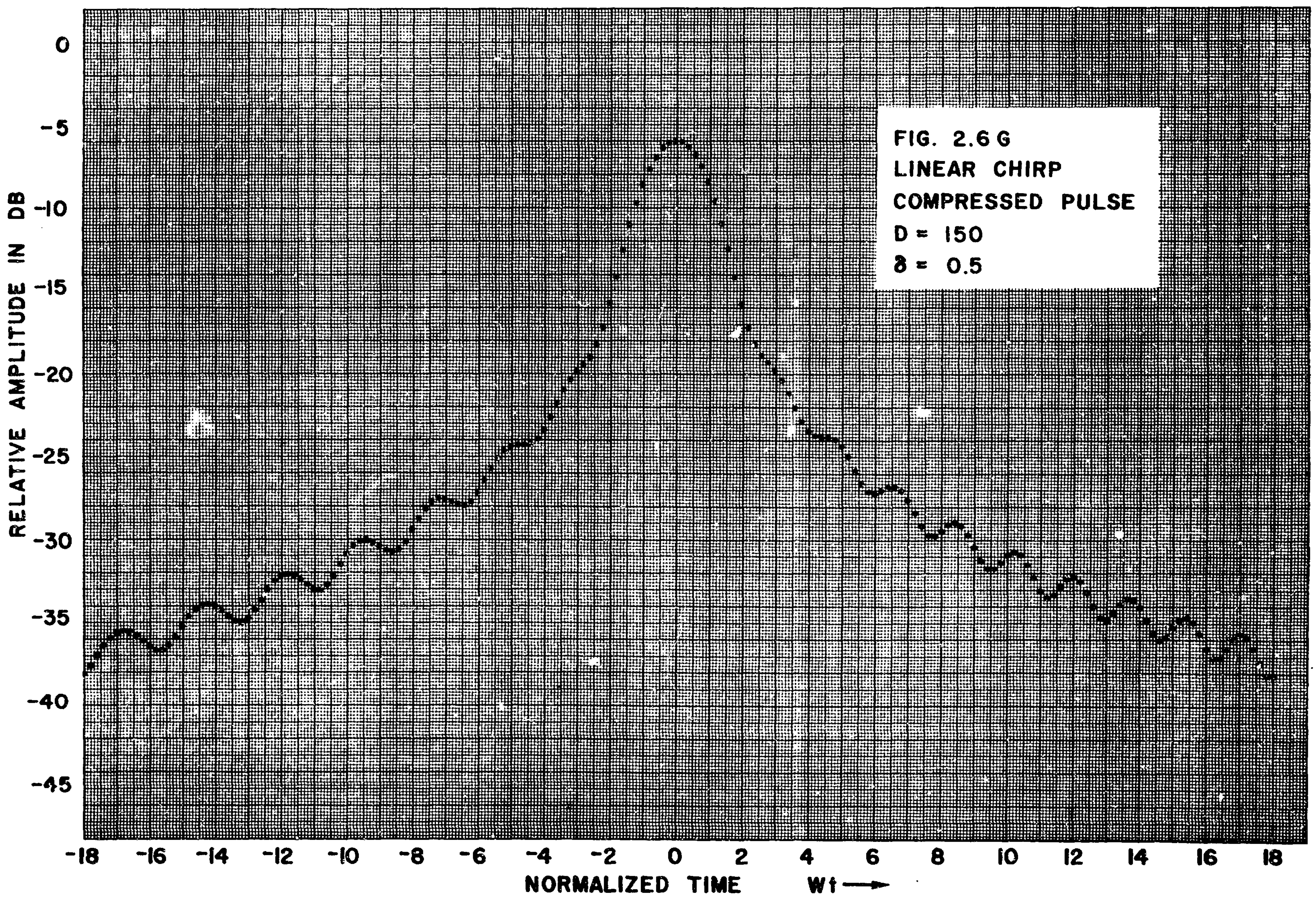
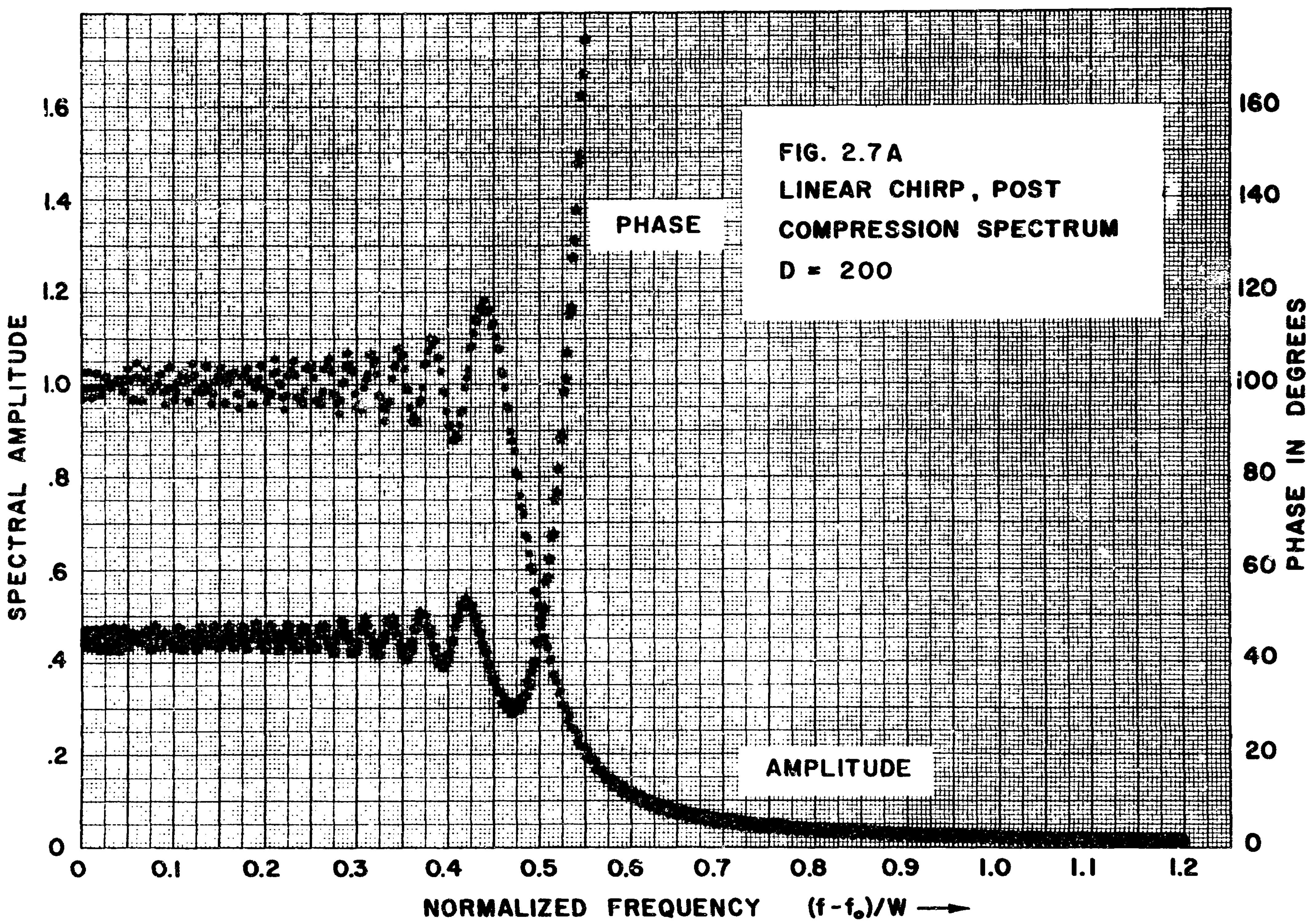


FIG. 2.6 G
LINEAR CHIRP
COMPRESSED PULSE
D = 150
δ = 0.5



K+E 10 X 10 TO THE CM. 359-14 KEUFFEL & ESSER CO.

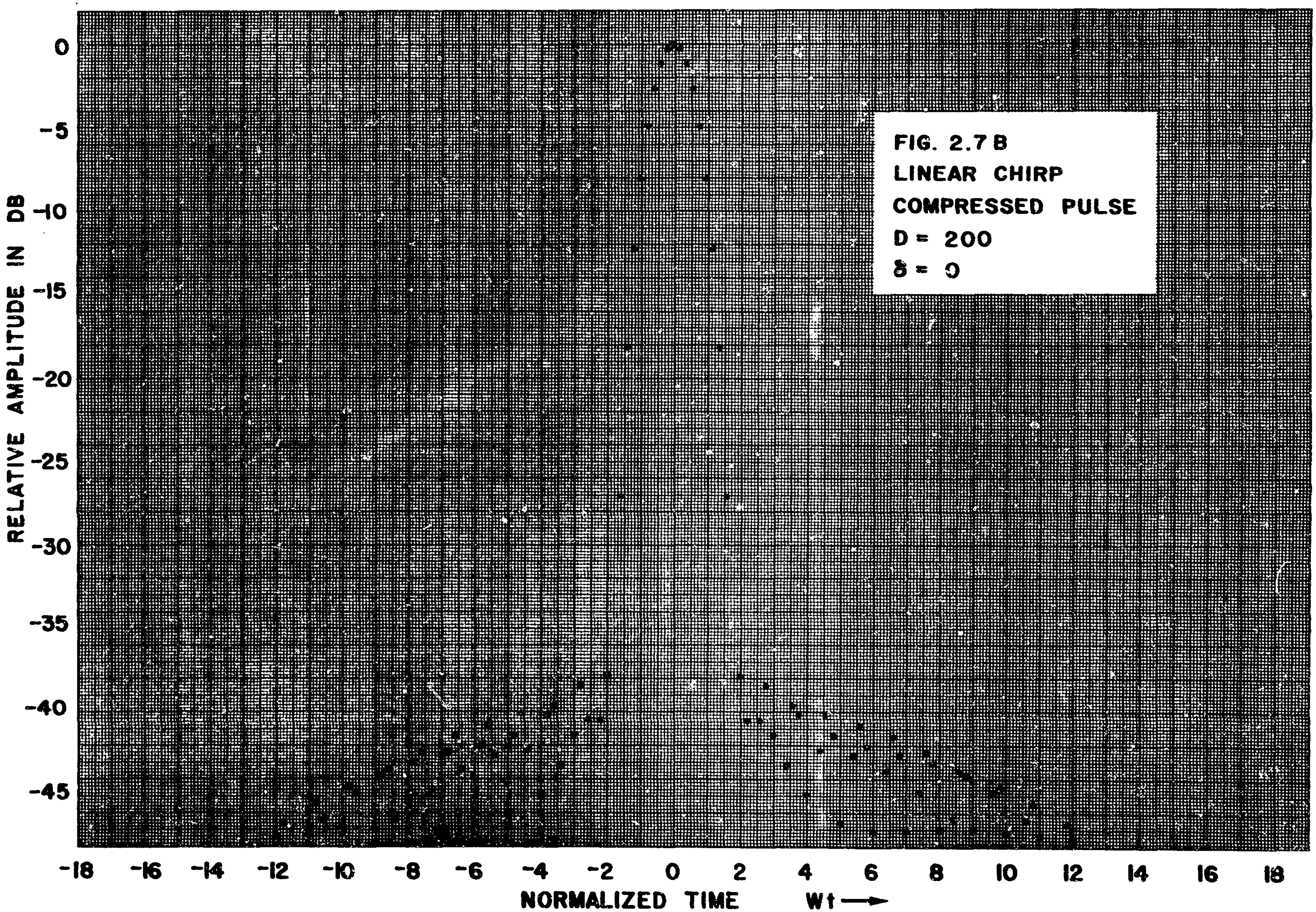


FIG. 2.7 B
LINEAR CHIRP
COMPRESSED PULSE
D = 200
δ = 0

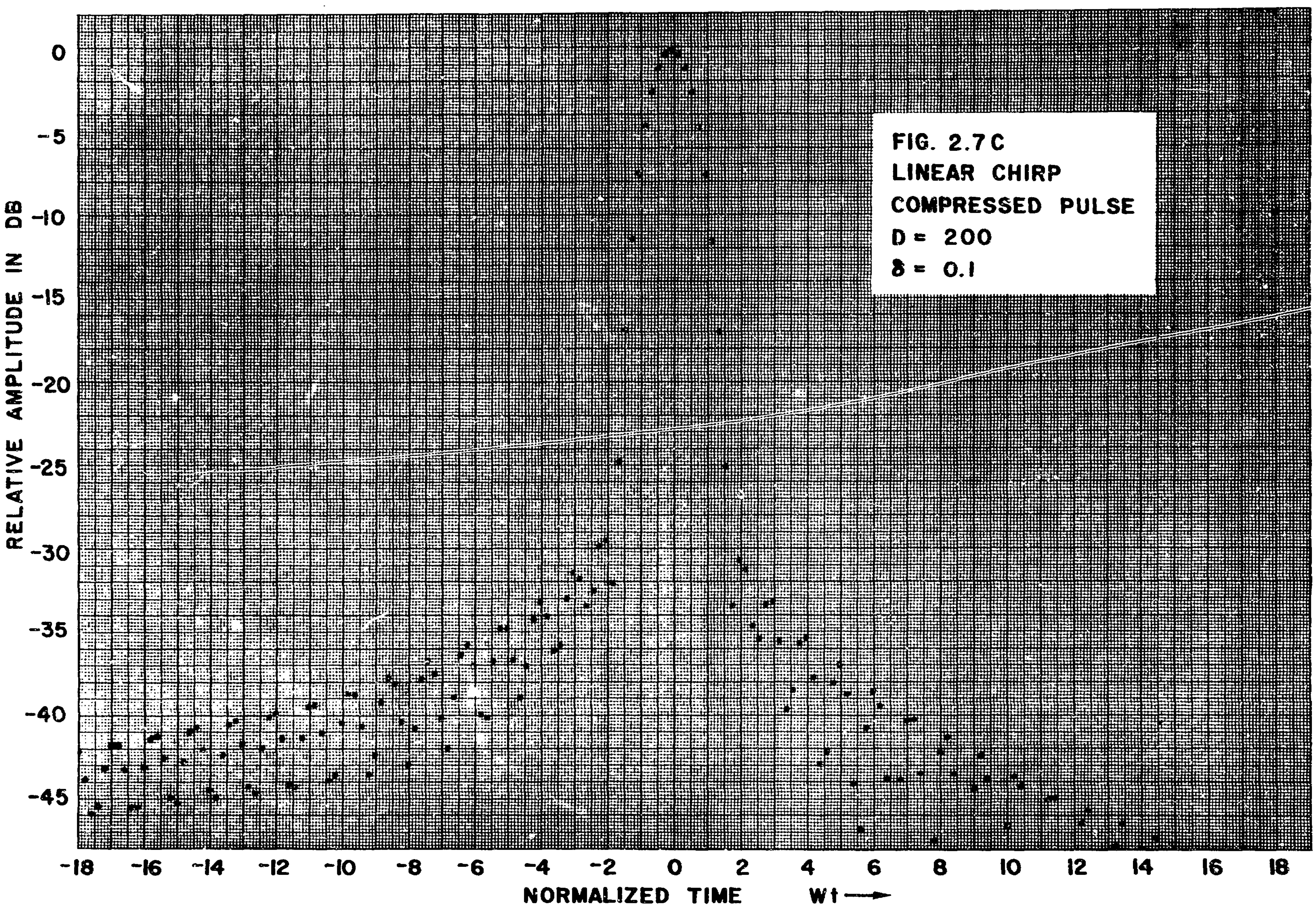


FIG. 2.7C
LINEAR CHIRP
COMPRESSED PULSE
D = 200
δ = 0.1
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME Wt

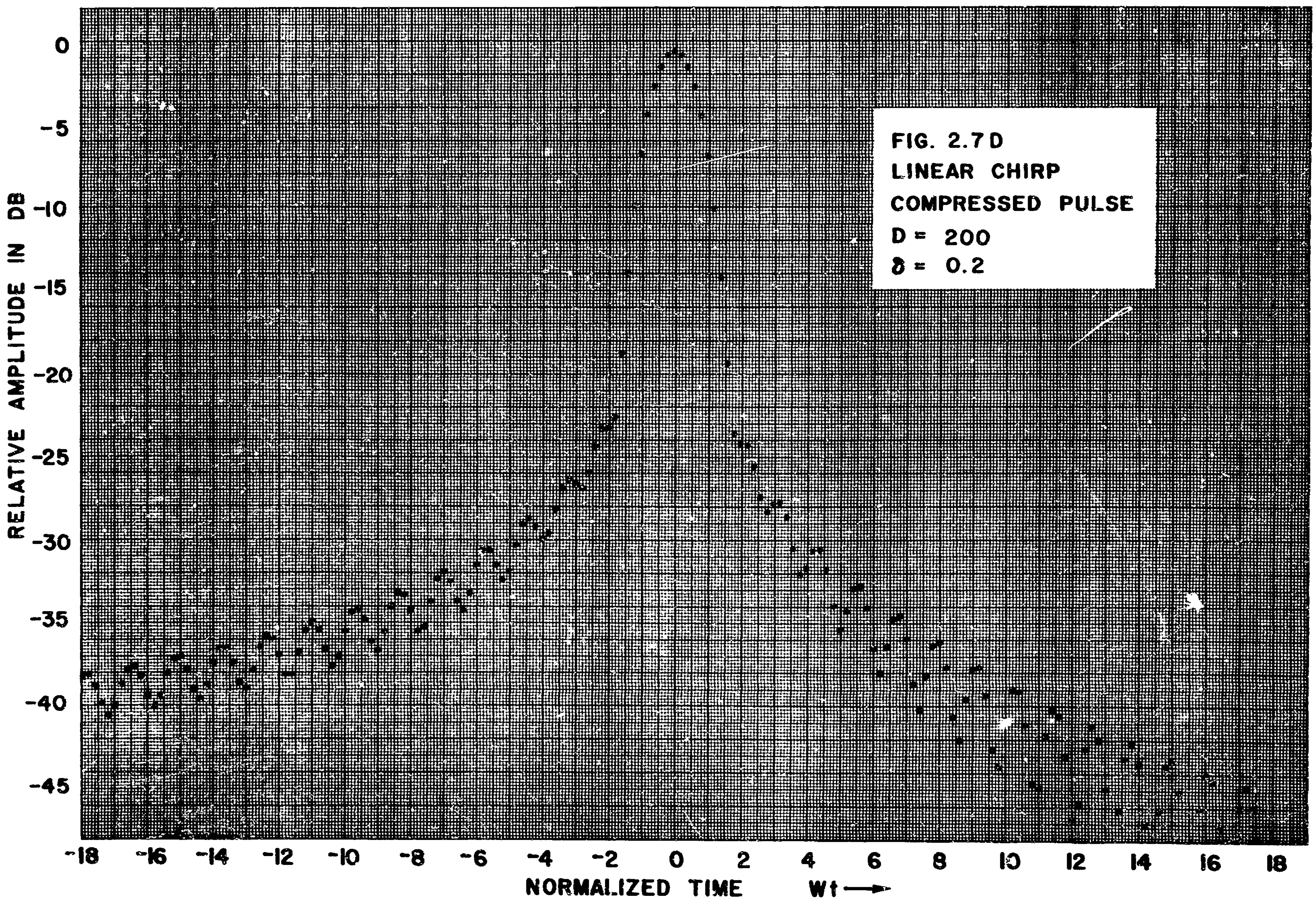


FIG. 2.7 D
LINEAR CHIRP
COMPRESSED PULSE
D = 200
δ = 0.2
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

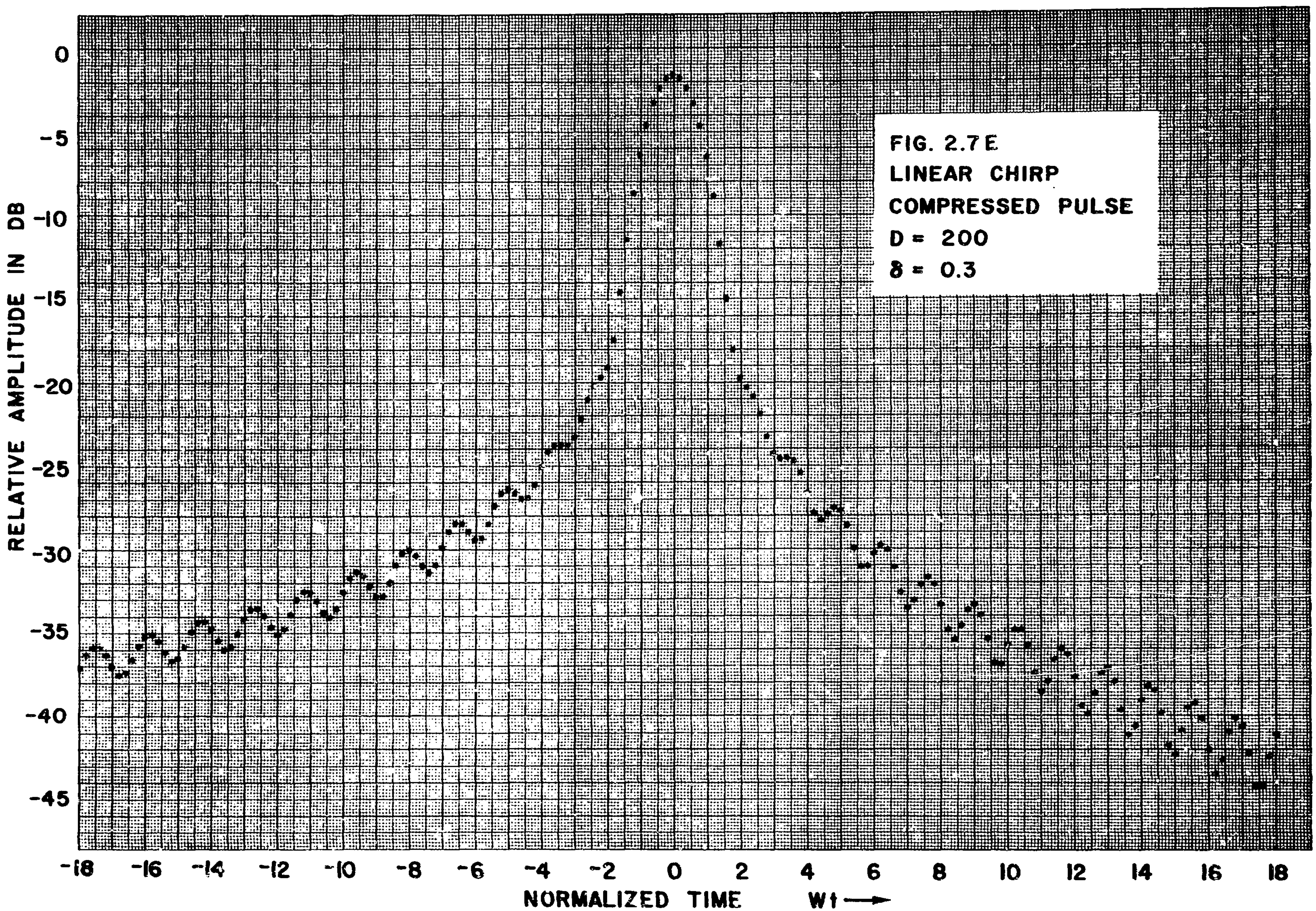


FIG. 2.7E
LINEAR CHIRP
COMPRESSED PULSE
D = 200
δ = 0.3
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

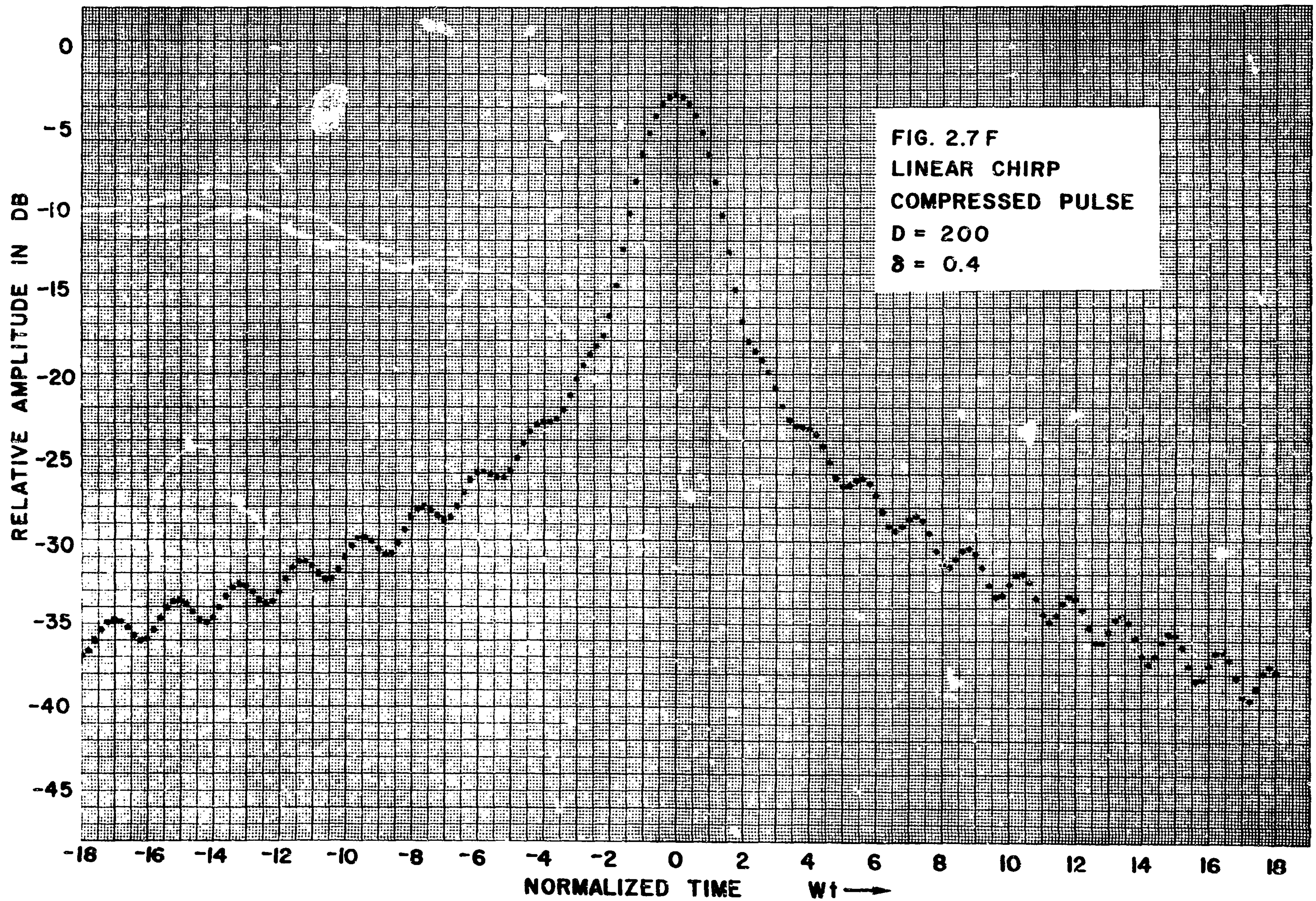


FIG. 2.7 F
LINEAR CHIRP
COMPRESSED PULSE
D = 200
δ = 0.4
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

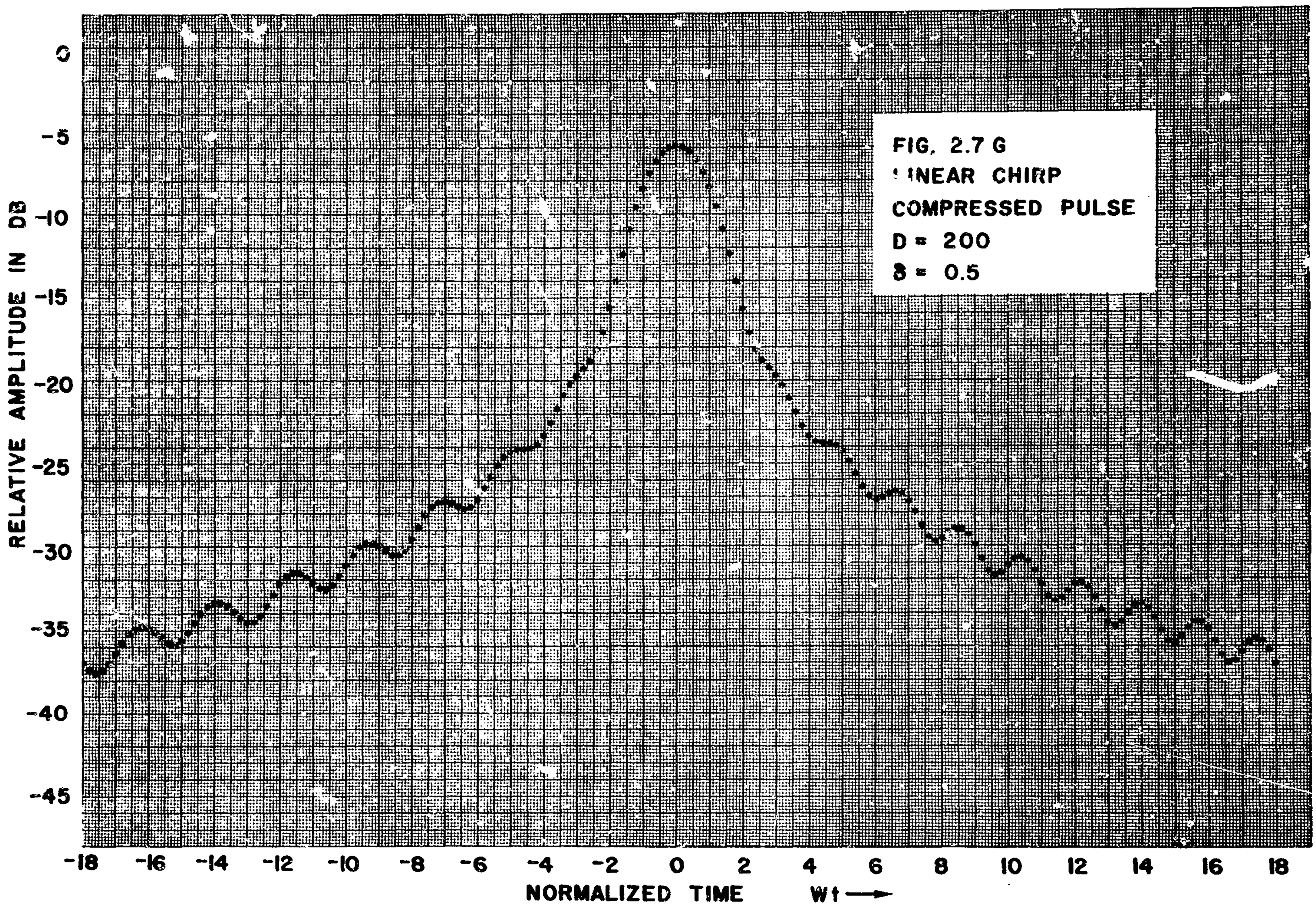


FIG. 2.7 G
LINEAR CHIRP
COMPRESSED PULSE
D = 200
δ = 0.5
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

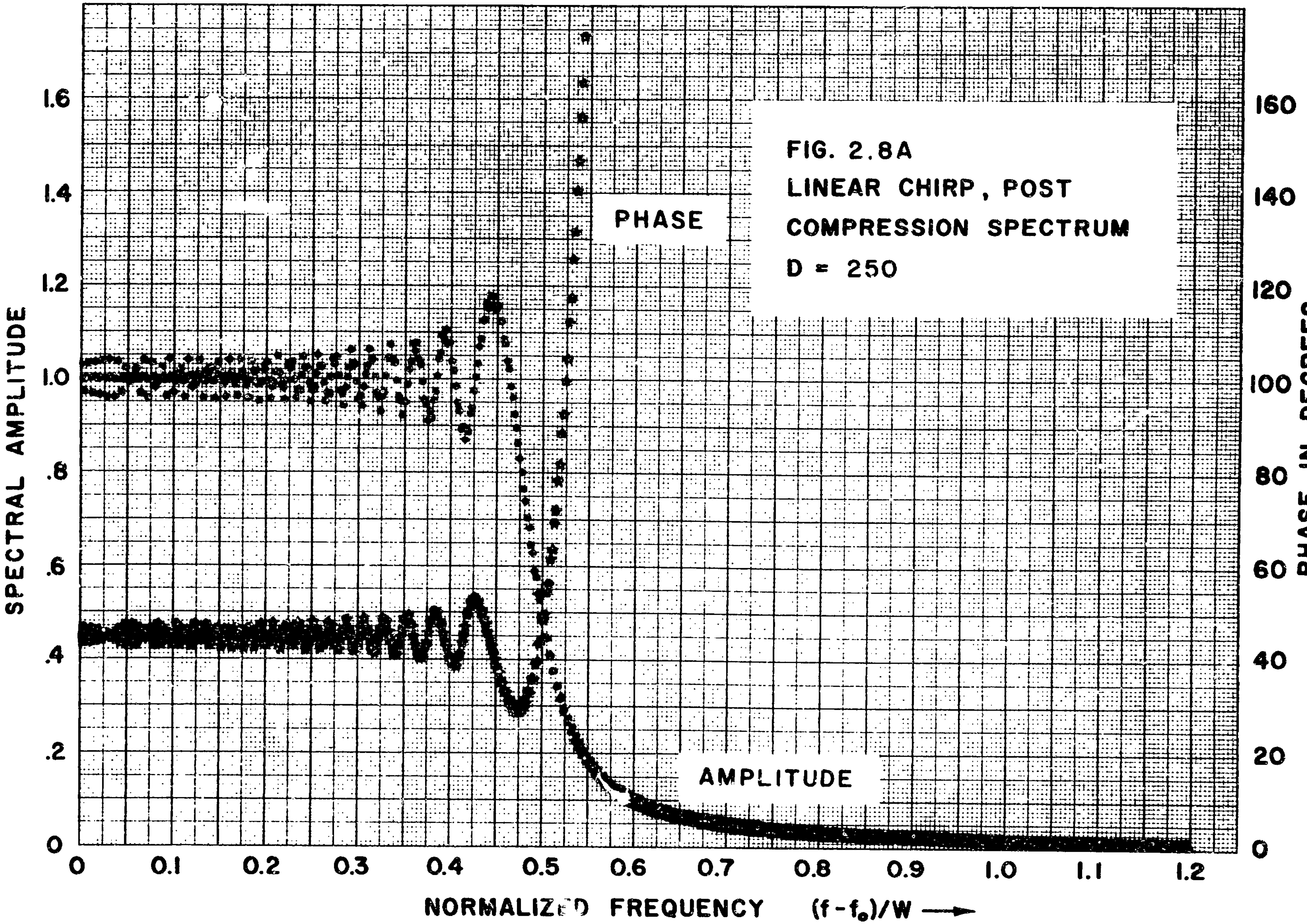


FIG. 2.8A
LINEAR CHIRP, POST
COMPRESSION SPECTRUM
D = 250
PHASE
AMPLITUDE
SPECTRAL AMPLITUDE
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
PHASE IN DEGREES
0
20
40
60
80
100
120
140
160
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
NORMALIZED FREQUENCY (f-f₀)/W

9.B D = 250

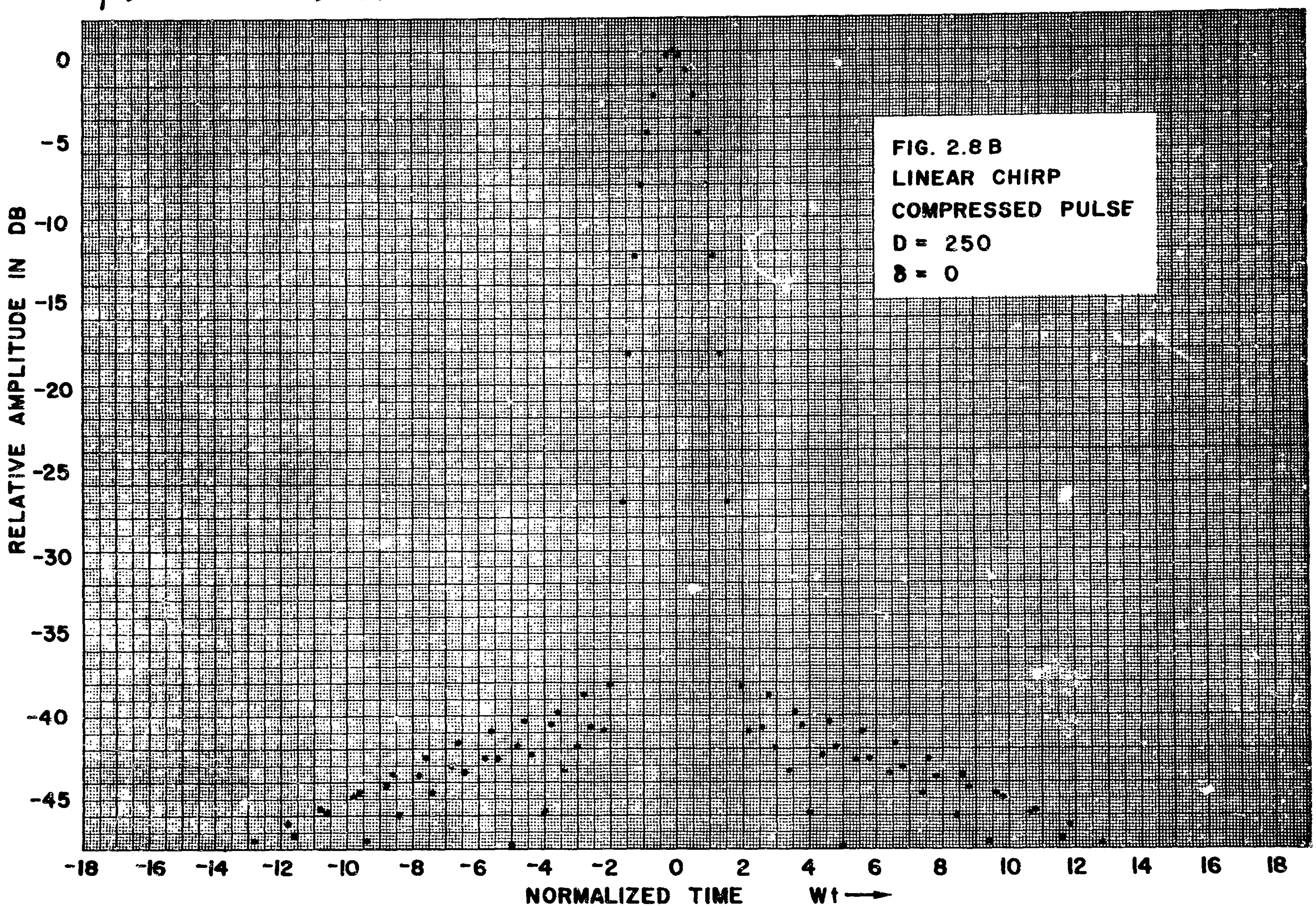


FIG. 2.8 B
LINEAR CHIRP
COMPRESSED PULSE
D = 250
δ = 0
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

8-C D = 250.0

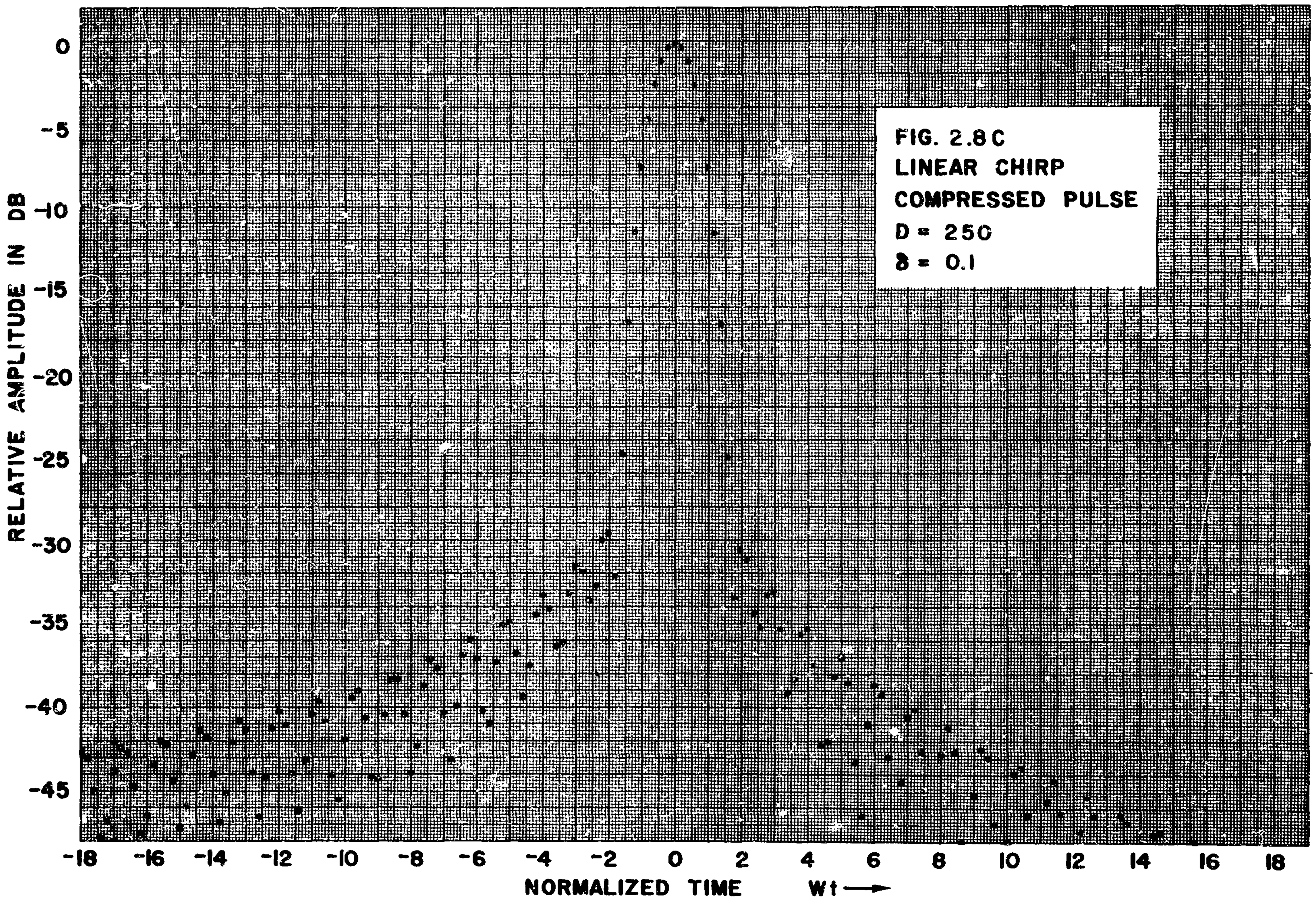


FIG. 2.8C
LINEAR CHIRP
COMPRESSED PULSE
D = 250
δ = 0.1
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

2-D

D = 2500

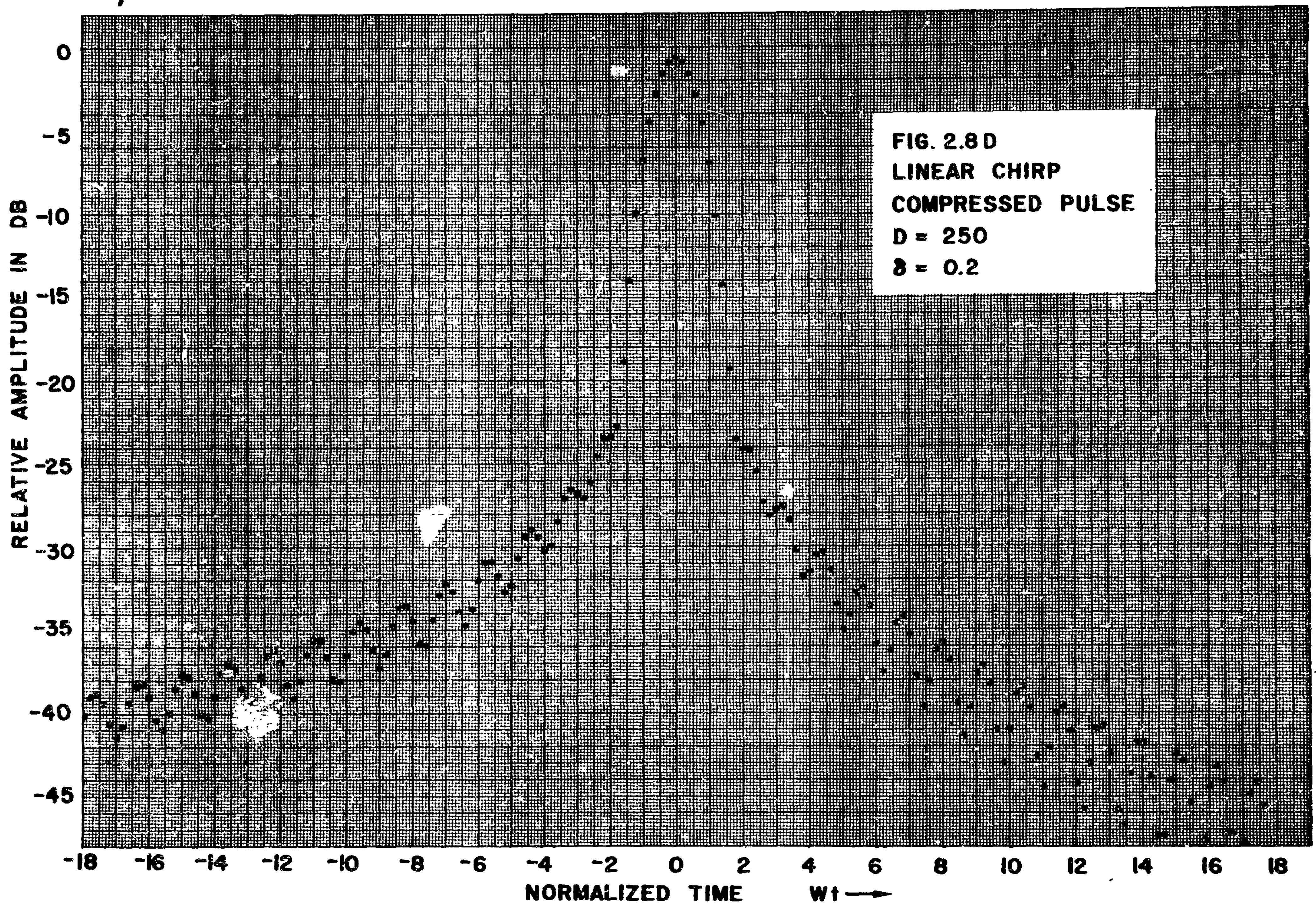


FIG. 2.8D
LINEAR CHIRP
COMPRESSED PULSE
D = 250
δ = 0.2
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

E D = 250.0

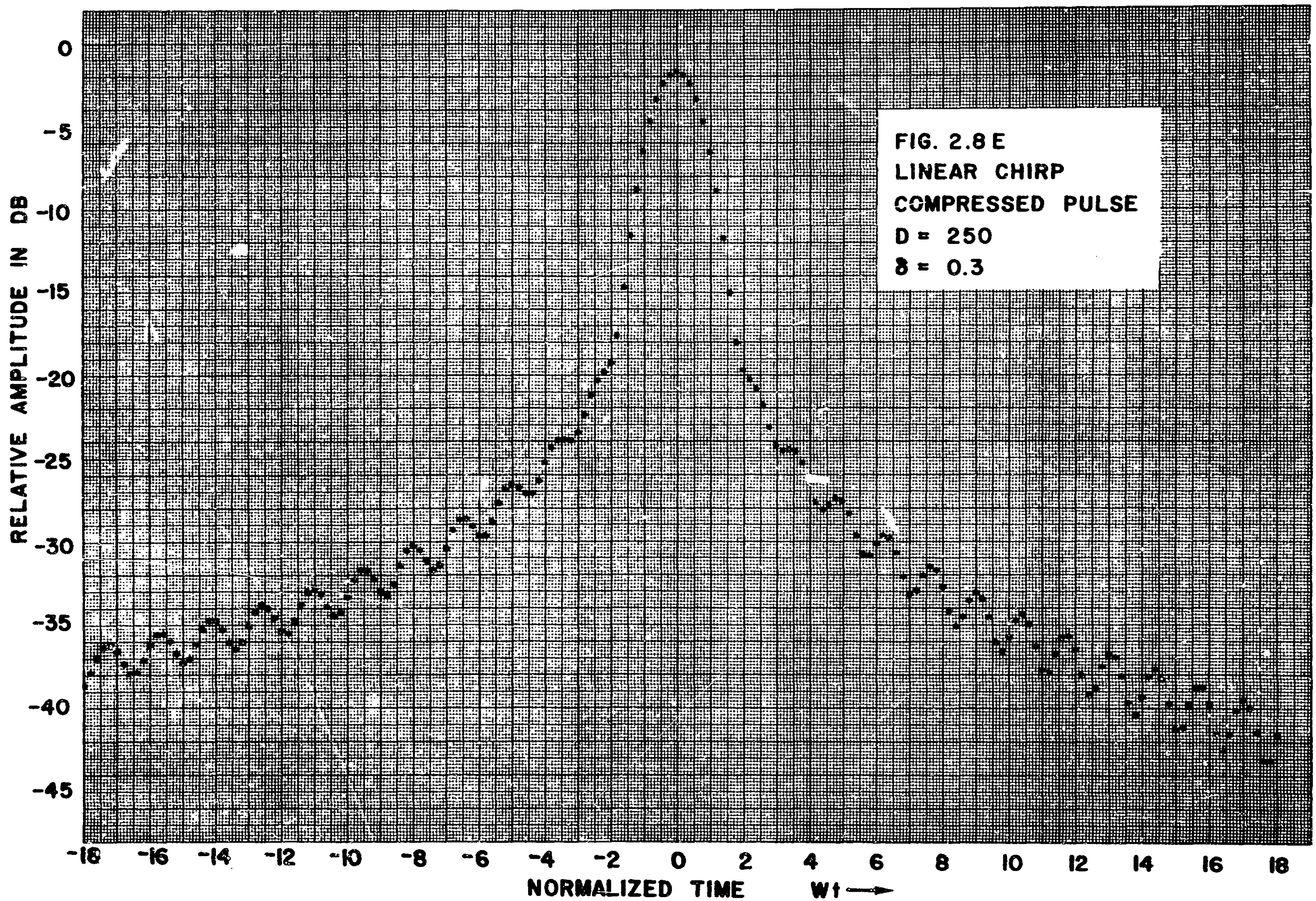
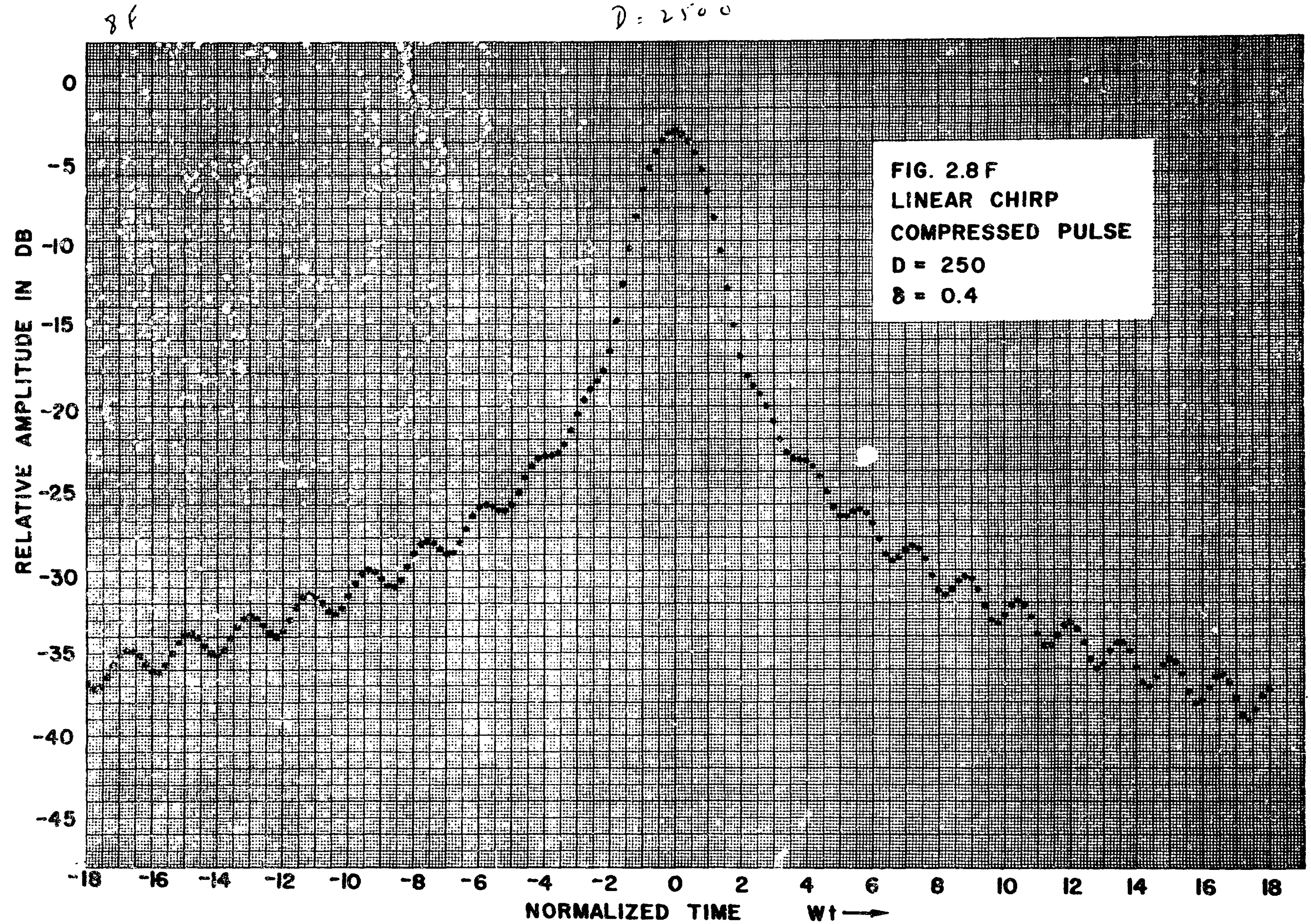


FIG. 2.8 E
LINEAR CHIRP
COMPRESSED PULSE
D = 250
δ = 0.3
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt


8F
D = 2500
FIG. 2.8 F
LINEAR CHIRP
COMPRESSED PULSE
D = 250
δ = 0.4
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt

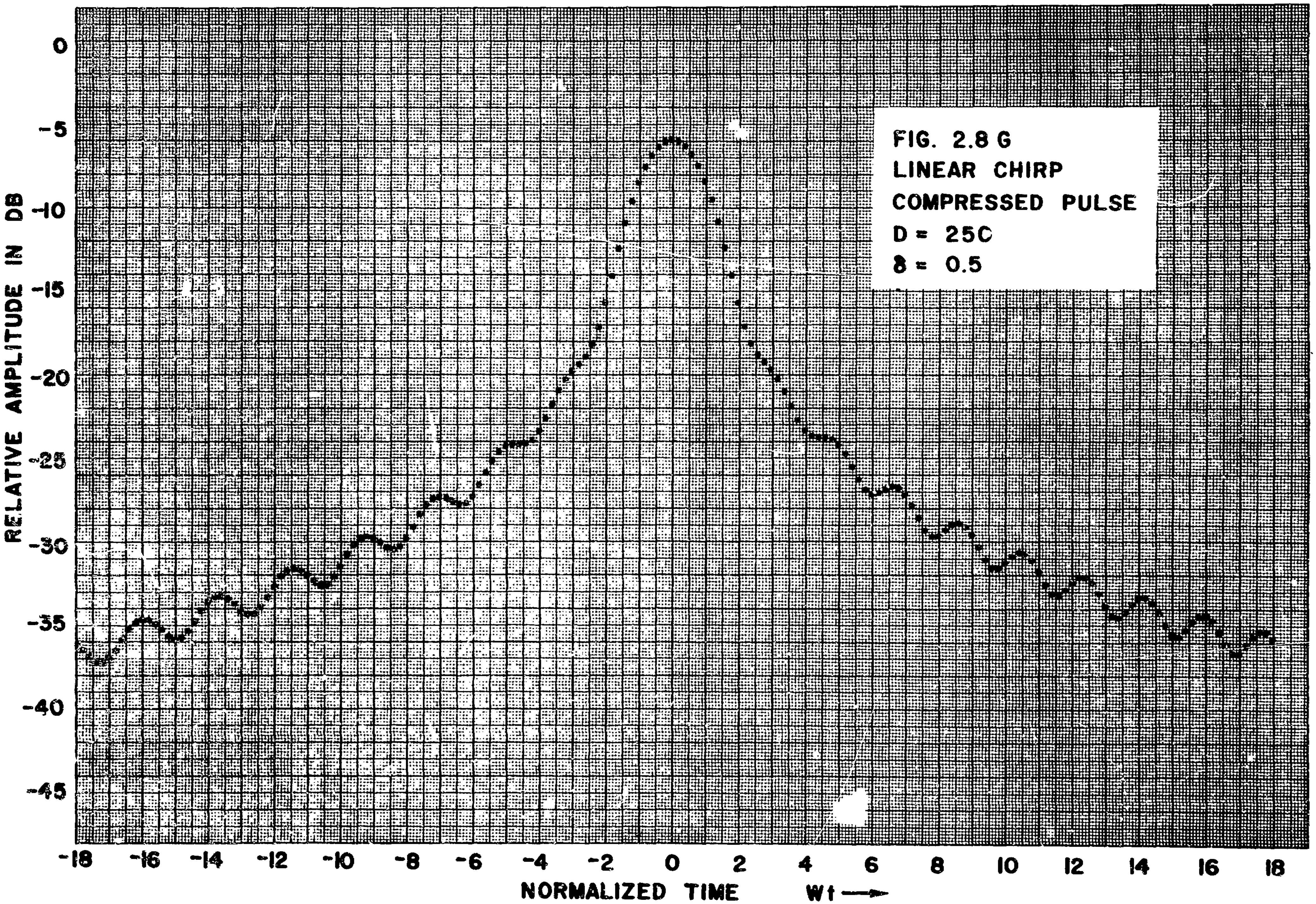


FIG. 2.8 G
LINEAR CHIRP
COMPRESSED PULSE
D = 25C
δ = 0.5

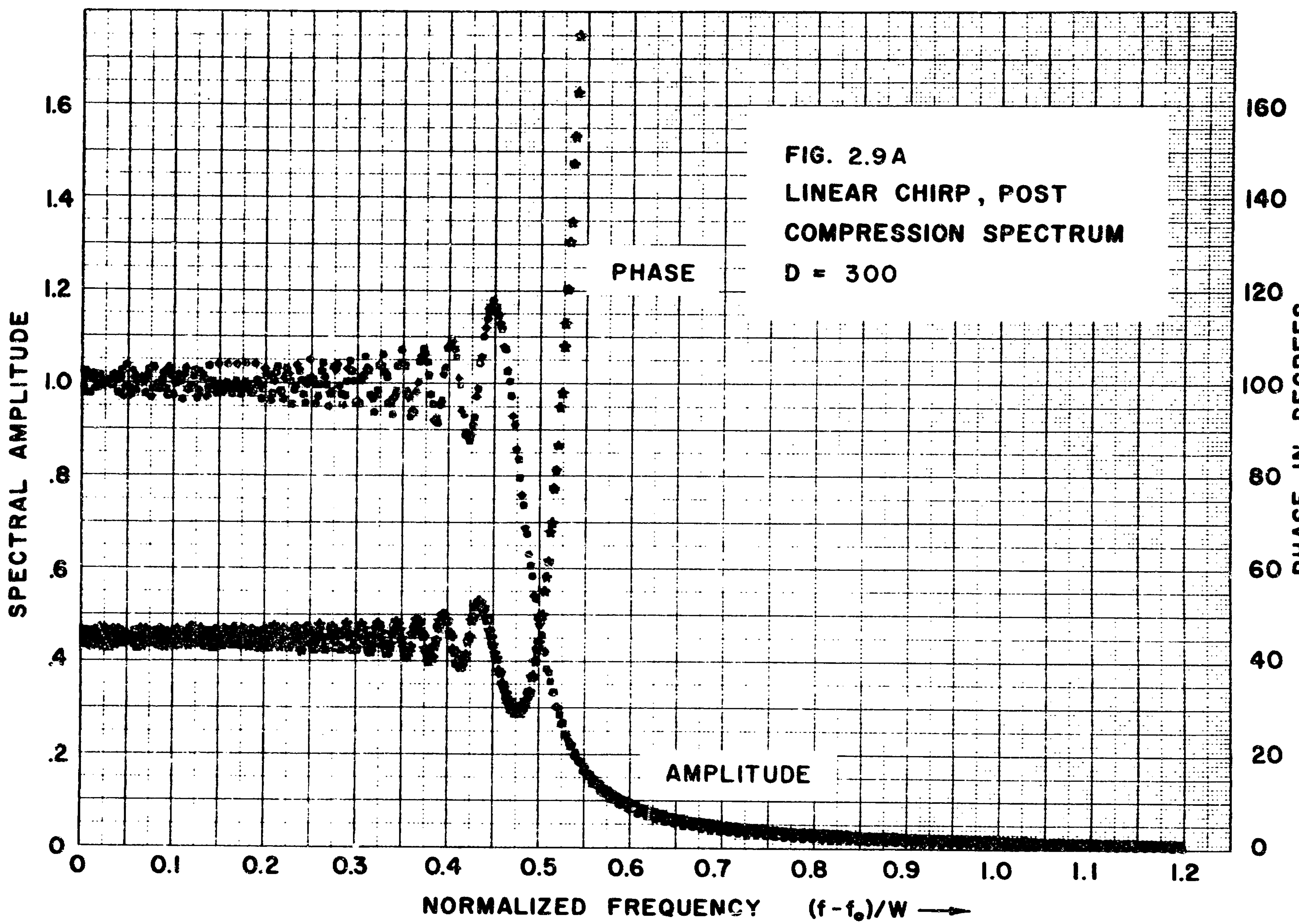


FIG. 2.9A
LINEAR CHIRP, POST
COMPRESSION SPECTRUM
D = 300

9-B D=300

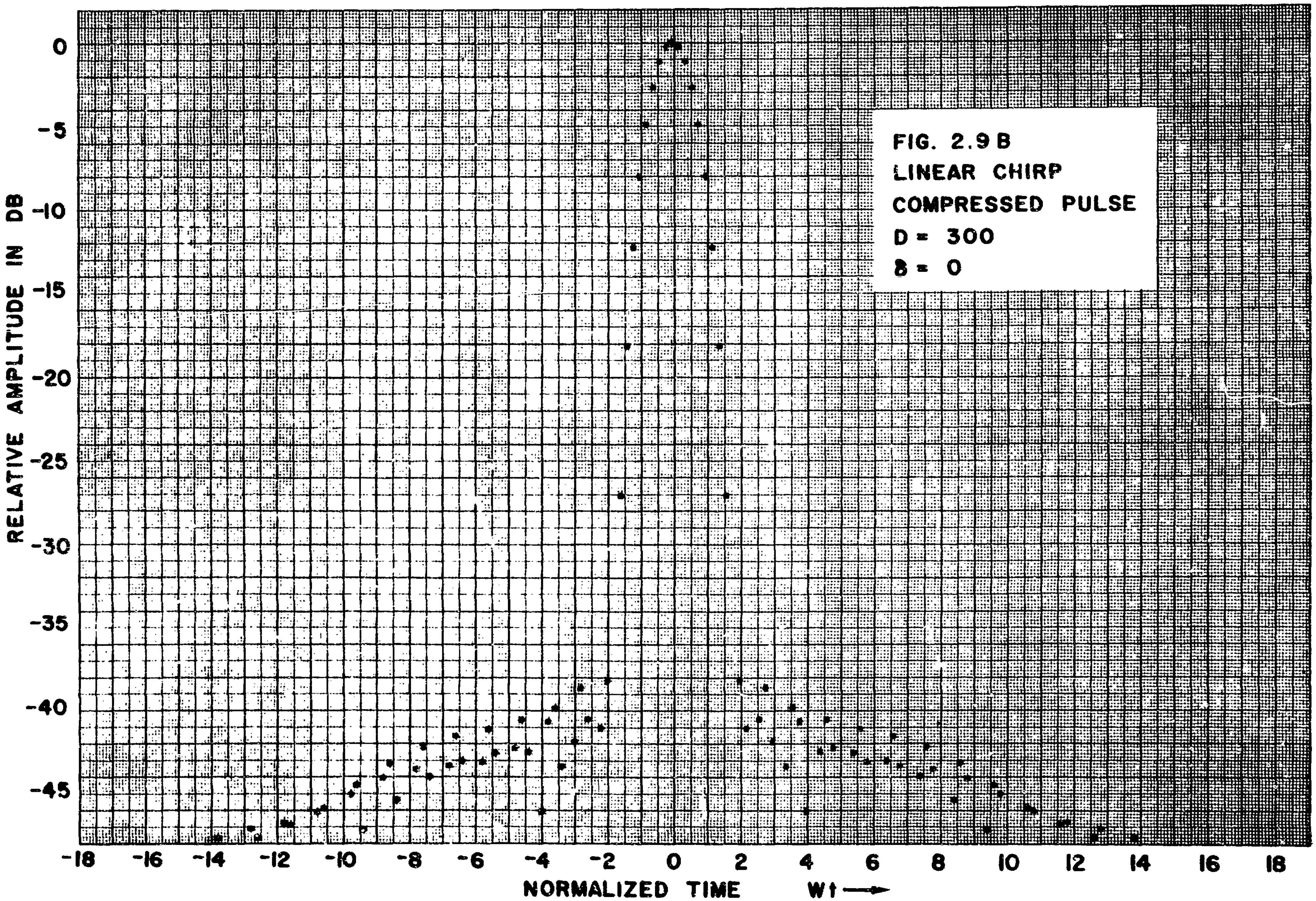


FIG. 2.9 B
LINEAR CHIRP
COMPRESSED PULSE
D = 300
δ = 0
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME Wt

9-C D-300 C

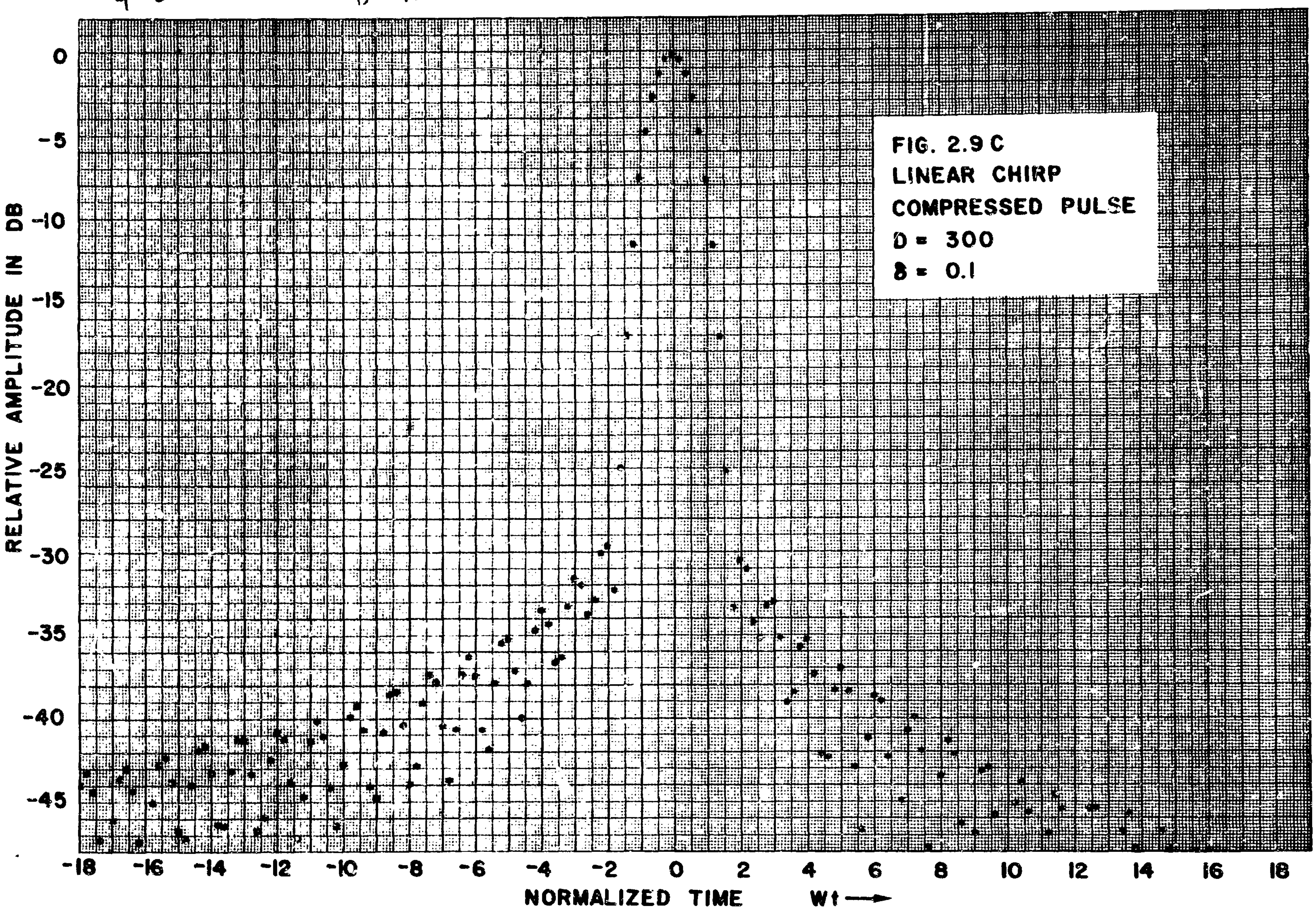


FIG. 2.9 C
LINEAR CHIRP
COMPRESSED PULSE
D = 300
δ = 0.1
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt ⟶

9-D

D = 3000

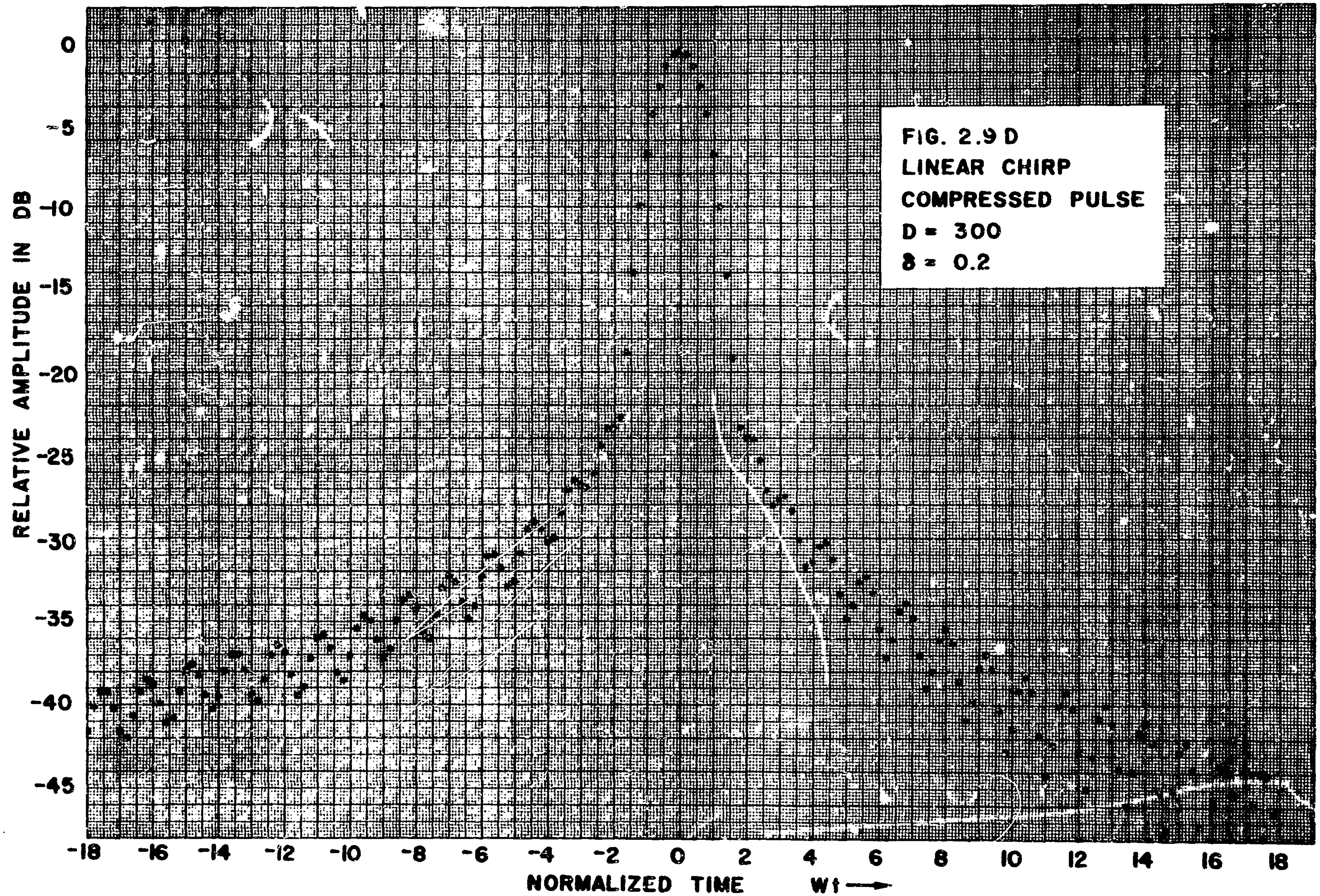


FIG. 2.9 D
LINEAR CHIRP
COMPRESSED PULSE
D = 300
δ = 0.2
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt →

9-E

D=300

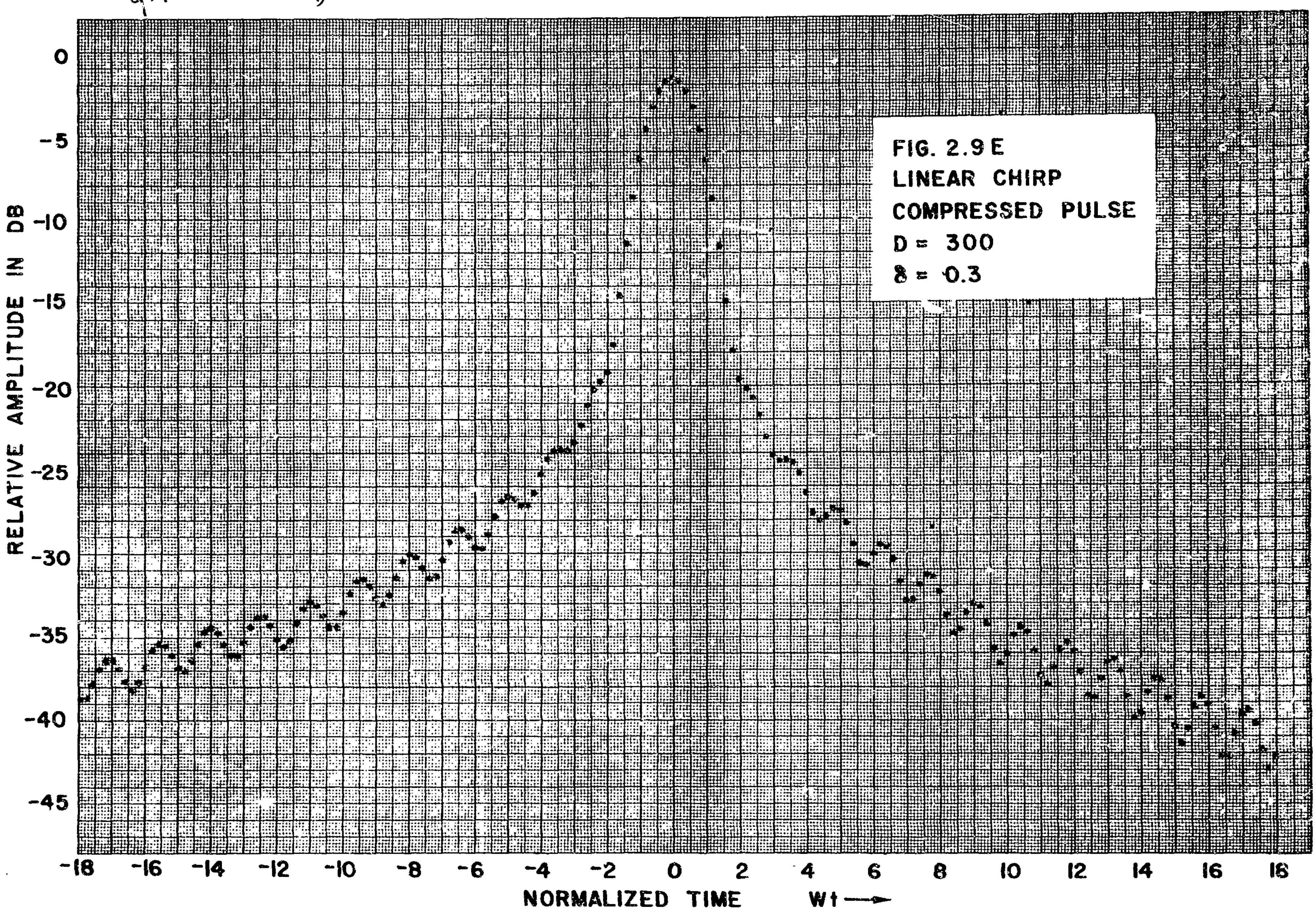


FIG. 2.9 E
LINEAR CHIRP
COMPRESSED PULSE
D = 300
δ = 0.3
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

9.F D = 3000

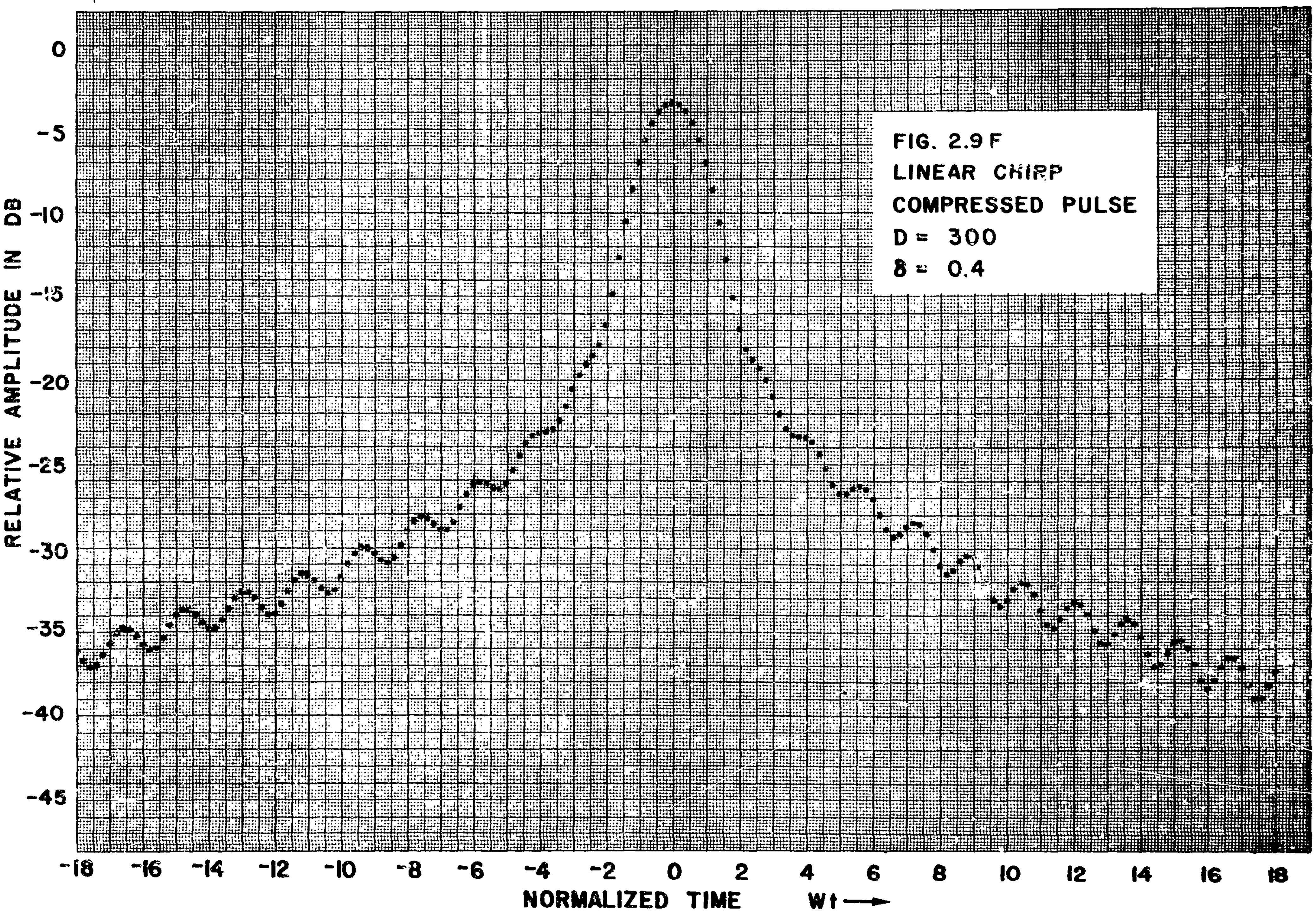


FIG. 2.9 F
LINEAR CHIRP
COMPRESSED PULSE
D = 300
δ = 0.4
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

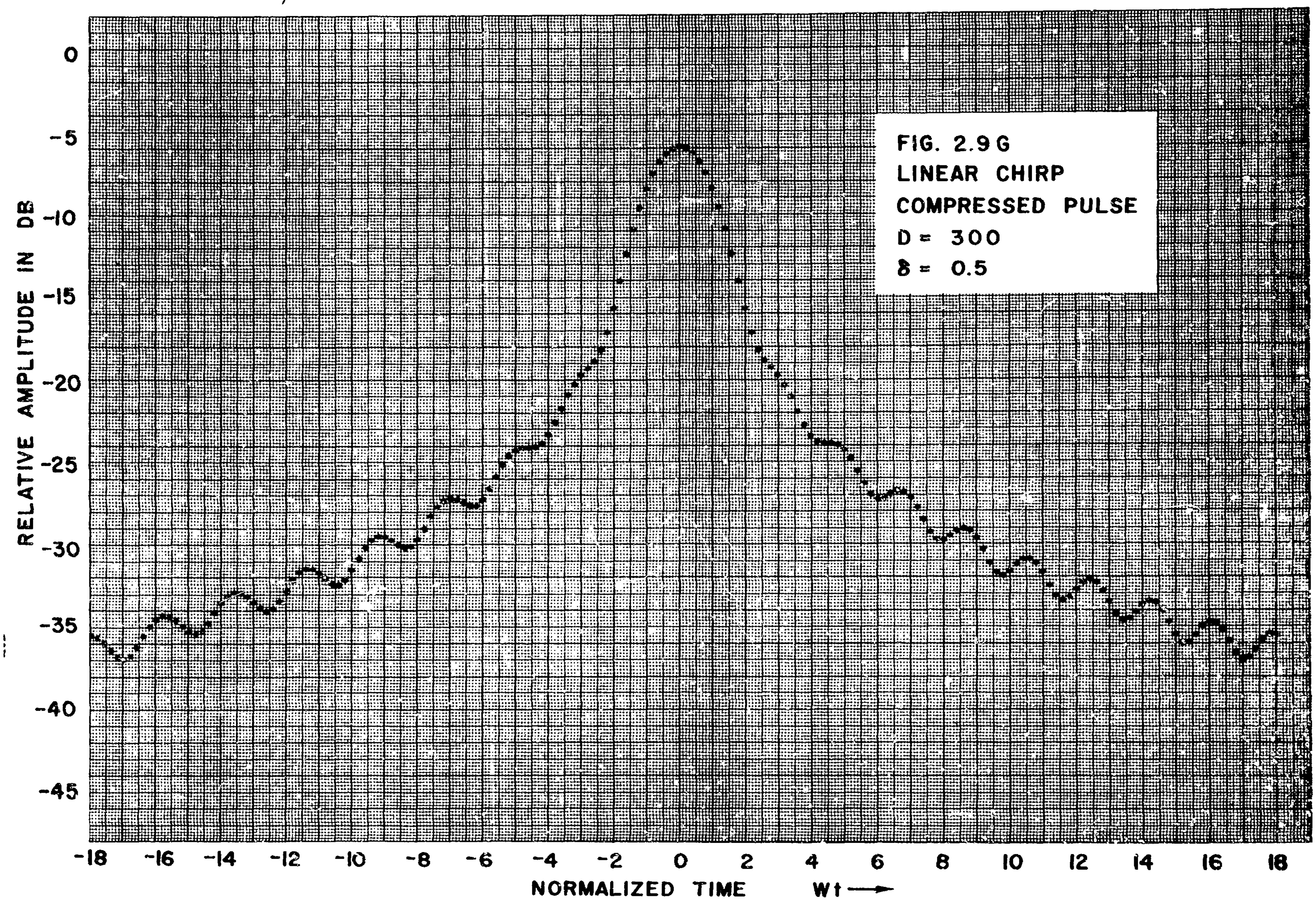


FIG. 2.9G
LINEAR CHIRP
COMPRESSED PULSE
D = 300
δ = 0.5
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt →

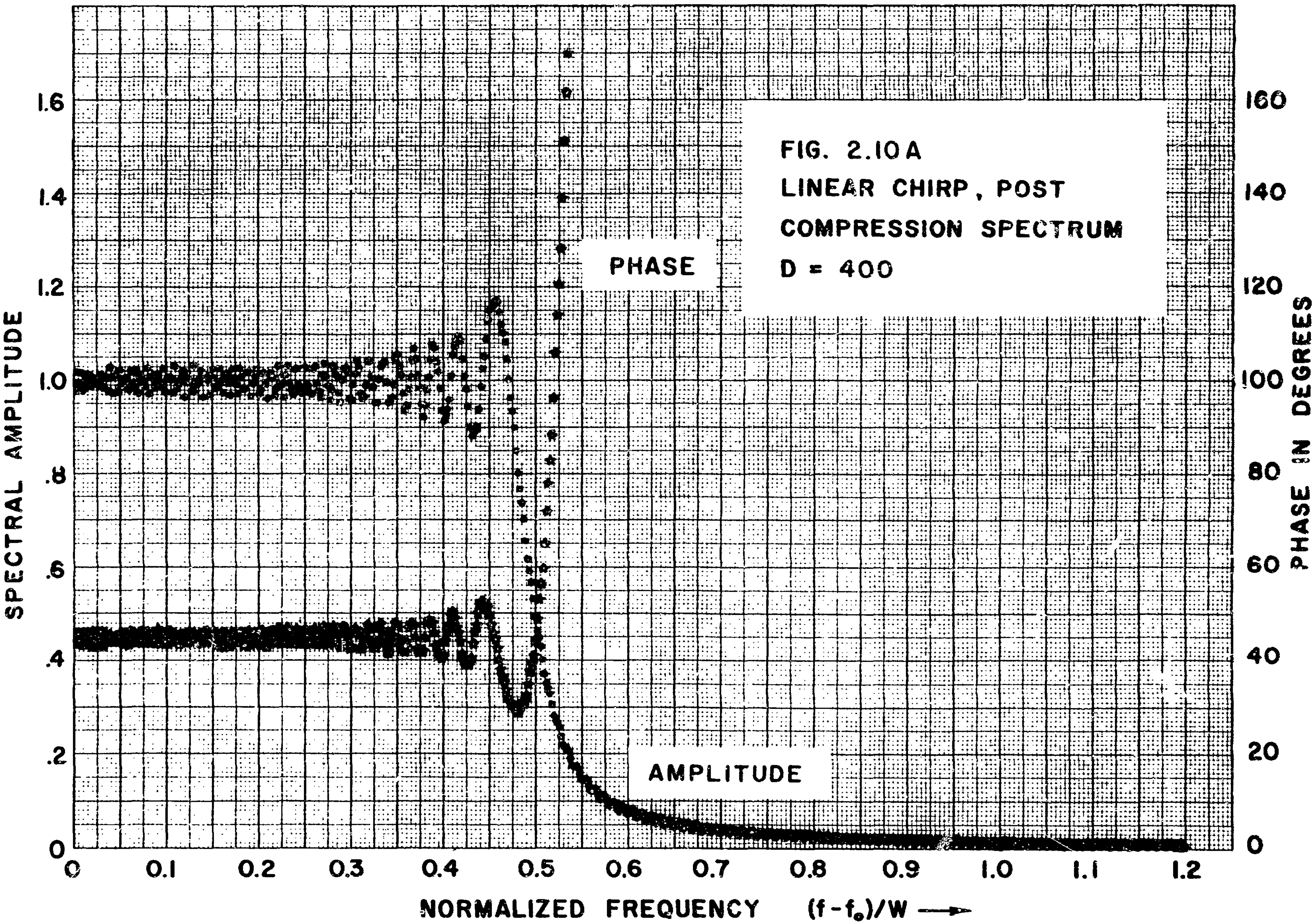


FIG. 2.10A
LINEAR CHIRP, POST
COMPRESSION SPECTRUM
D = 400
PHASE
AMPLITUDE
SPECTRAL AMPLITUDE
PHASE IN DEGREES
NORMALIZED FREQUENCY (f-f₀)/W
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
20
40
60
80
100
120
140
160

1-8 D=400.0

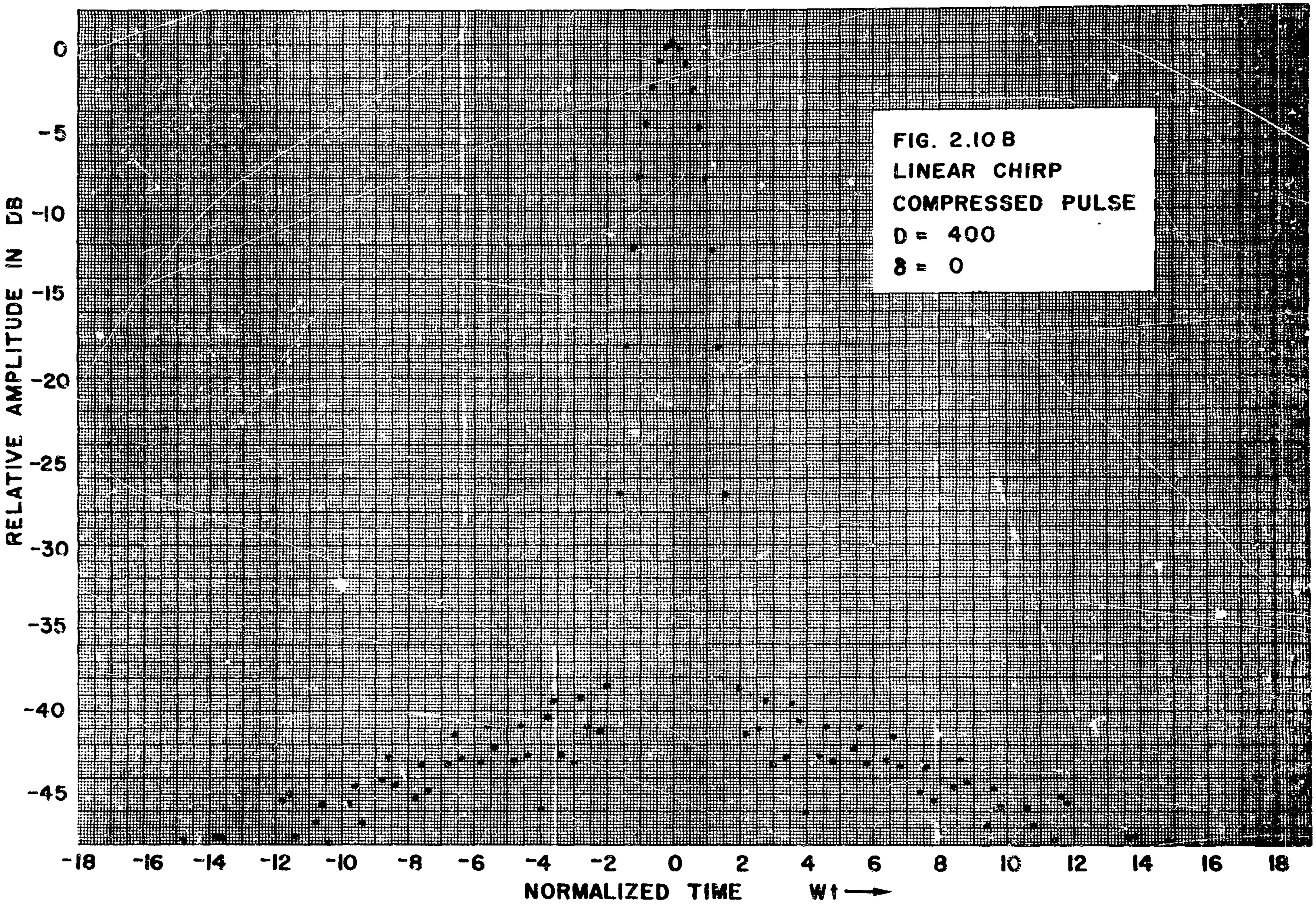


FIG. 2.10 B
LINEAR CHIRP
COMPRESSED PULSE
D = 400
δ = 0
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

10-C D = 400.0

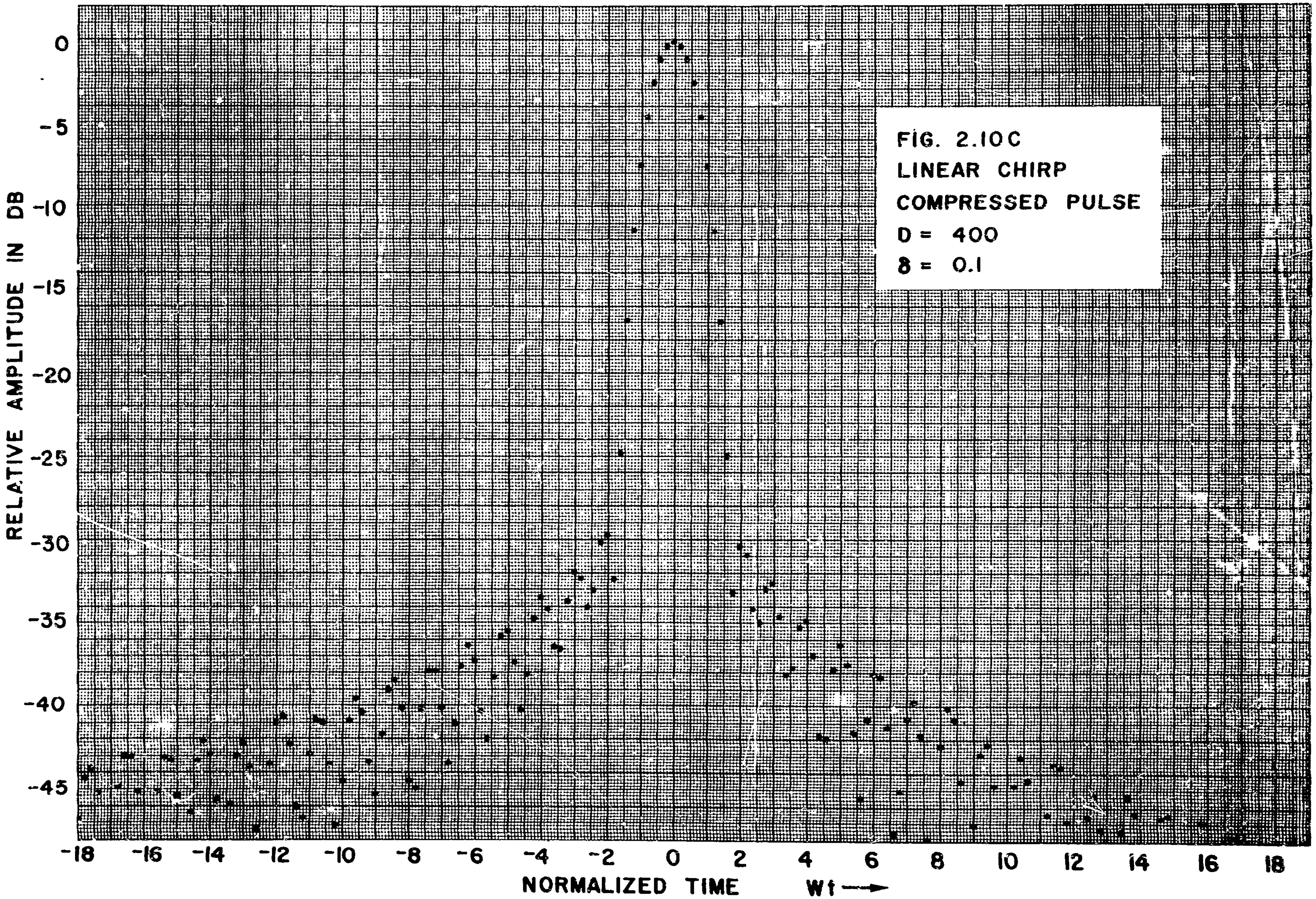


FIG. 2.10C
LINEAR CHIRP
COMPRESSED PULSE
D = 400
δ = 0.1
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt →

D = 400.0

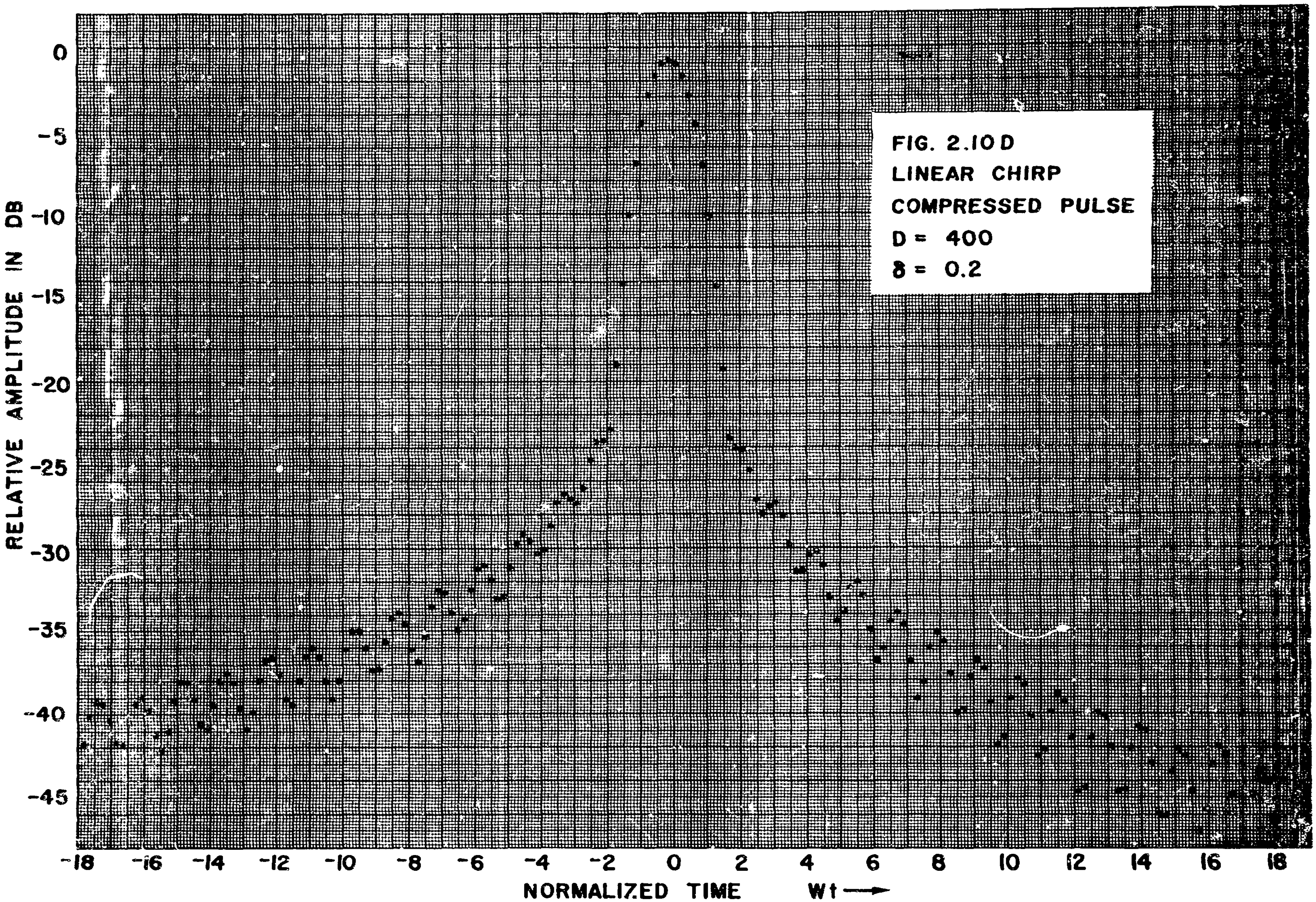


FIG. 2.10 D
LINEAR CHIRP
COMPRESSED PULSE
D = 400
δ = 0.2
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

E D=400

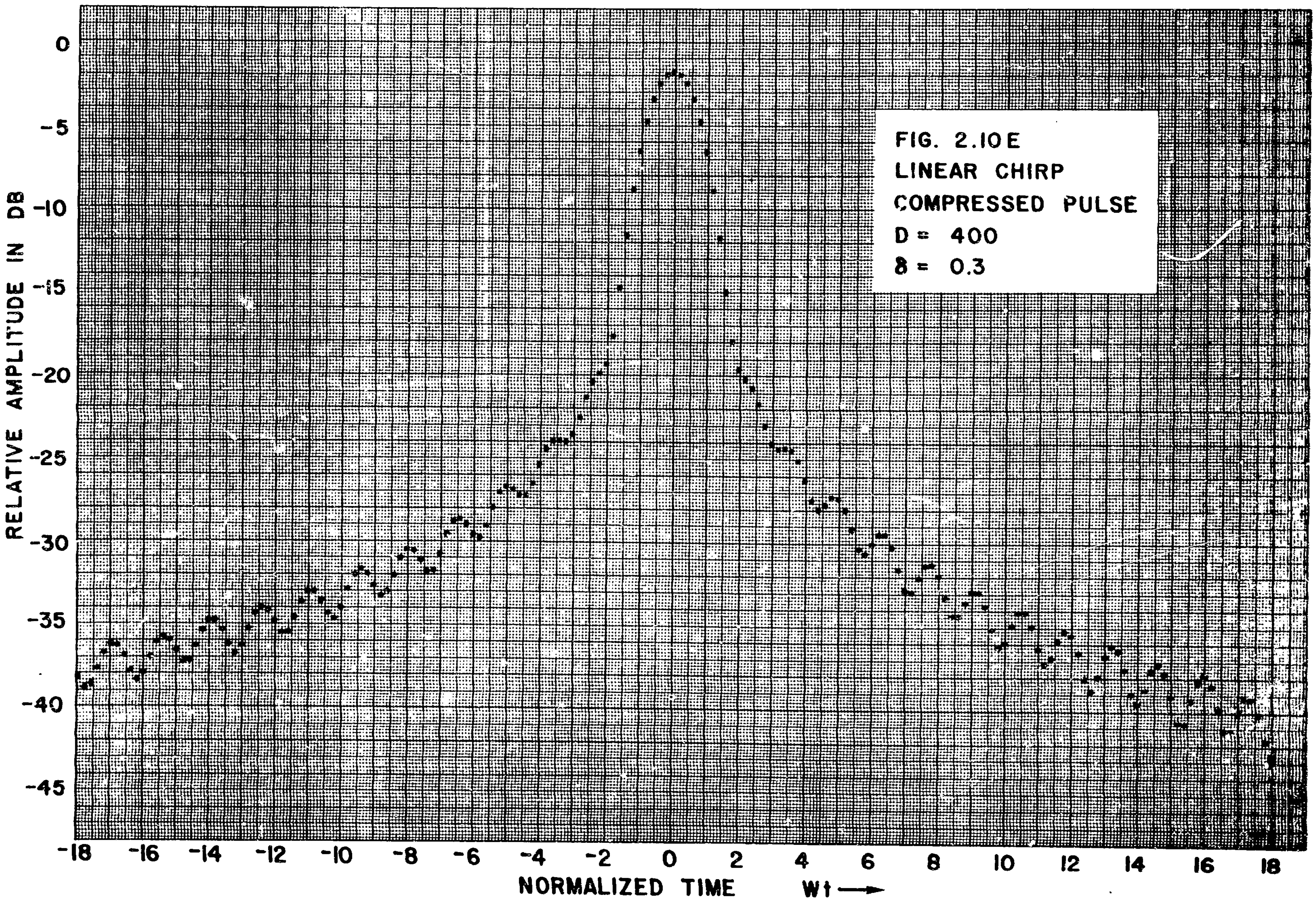


FIG. 2.10E
LINEAR CHIRP
COMPRESSED PULSE
D = 400
δ = 0.3

D-4070

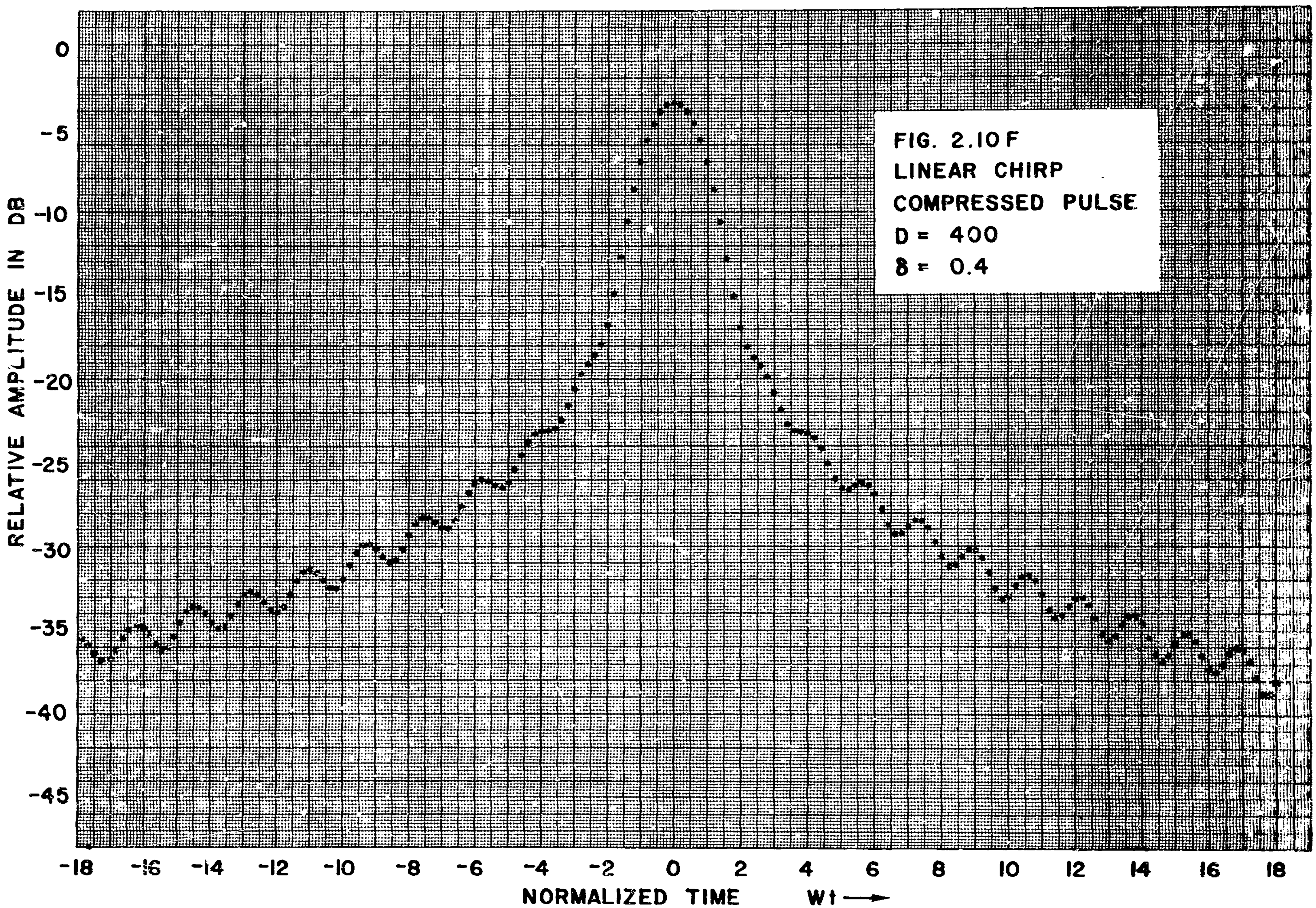


FIG. 2.10F
LINEAR CHIRP
COMPRESSED PULSE
D = 400
δ = 0.4
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt →

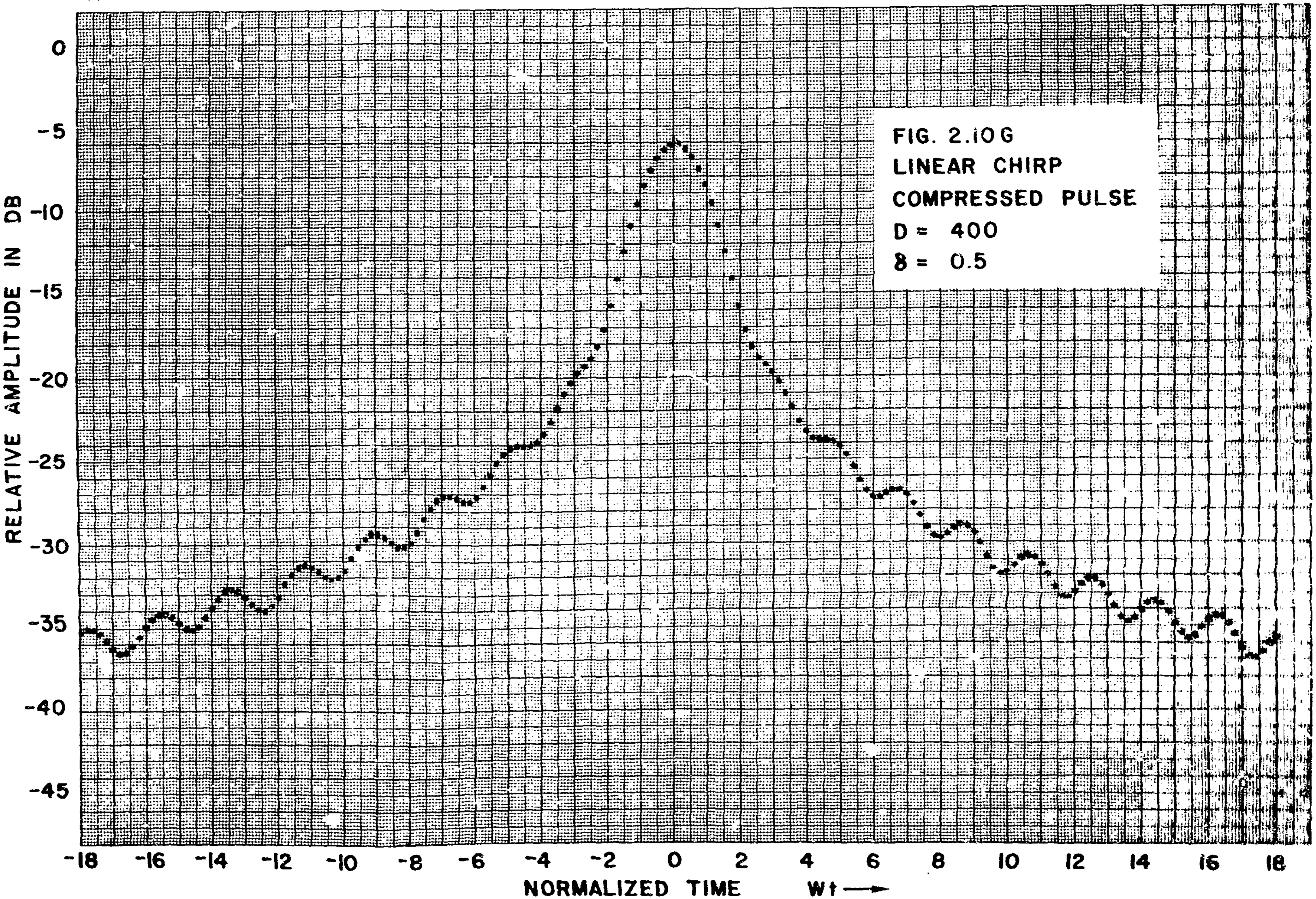
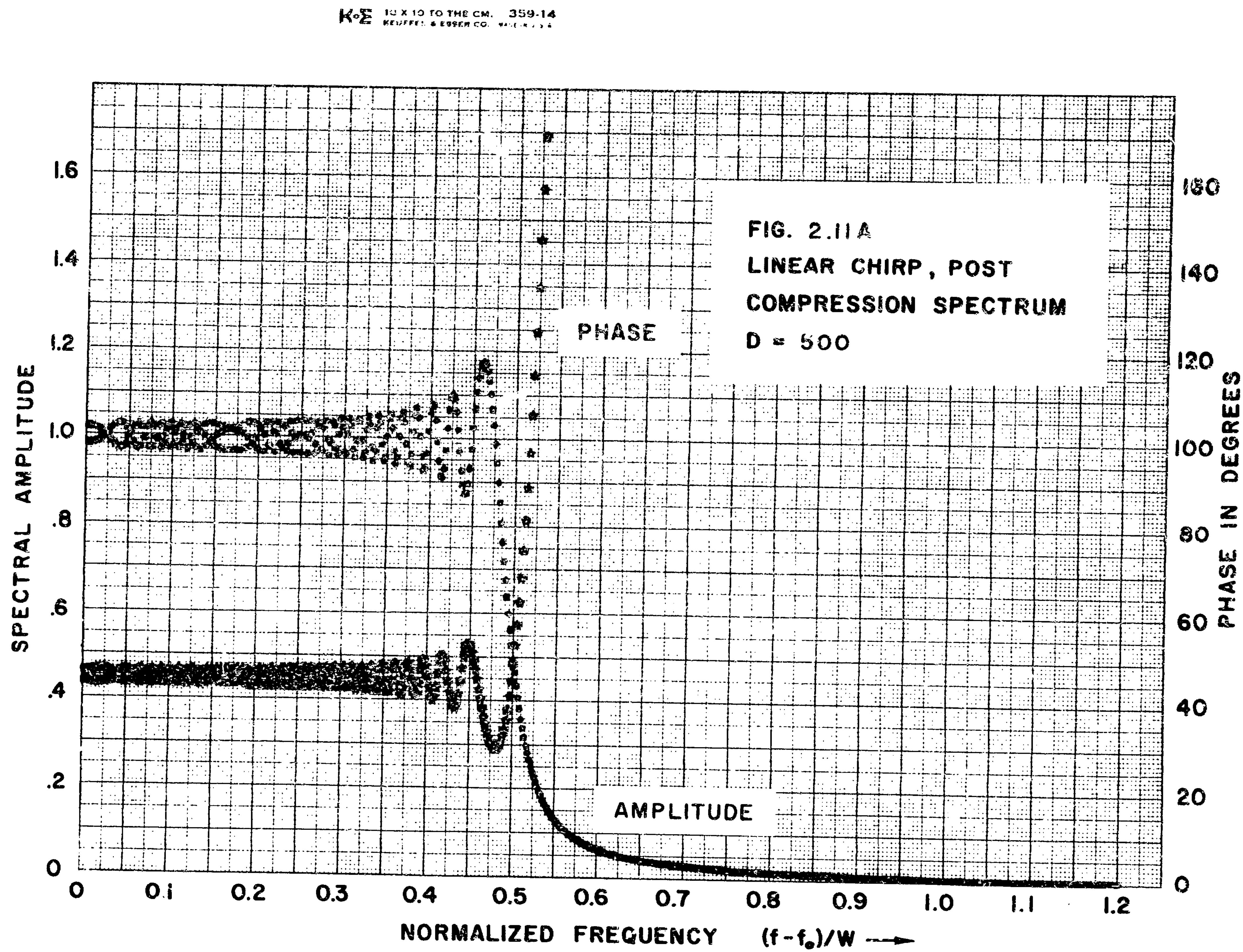


FIG. 2.10G
LINEAR CHIRP
COMPRESSED PULSE
D = 400
δ = 0.5



K&E 10 X 10 TO THE CM. 359-14
FIG. 2.11A
LINEAR CHIRP, POST
COMPRESSION SPECTRUM
D = 500
PHASE
AMPLITUDE
SPECTRAL AMPLITUDE
PHASE IN DEGREES
NORMALIZED FREQUENCY (f - f₀)/W
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
0
20
40
60
80
100
120
140
180
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2

11-B

D= 1000

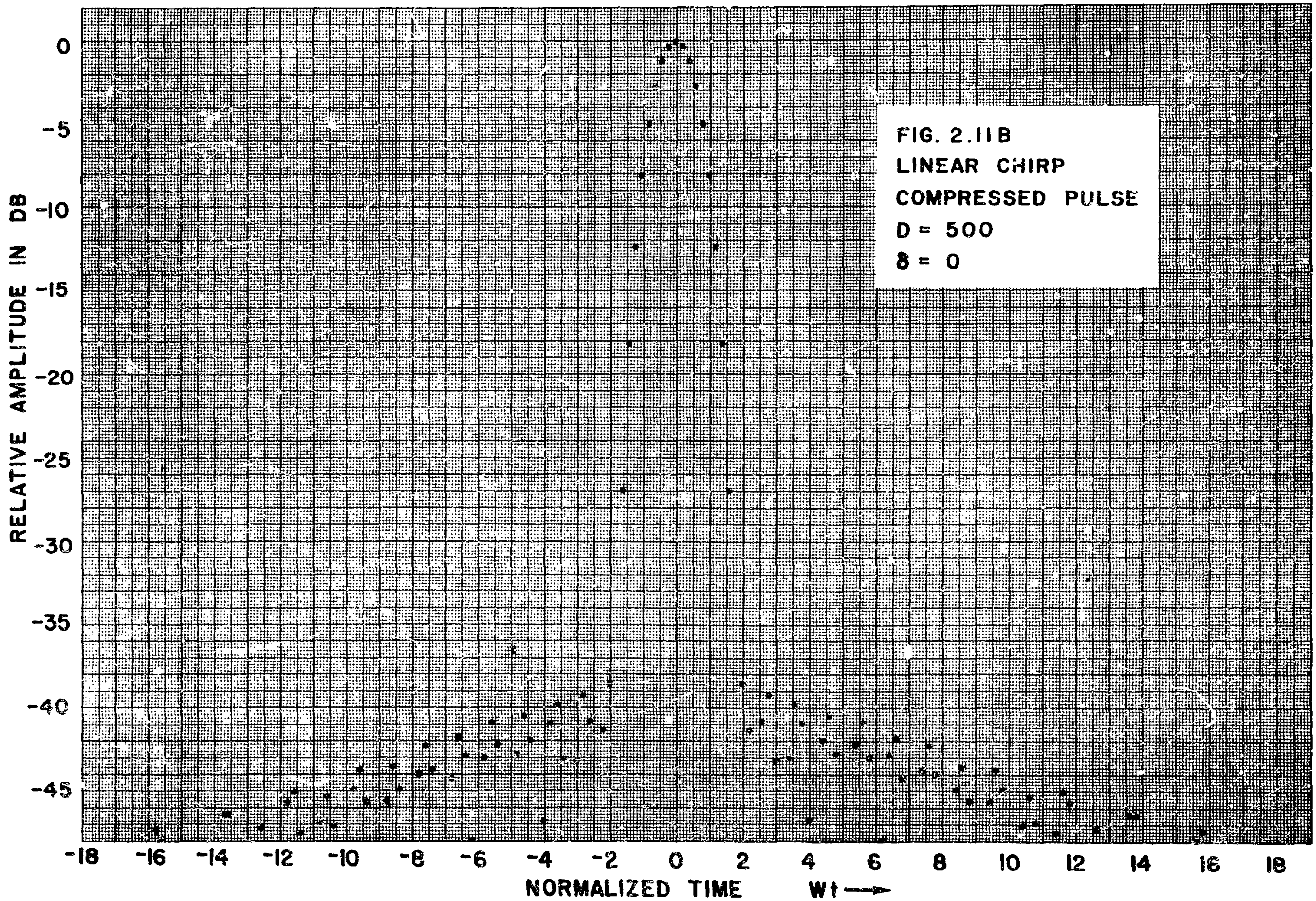
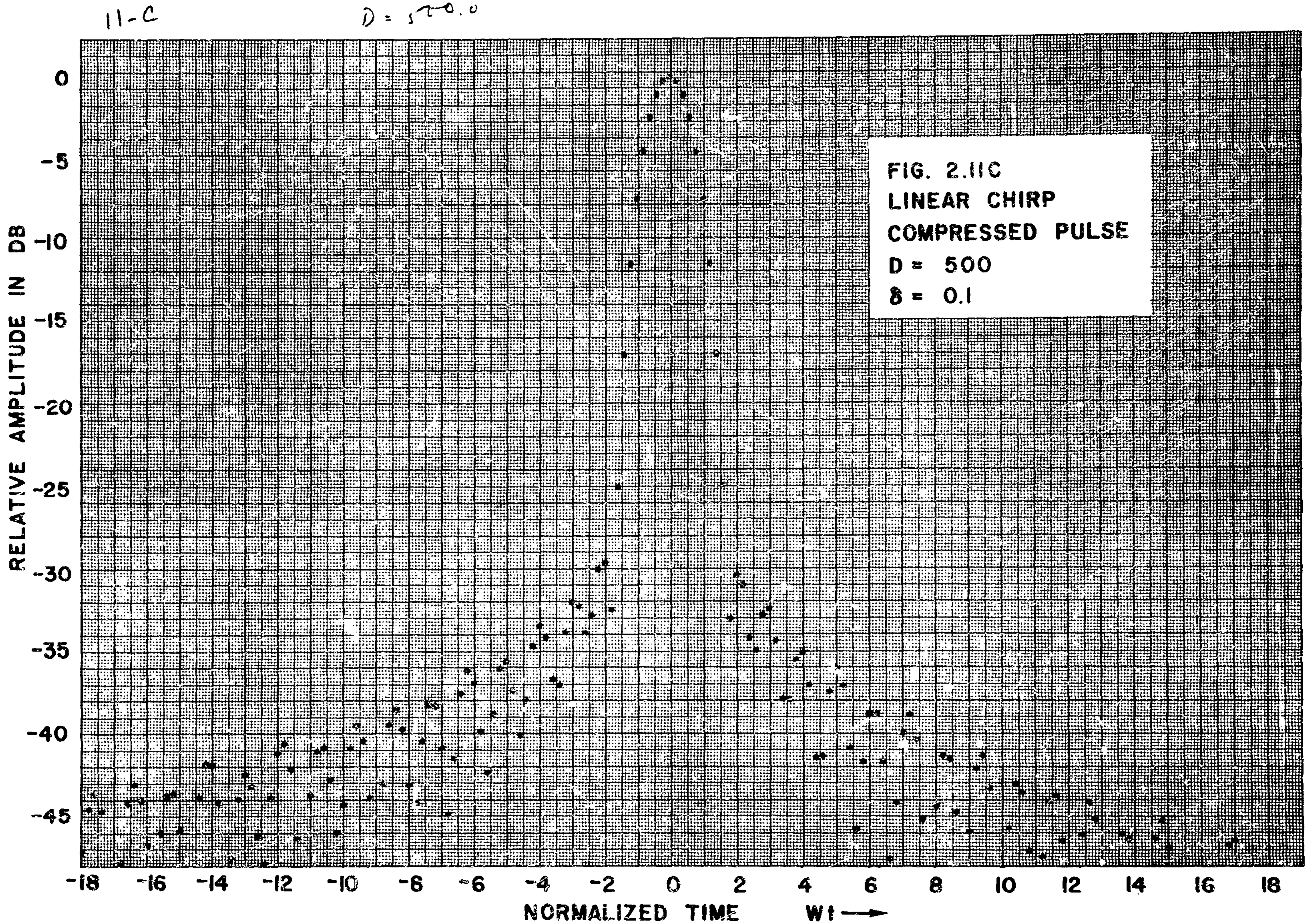


FIG. 2.11B
LINEAR CHIRP
COMPRESSED PULSE
D = 500
8 = 0
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt →


11-C
D = 500.0
FIG. 2.11C
LINEAR CHIRP
COMPRESSED PULSE
D = 500
δ = 0.1
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

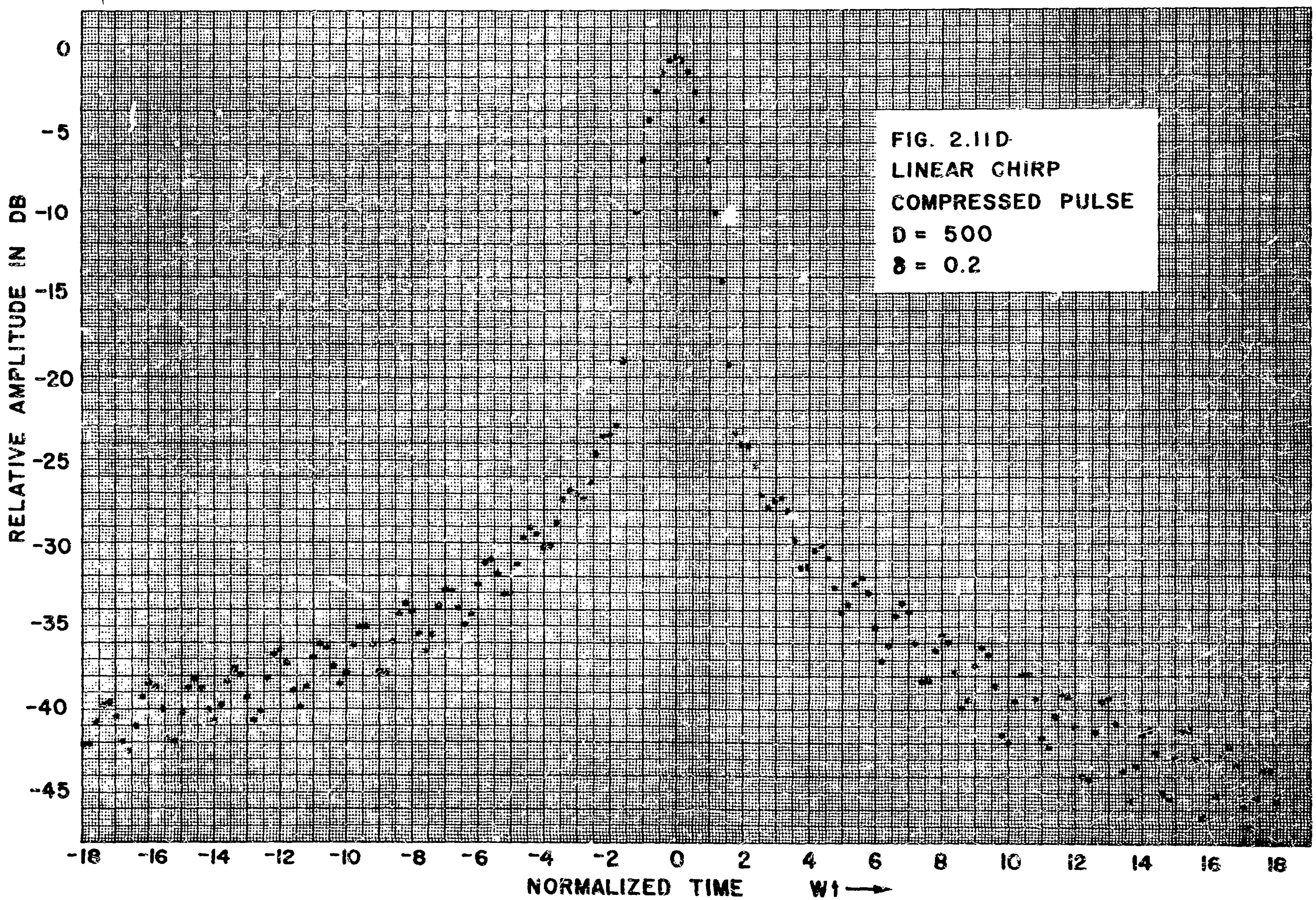


FIG. 2.11D
LINEAR CHIRP
COMPRESSED PULSE
D = 500
δ = 0.2
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

11-E D: 500 0

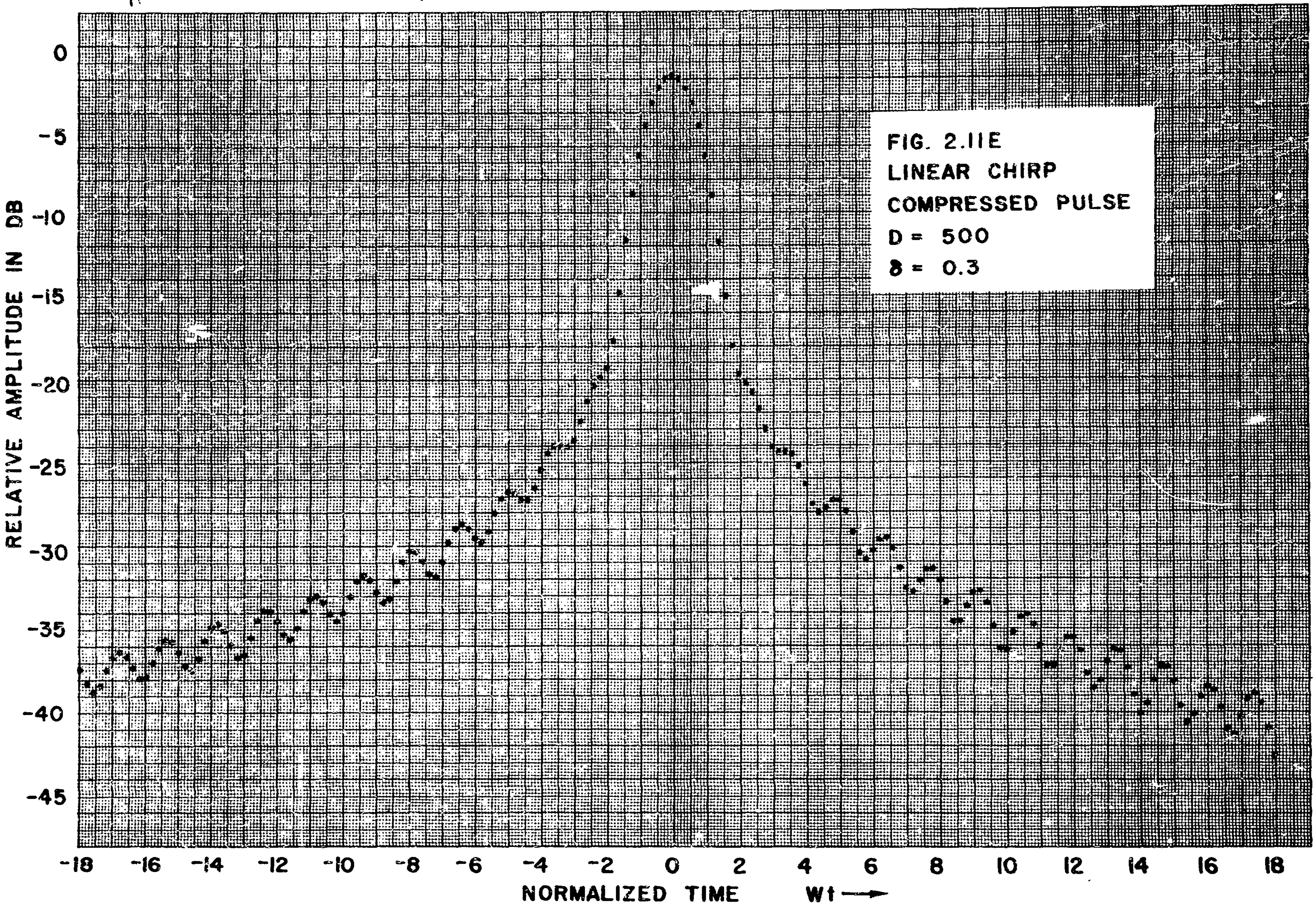


FIG. 2.11E
LINEAR CHIRP
COMPRESSED PULSE
D = 500
δ = 0.3

11-F

D = 500.0

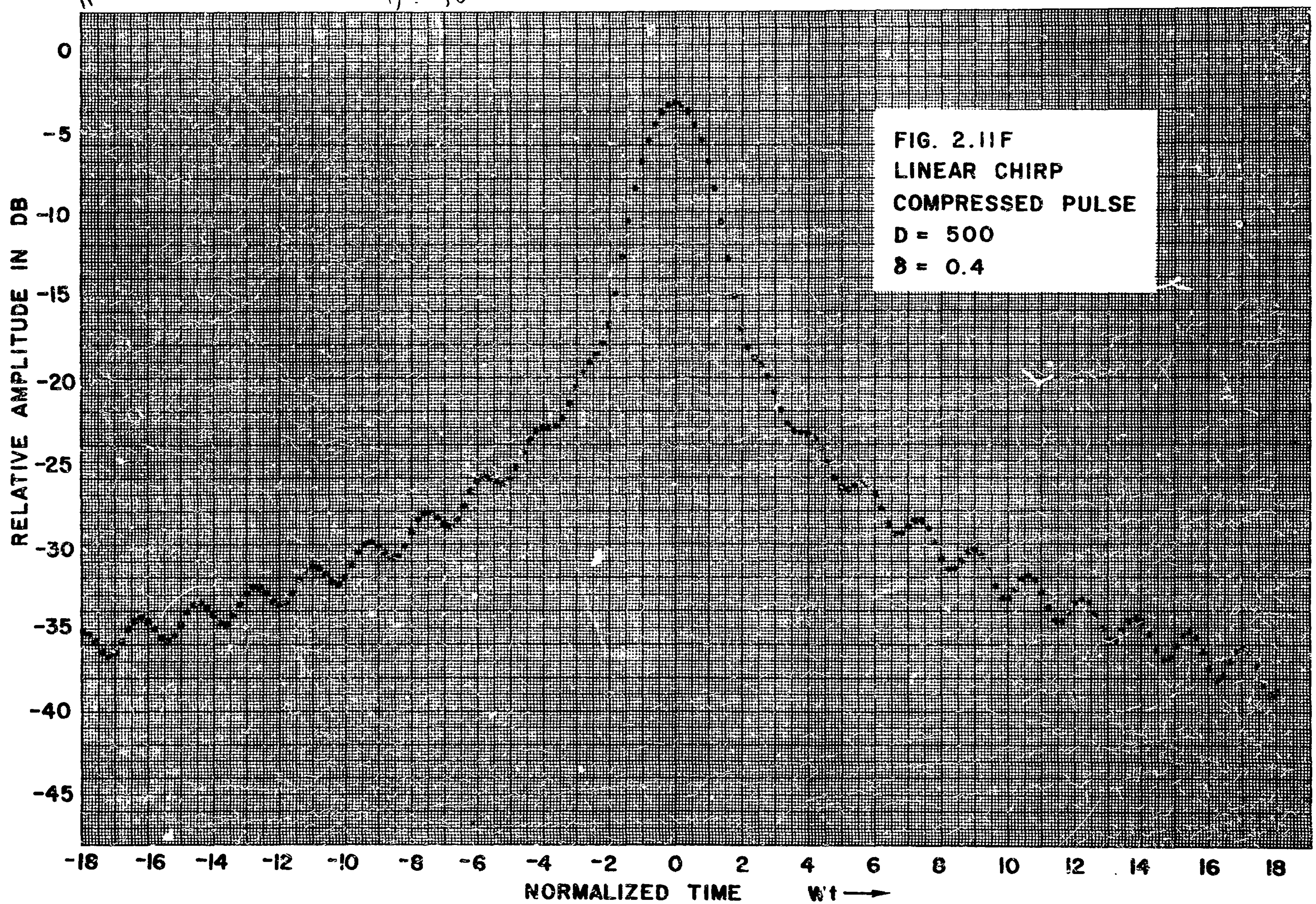


FIG. 2.11F
LINEAR CHIRP
COMPRESSED PULSE
D = 500
δ = 0.4
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt

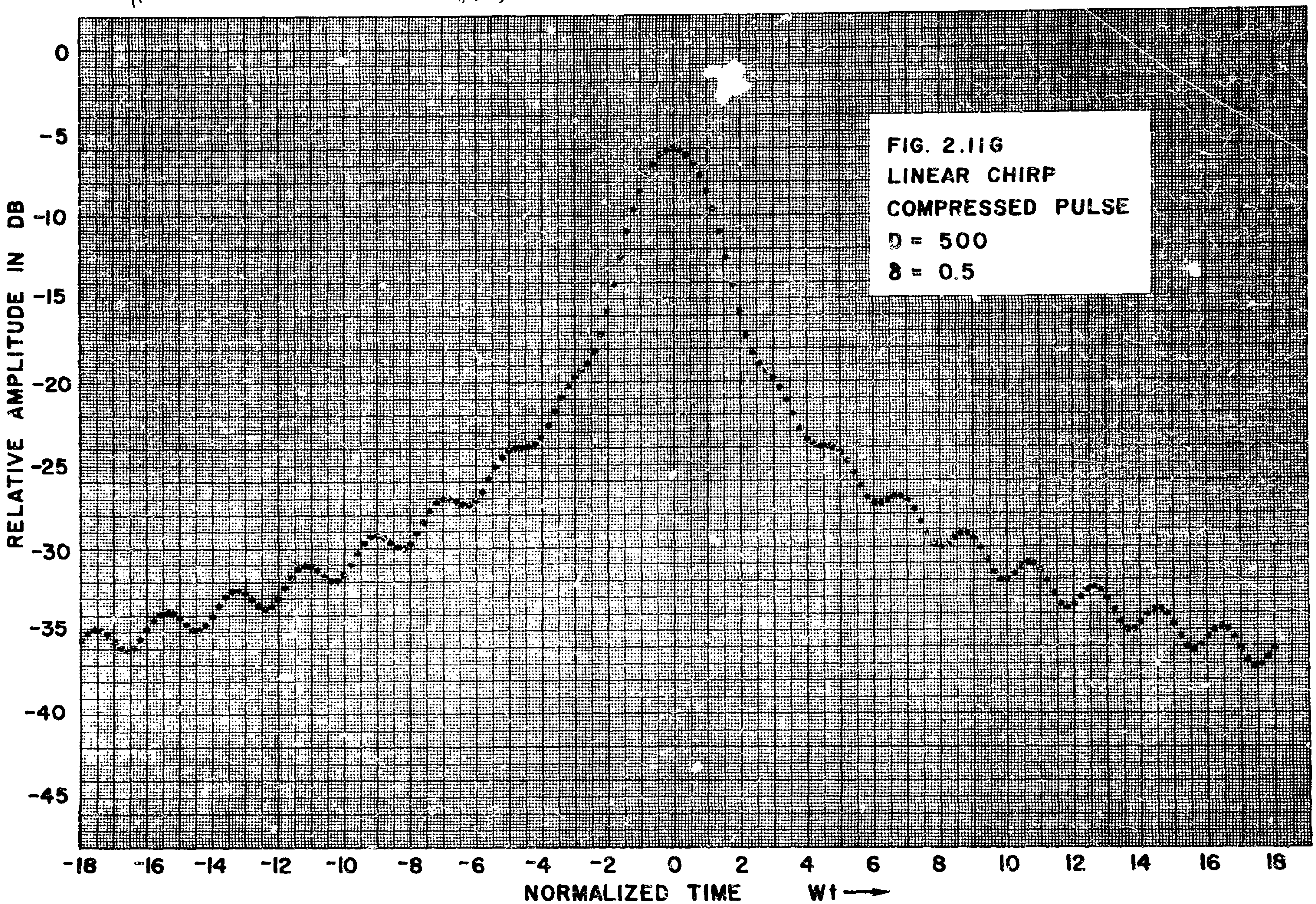


FIG. 2.11G
LINEAR CHIRP
COMPRESSED PULSE
D = 500
δ = 0.5

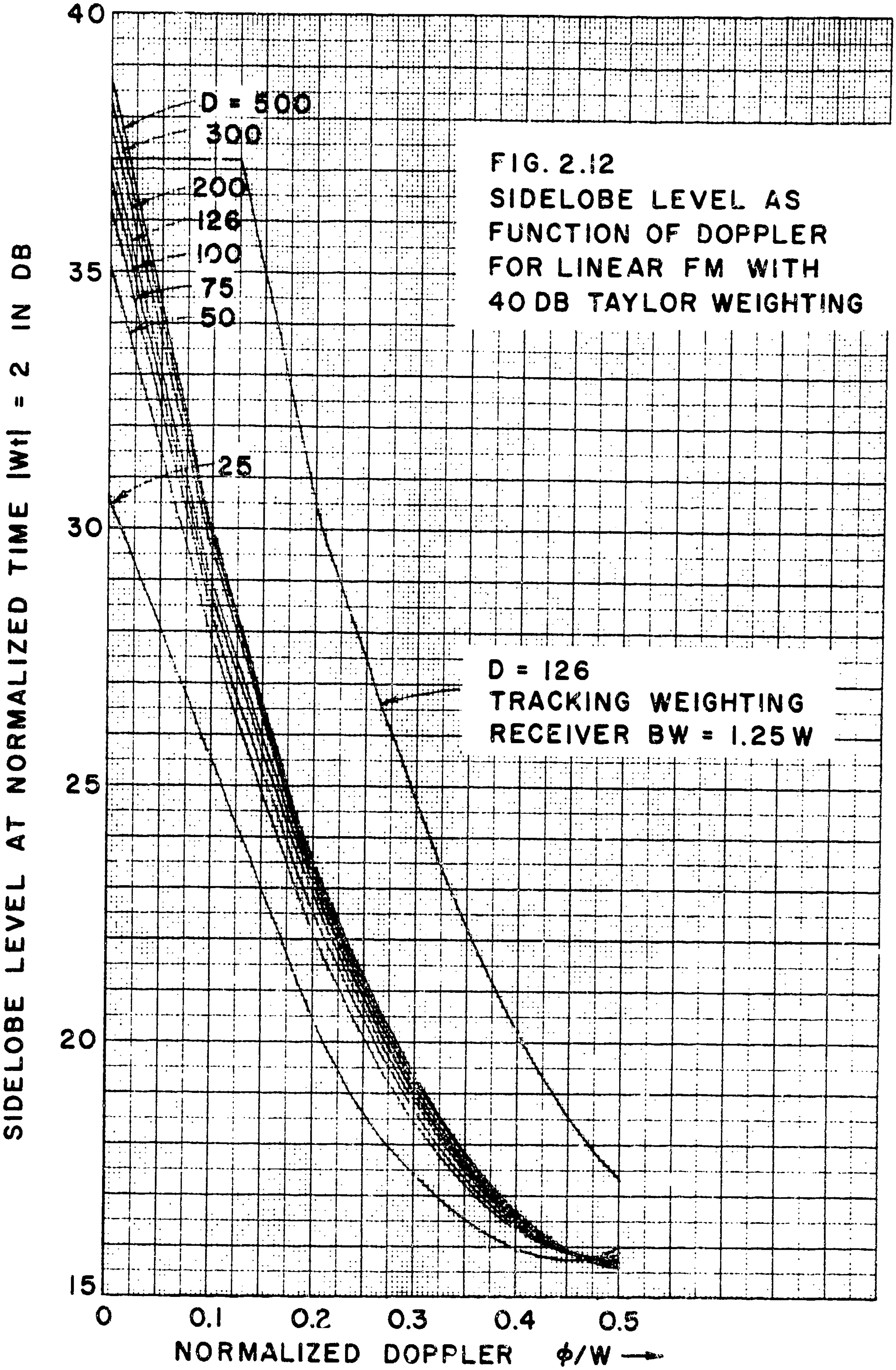


FIG. 2.12
SIDELOBE LEVEL AS FUNCTION OF DOPPLER FOR LINEAR FM WITH 40 DB TAYLOR WEIGHTING

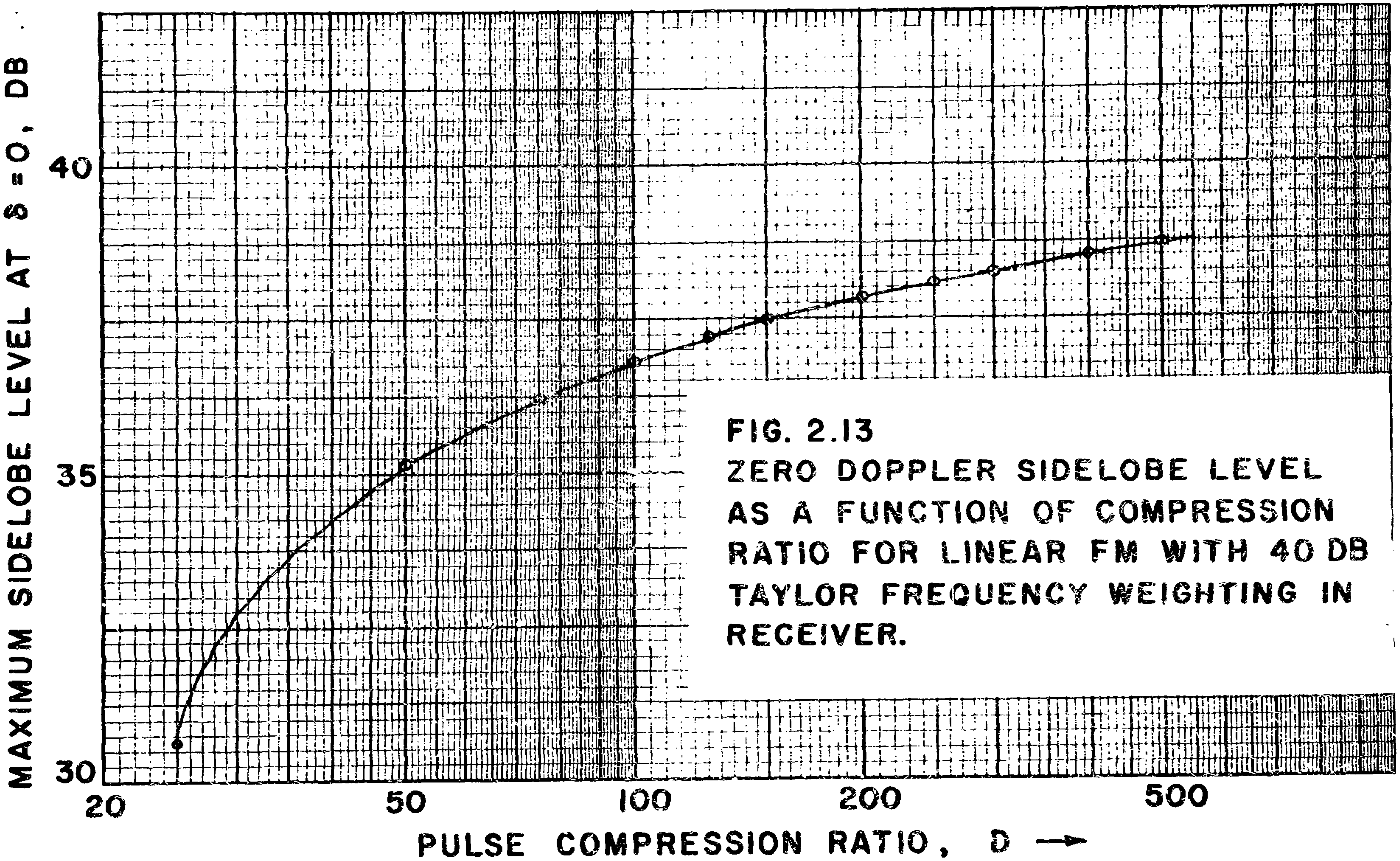


FIG. 2.13
ZERO DOPPLER SIDELOBE LEVEL AS A FUNCTION OF COMPRESSION RATIO FOR LINEAR FM WITH 40 DB TAYLOR FREQUENCY WEIGHTING IN RECEIVER.

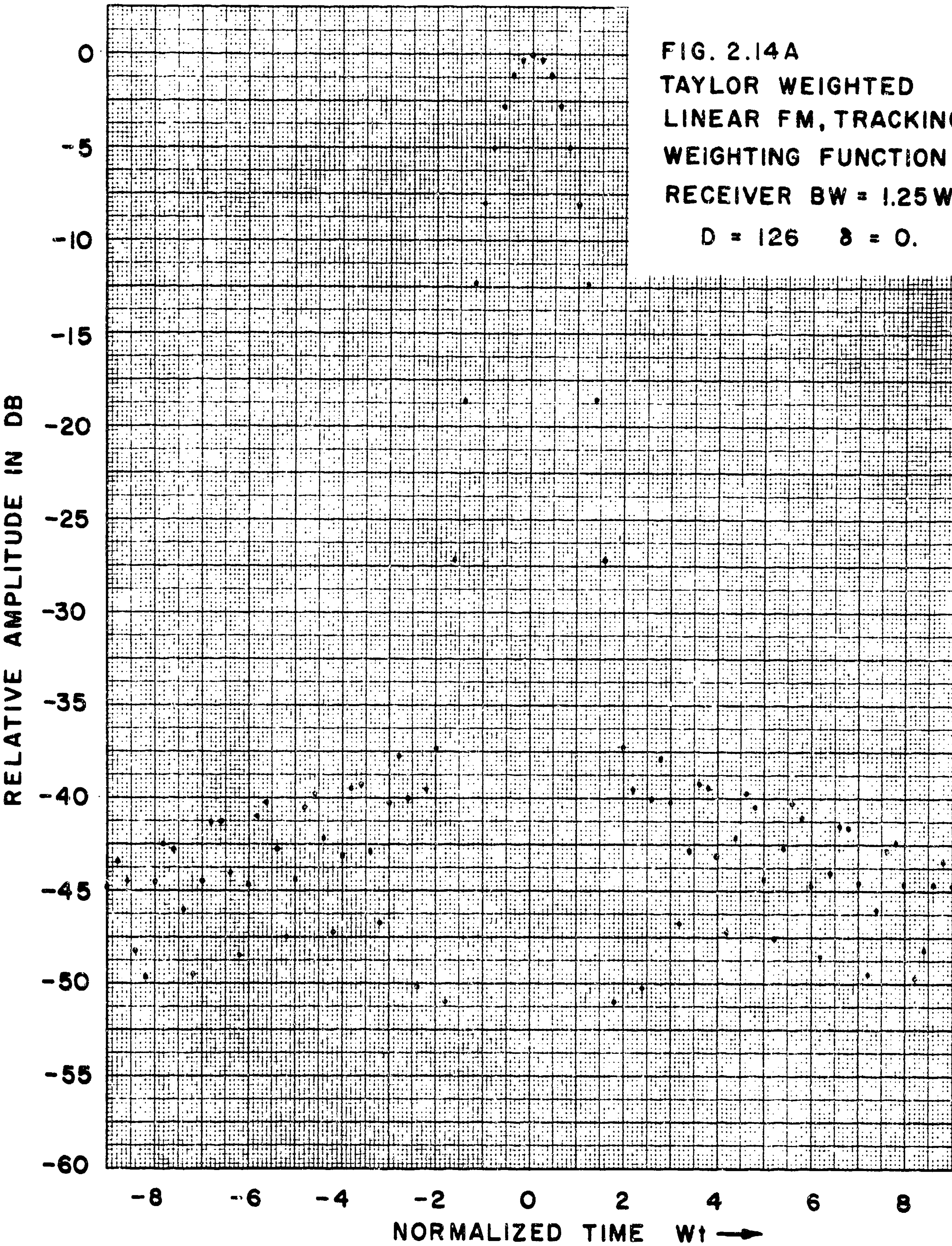


FIG. 2.14A
TAYLOR WEIGHTED
LINEAR FM, TRACKING
WEIGHTING FUNCTION
RECEIVER BW = 1.25 W
D = 126 δ = 0.

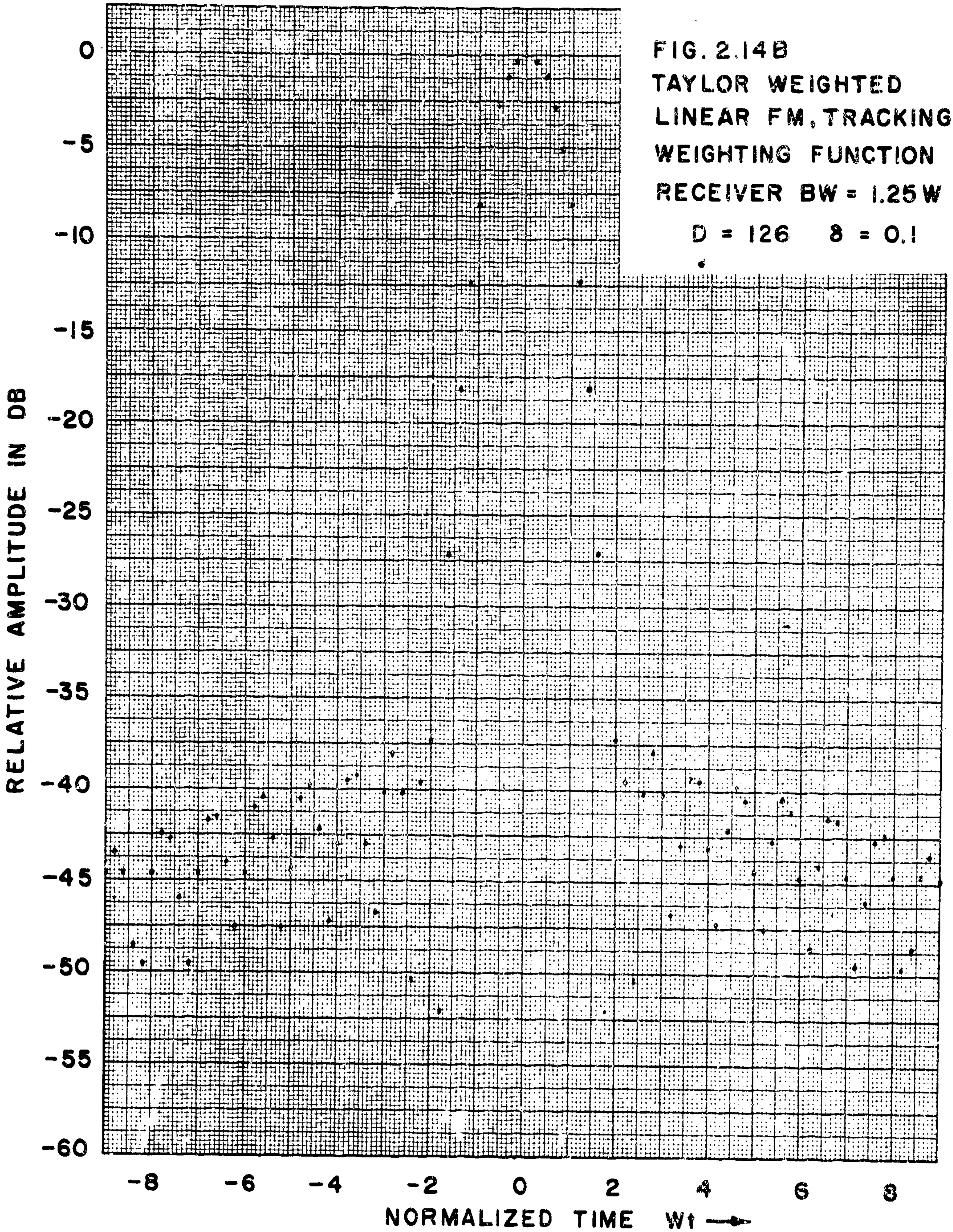


FIG. 2.14B
TAYLOR WEIGHTED
LINEAR FM, TRACKING
WEIGHTING FUNCTION
RECEIVER BW = 1.25 W
D = 126 δ = 0.1
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
-55
-60
-8
-6
-4
-2
0
2
4
6
8
NORMALIZED TIME Wt

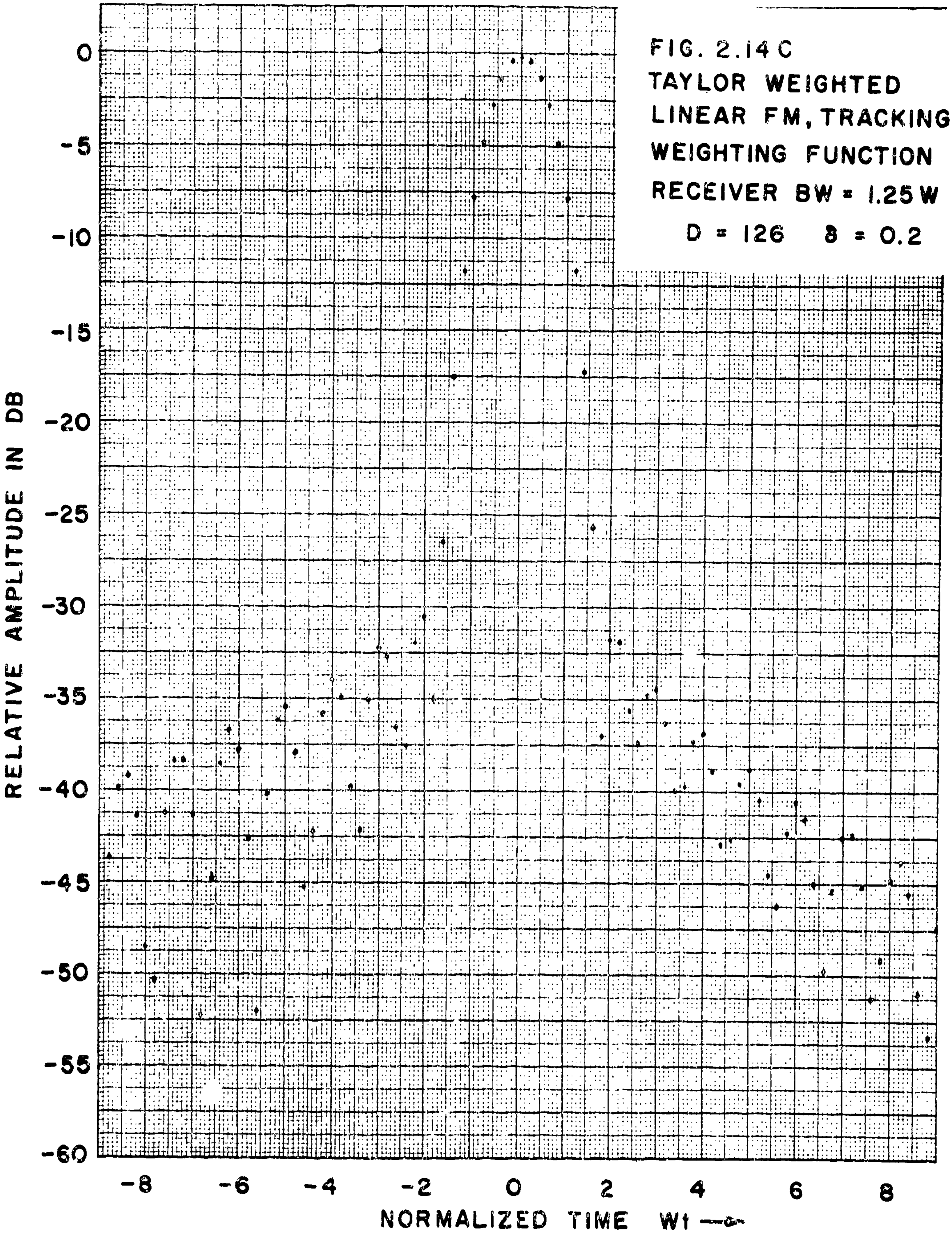


FIG. 2.14C
TAYLOR WEIGHTED
LINEAR FM, TRACKING
WEIGHTING FUNCTION
RECEIVER BW = 1.25W
D = 126 δ = 0.2

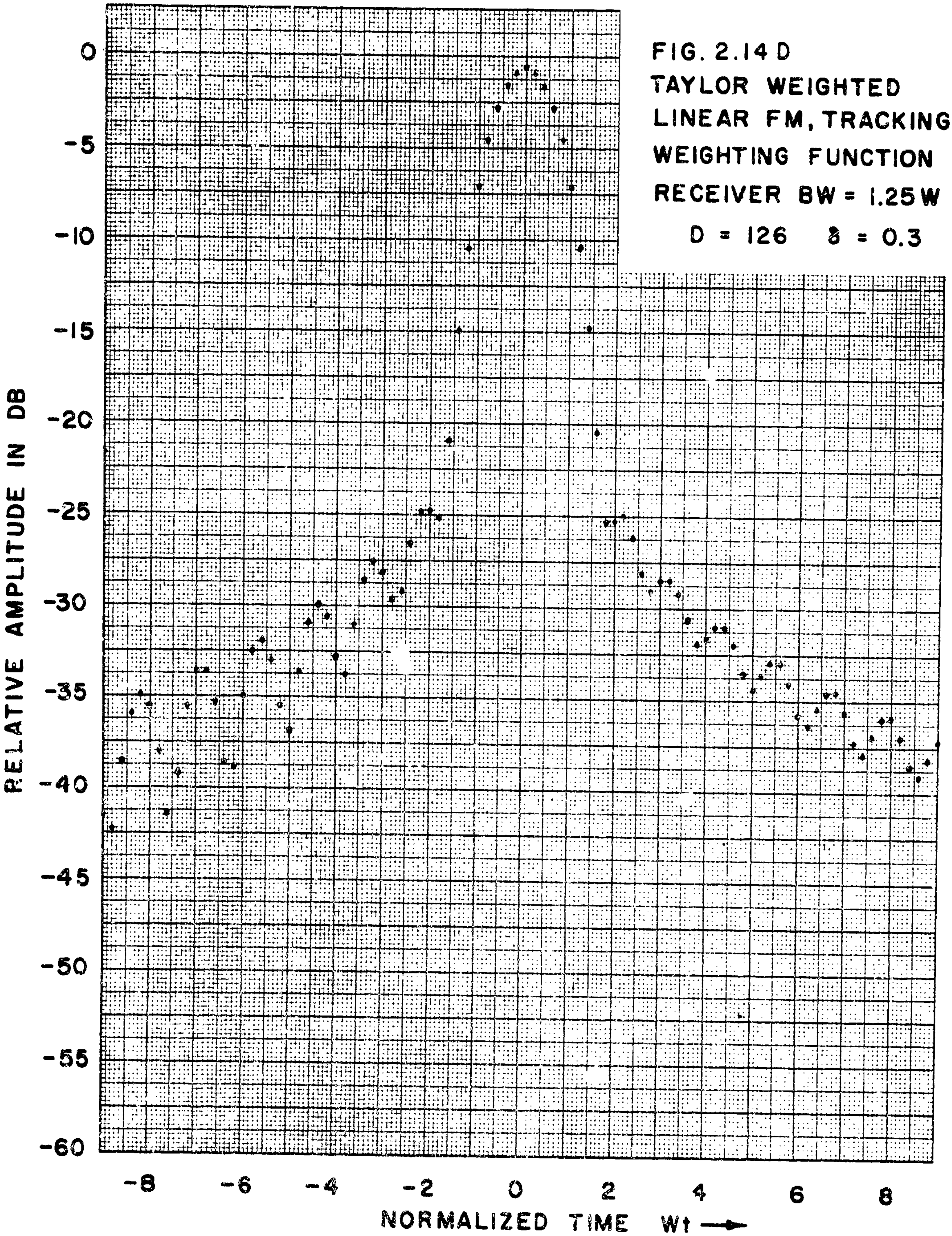


FIG. 2.14 D
TAYLOR WEIGHTED
LINEAR FM, TRACKING
WEIGHTING FUNCTION
RECEIVER BW = 1.25 W
D = 126 δ = 0.3
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
-55
-60
-8
-6
-4
-2
0
2
4
6
8
NORMALIZED TIME Wt →

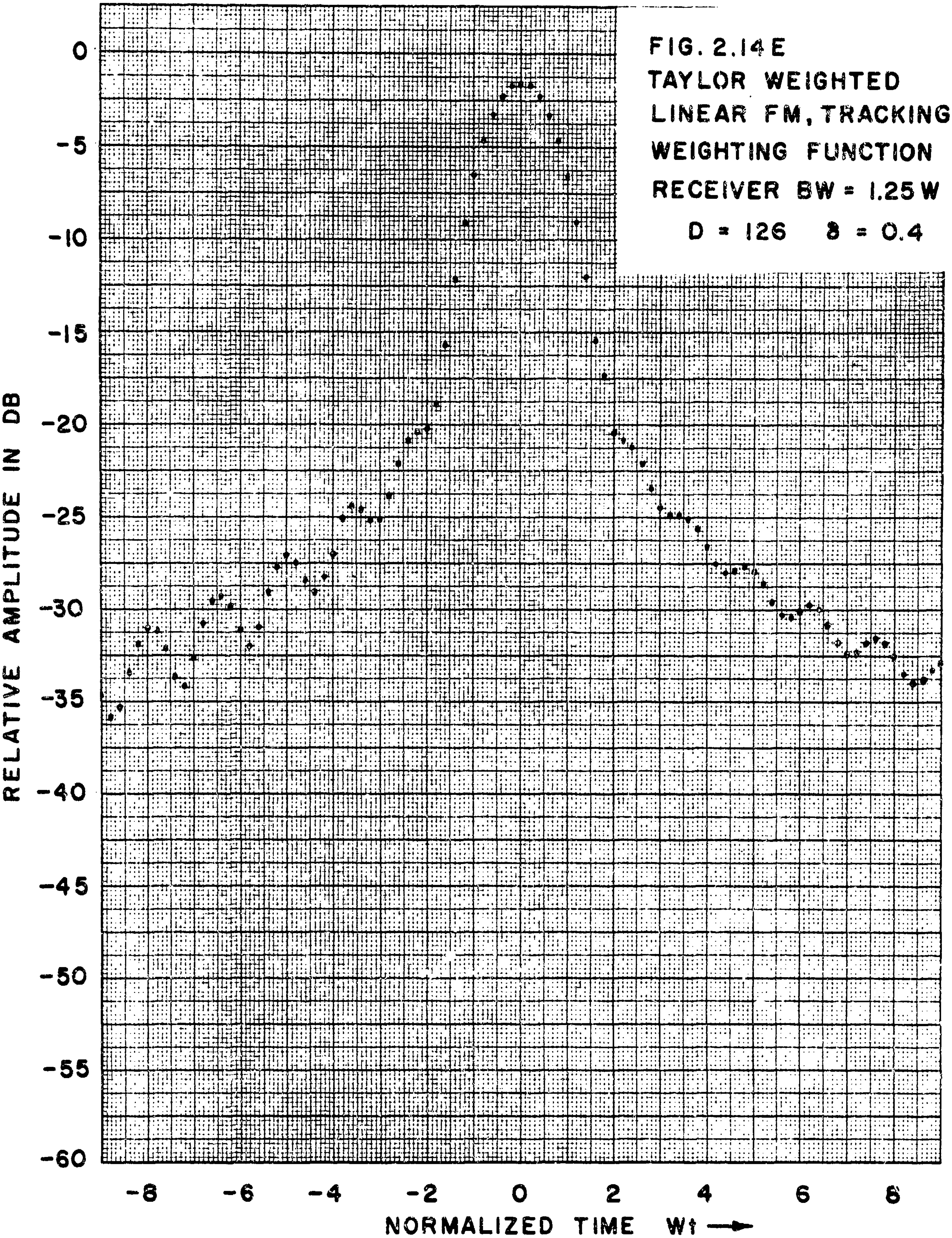


FIG. 2.14E
TAYLOR WEIGHTED
LINEAR FM, TRACKING
WEIGHTING FUNCTION
RECEIVER BW = 1.25 W
D = 126 δ = 0.4
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
-55
-60
-8
-6
-4
-2
0
2
4
6
8
NORMALIZED TIME Wt

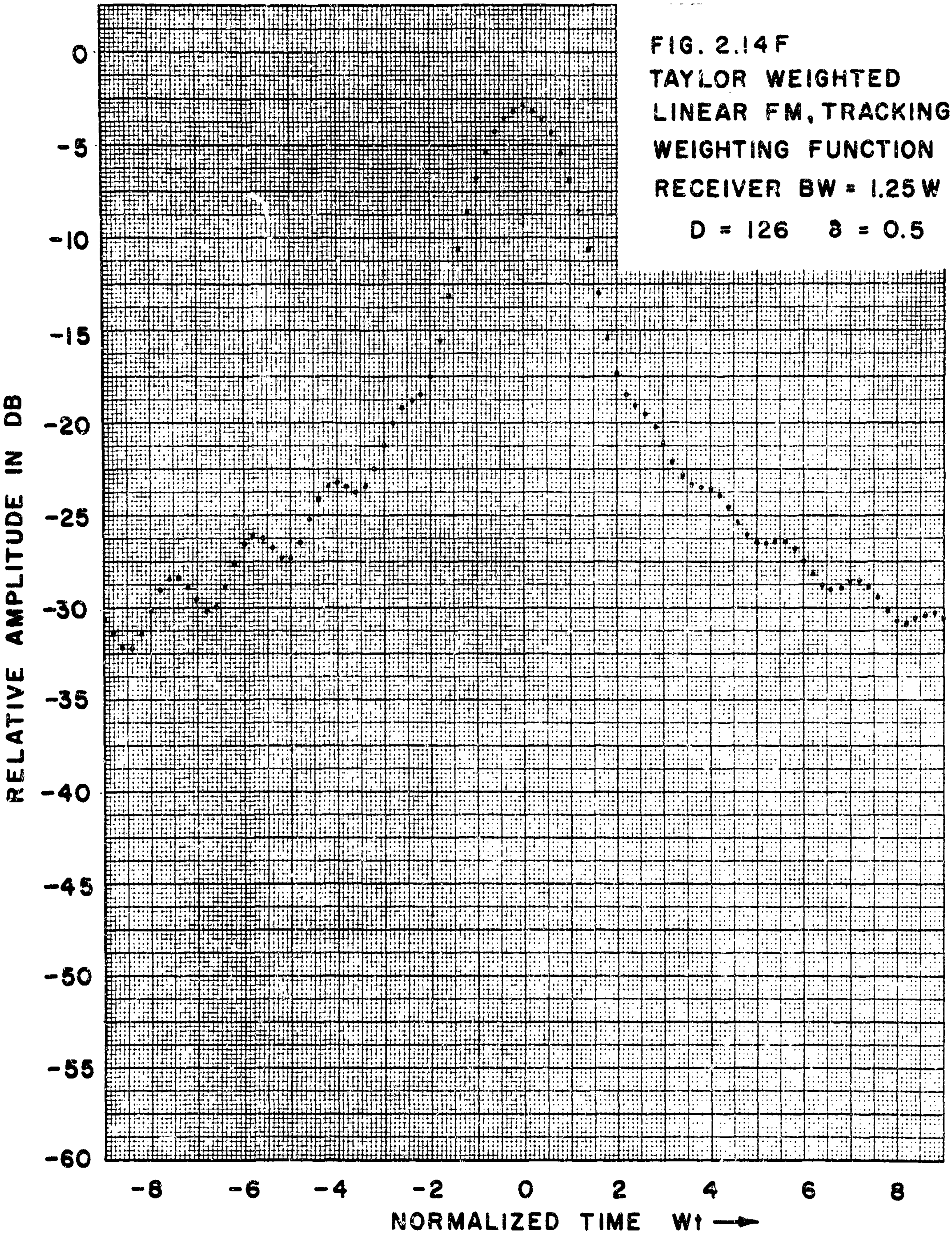


FIG. 2.14F
TAYLOR WEIGHTED
LINEAR FM, TRACKING
WEIGHTING FUNCTION
RECEIVER BW = 1.25 W
D = 126 δ = 0.5

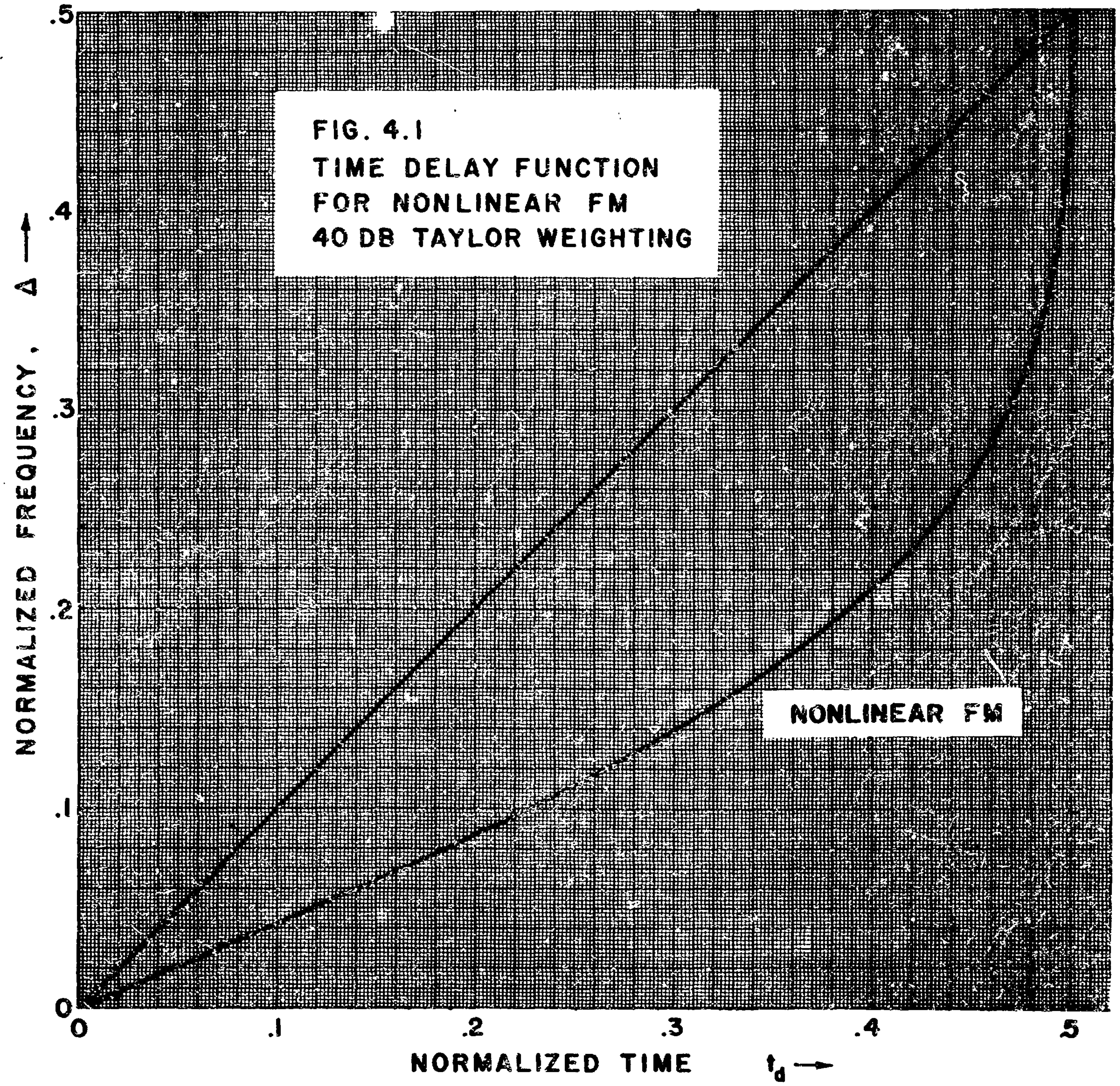


FIG. 4.1
TIME DELAY FUNCTION
FOR NONLINEAR FM
40 DB TAYLOR WEIGHTING

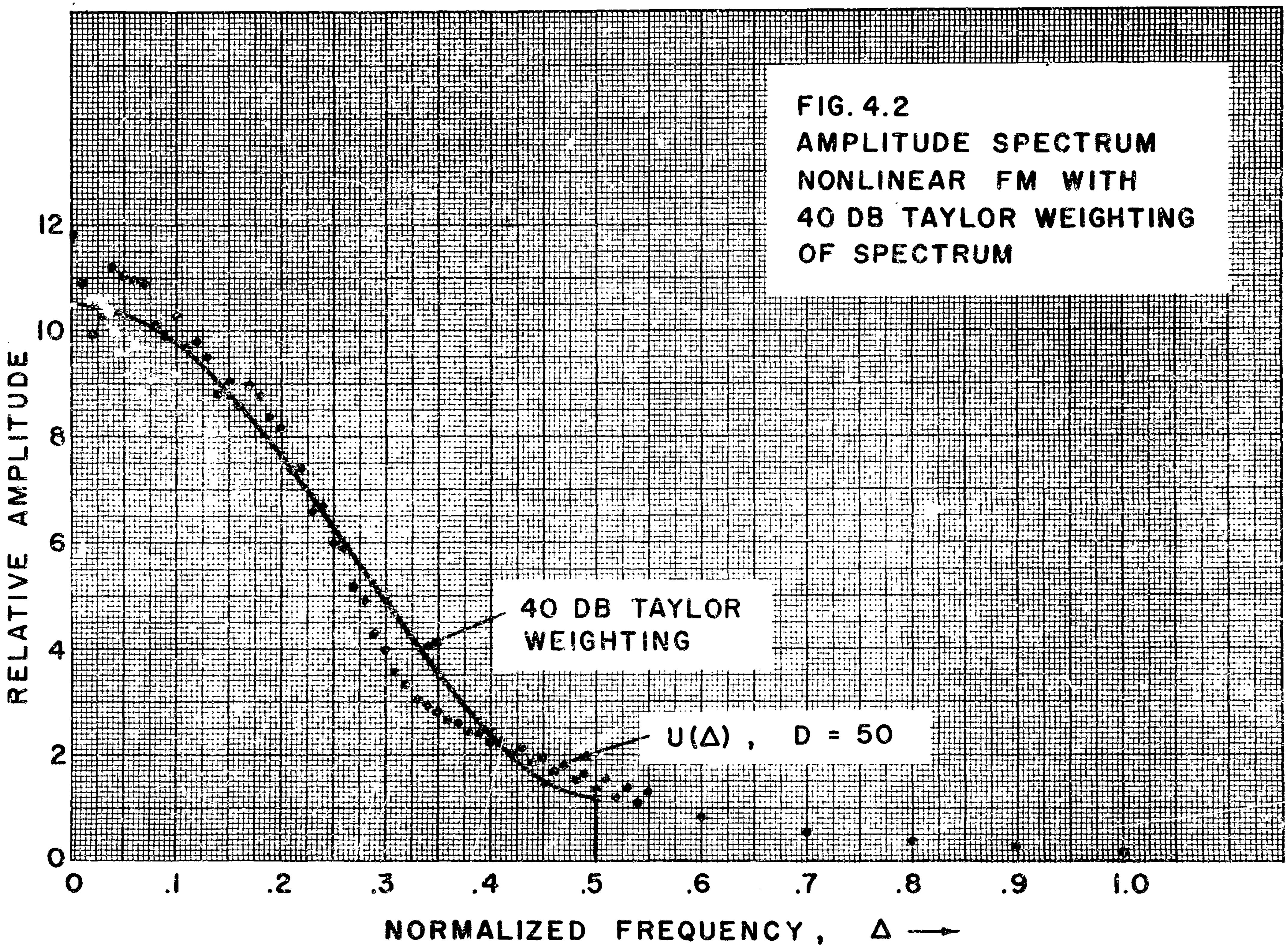


FIG. 4.2
AMPLITUDE SPECTRUM
NONLINEAR FM WITH
40 DB TAYLOR WEIGHTING
OF SPECTRUM

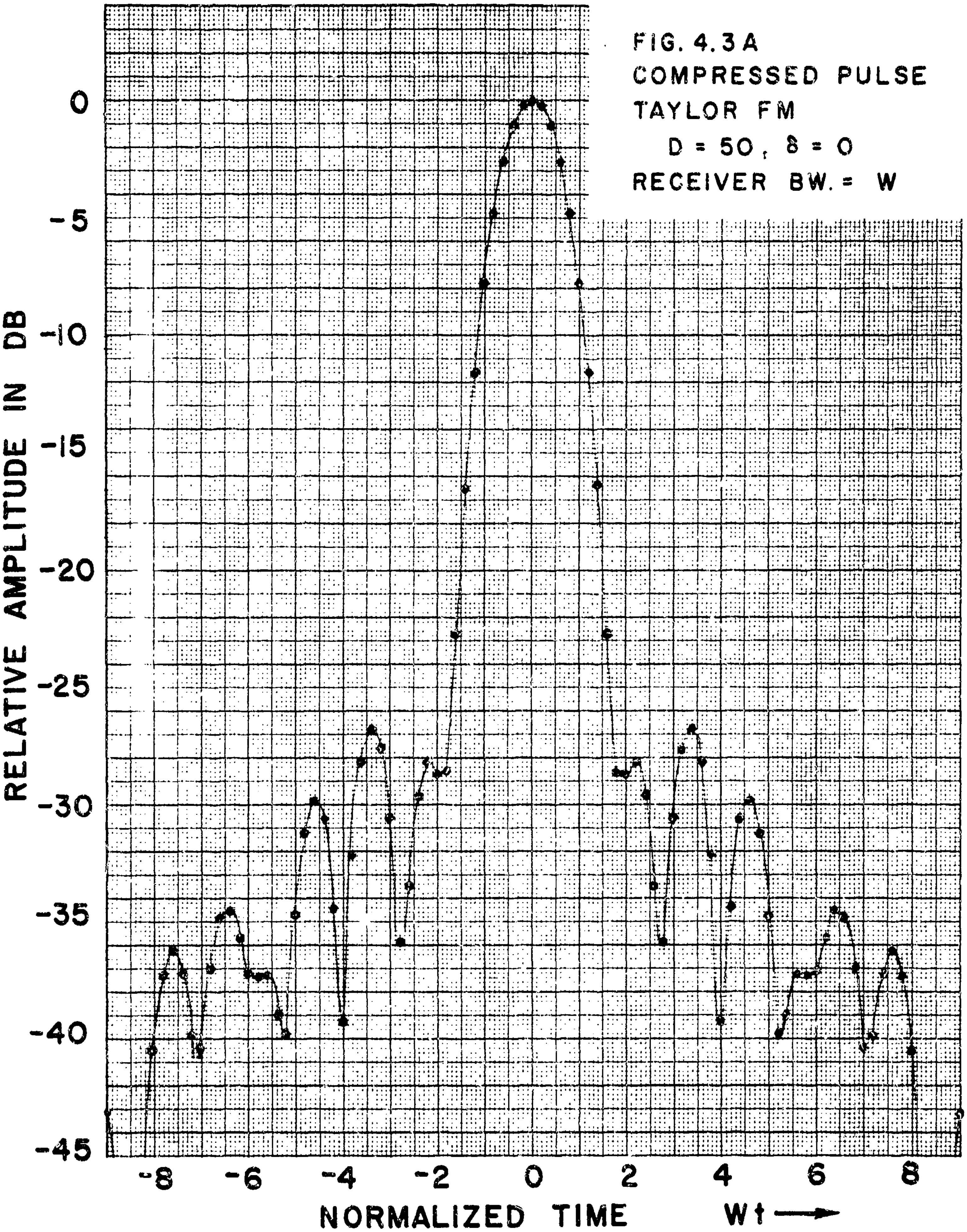


FIG. 4.3A
COMPRESSED PULSE
TAYLOR FM
D = 50, δ = 0
RECEIVER BW. = W

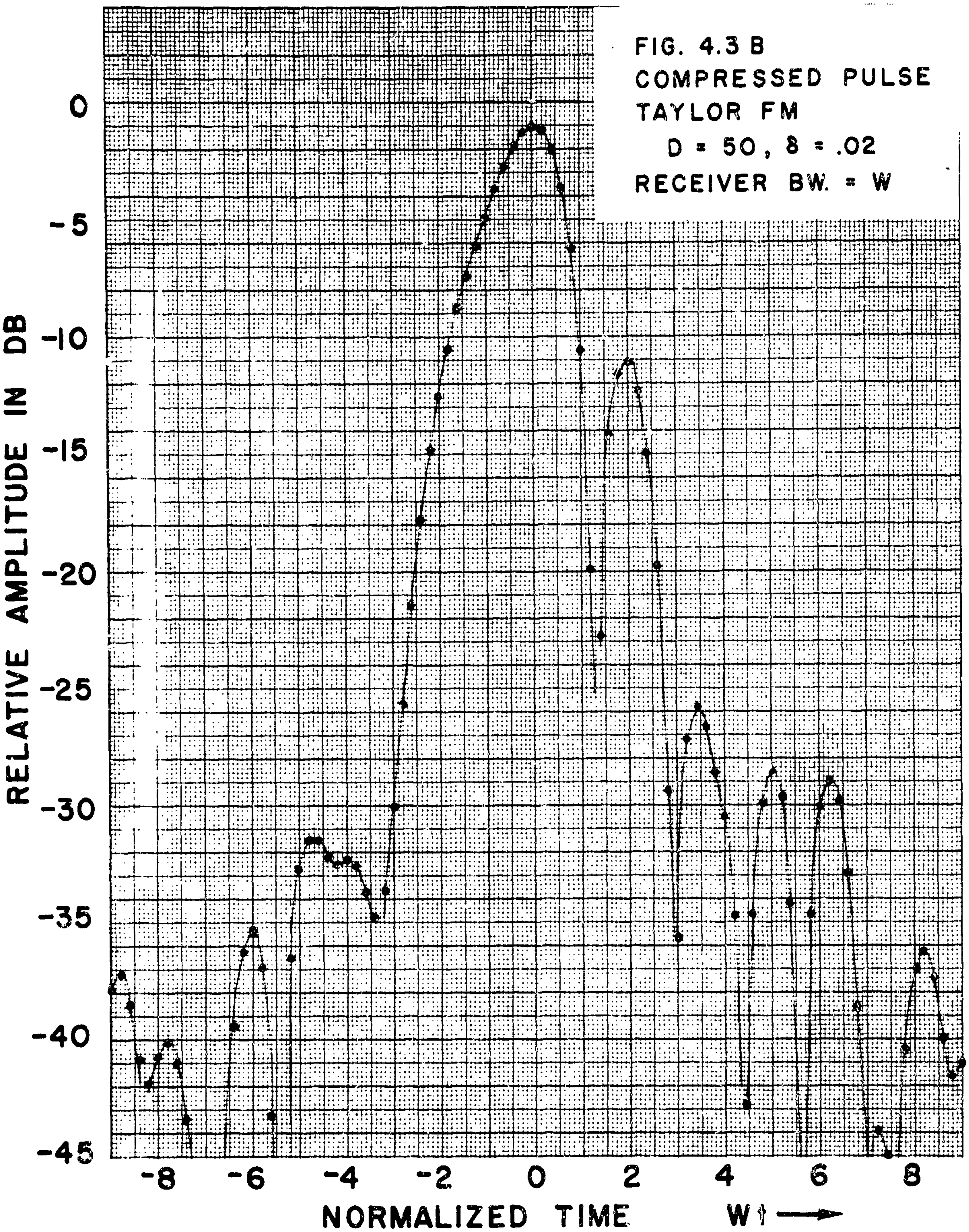


FIG. 4.3 B
COMPRESSED PULSE
TAYLOR FM
D = 50, δ = .02
RECEIVER BW. = W

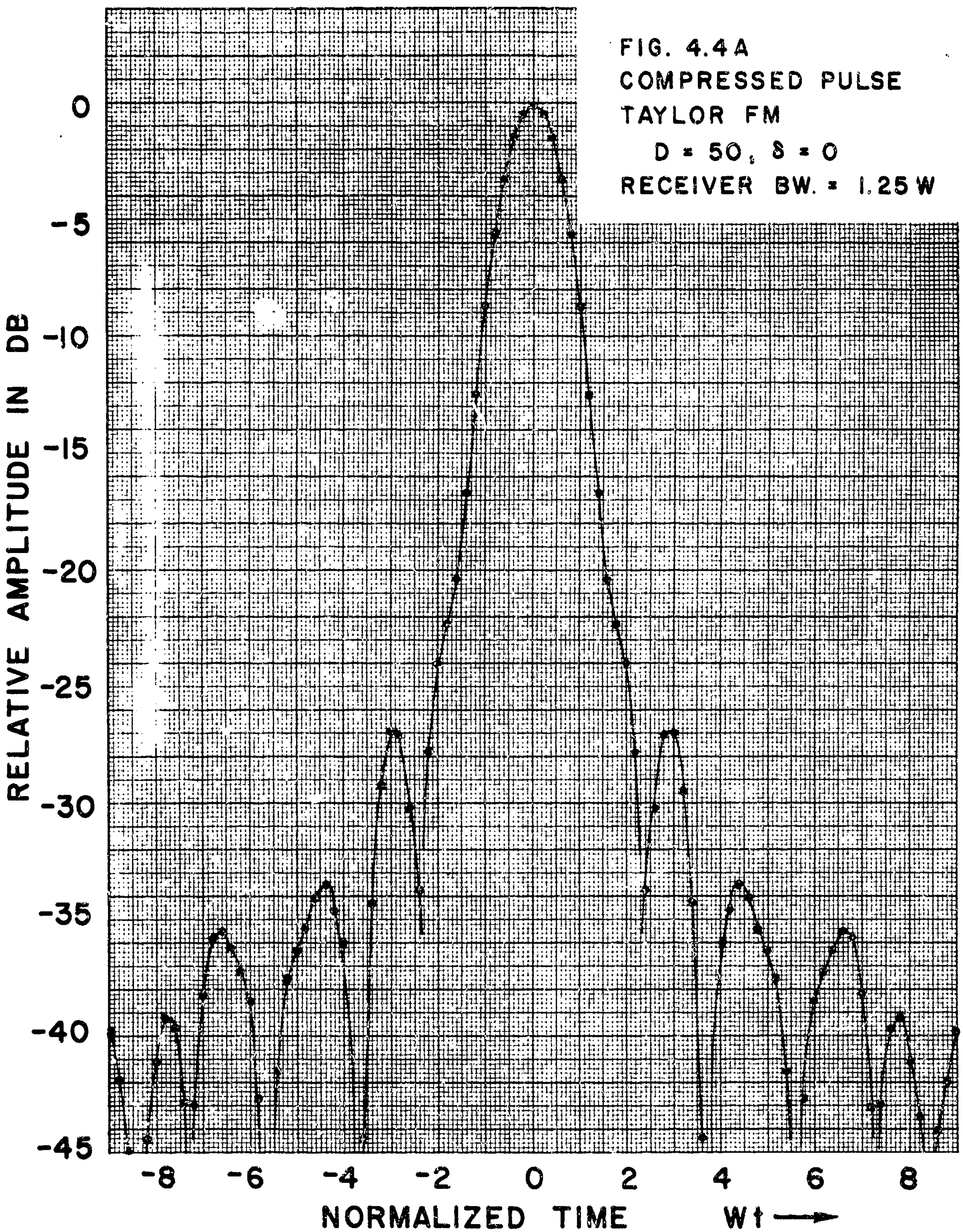


FIG. 4.4A
COMPRESSED PULSE
TAYLOR FM
D = 50, δ = 0
RECEIVER BW. = 1.25 W
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-8
-6
-4
-2
0
2
4
6
8
NORMALIZED TIME
Wt

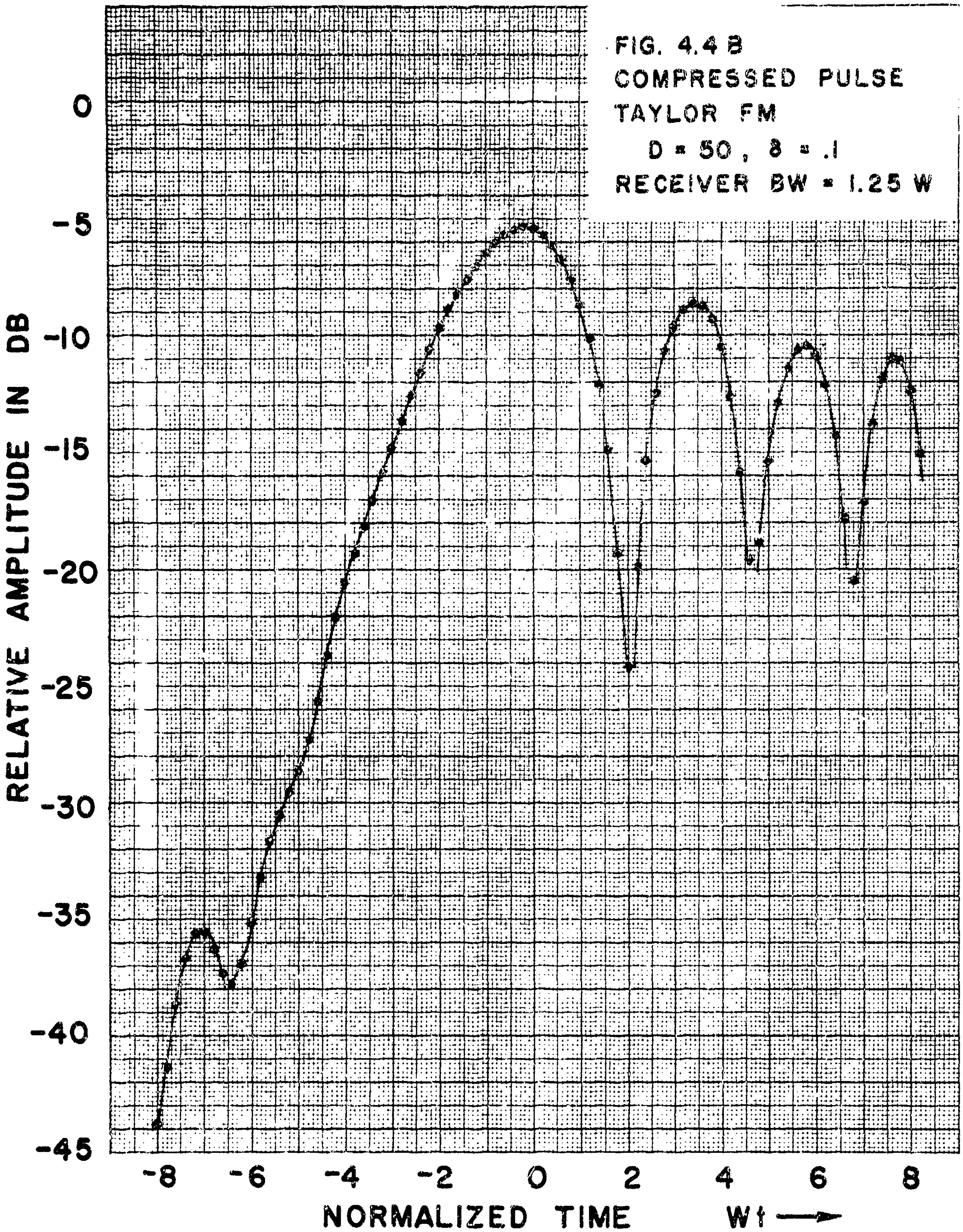


FIG. 4.4 B
COMPRESSED PULSE
TAYLOR FM
D = 50, δ = .1
RECEIVER BW = 1.25 W

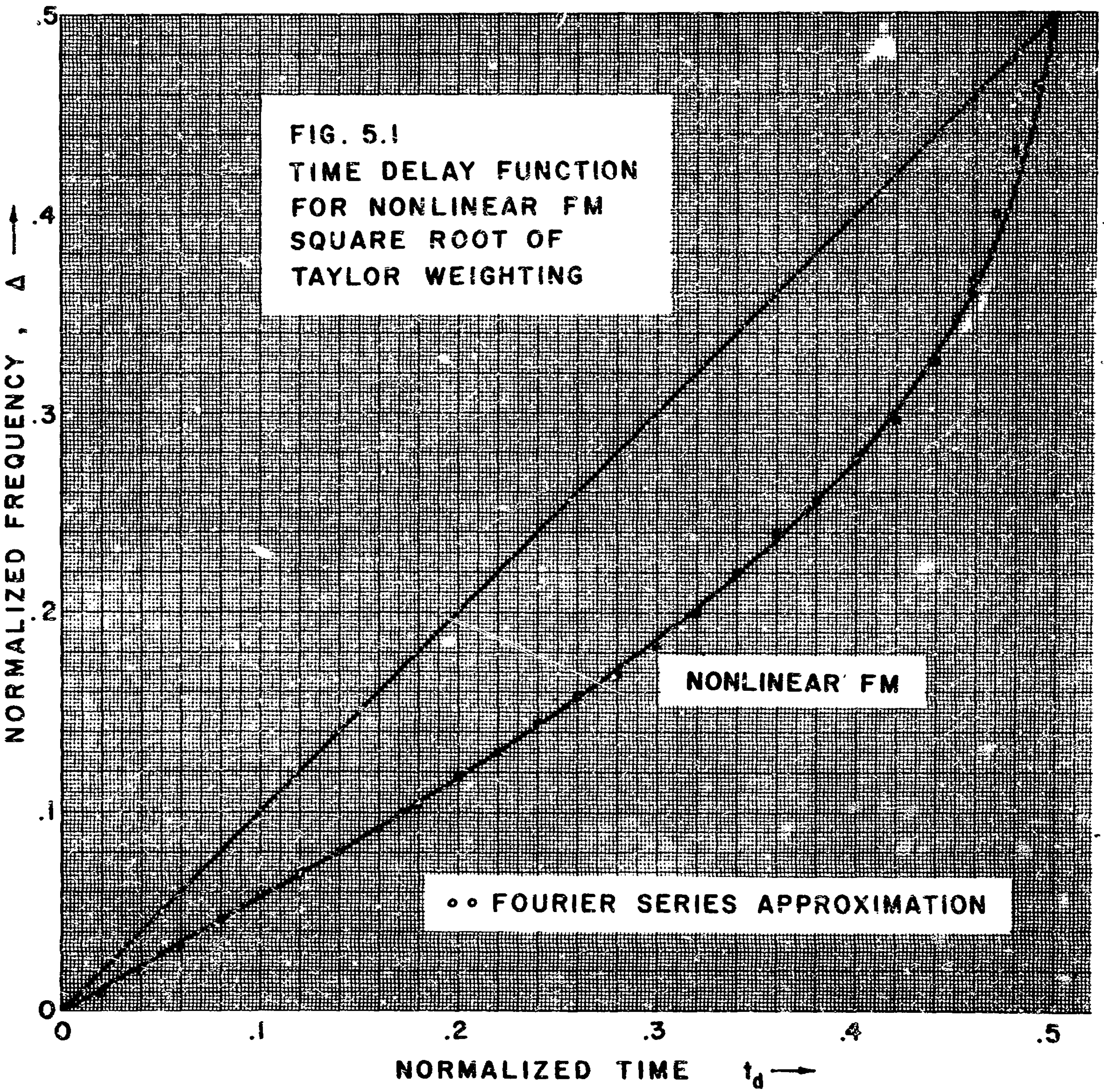


FIG. 5.1
TIME DELAY FUNCTION
FOR NONLINEAR FM
SQUARE ROOT OF
TAYLOR WEIGHTING

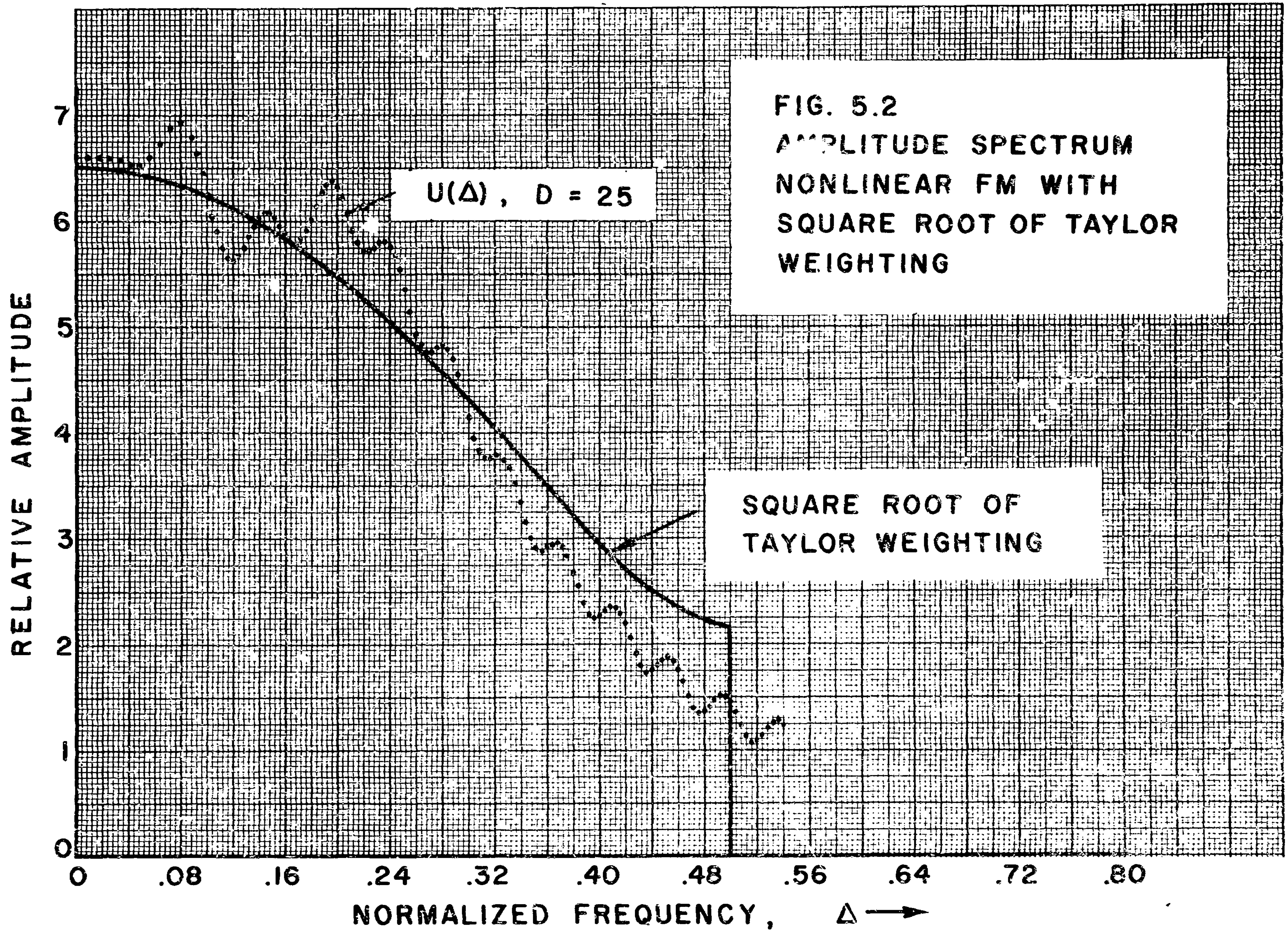


FIG. 5.2
AMPLITUDE SPECTRUM
NONLINEAR FM WITH
SQUARE ROOT OF TAYLOR
WEIGHTING
U(Δ), D = 25
SQUARE ROOT OF
TAYLOR WEIGHTING
RELATIVE AMPLITUDE
0
1
2
3
4
5
6
7
0
.08
.16
.24
.32
.40
.48
.56
.64
.72
.80
NORMALIZED FREQUENCY, Δ

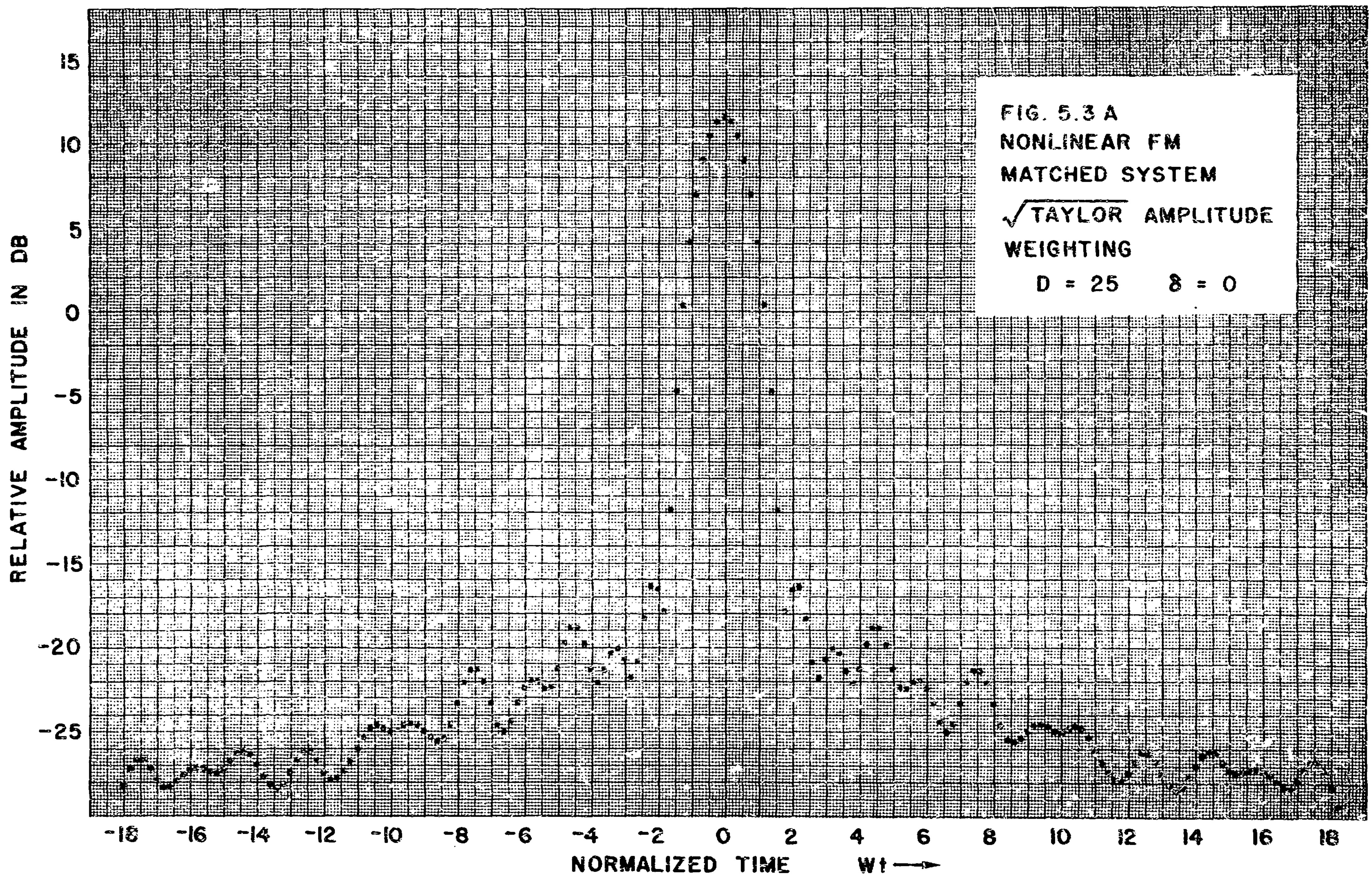


FIG. 5.3 A
NONLINEAR FM
MATCHED SYSTEM
√TAYLOR AMPLITUDE
WEIGHTING
D = 25 δ = 0
RELATIVE AMPLITUDE IN DB
15
10
5
0
-5
-10
-15
-20
-25
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME Wt

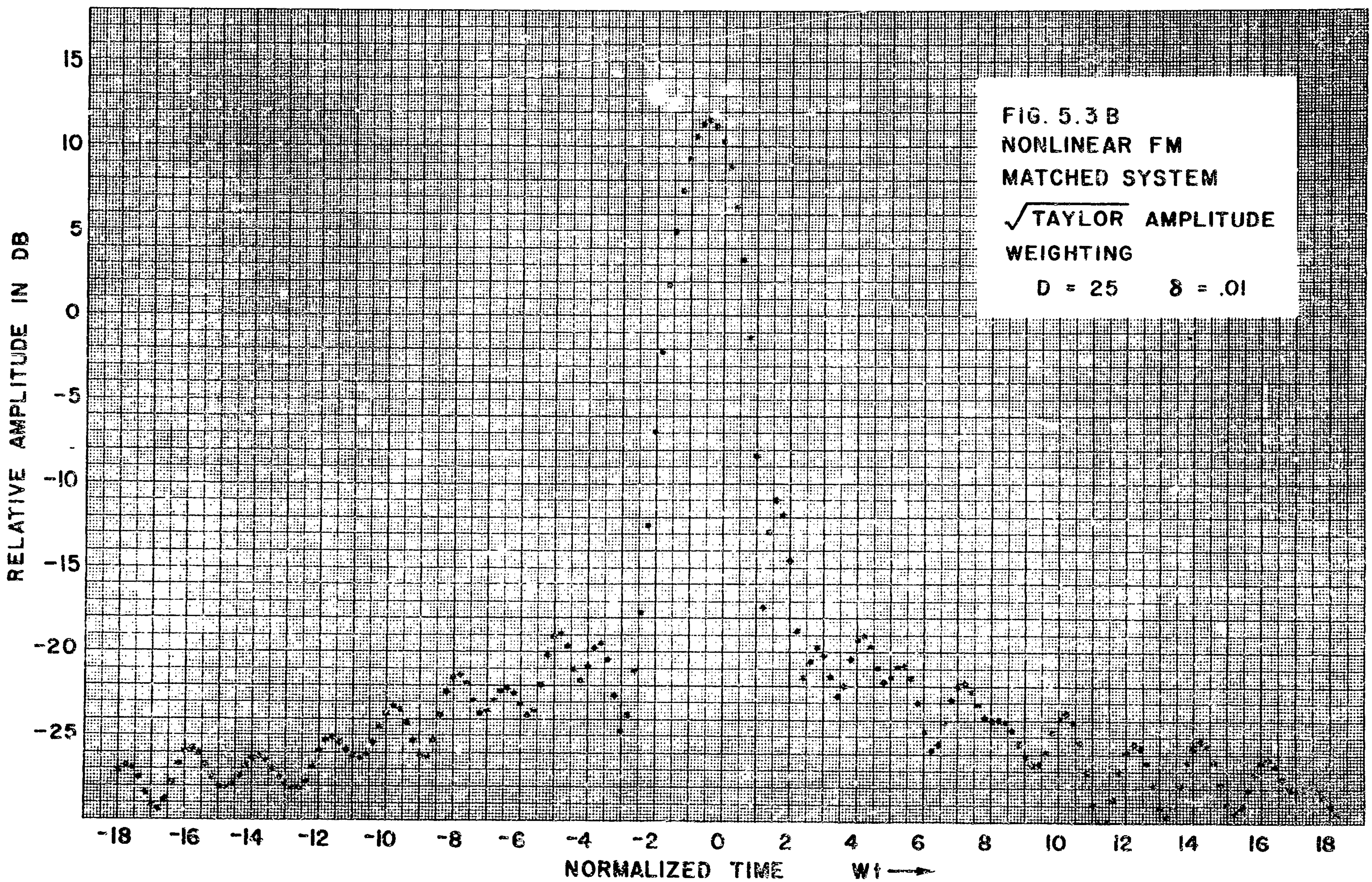


FIG. 5.3 B
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{TAYLOR}}$ AMPLITUDE
WEIGHTING
D = 25   δ = .01
RELATIVE AMPLITUDE IN DB
15
10
5
0
-5
-10
-15
-20
-25
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt

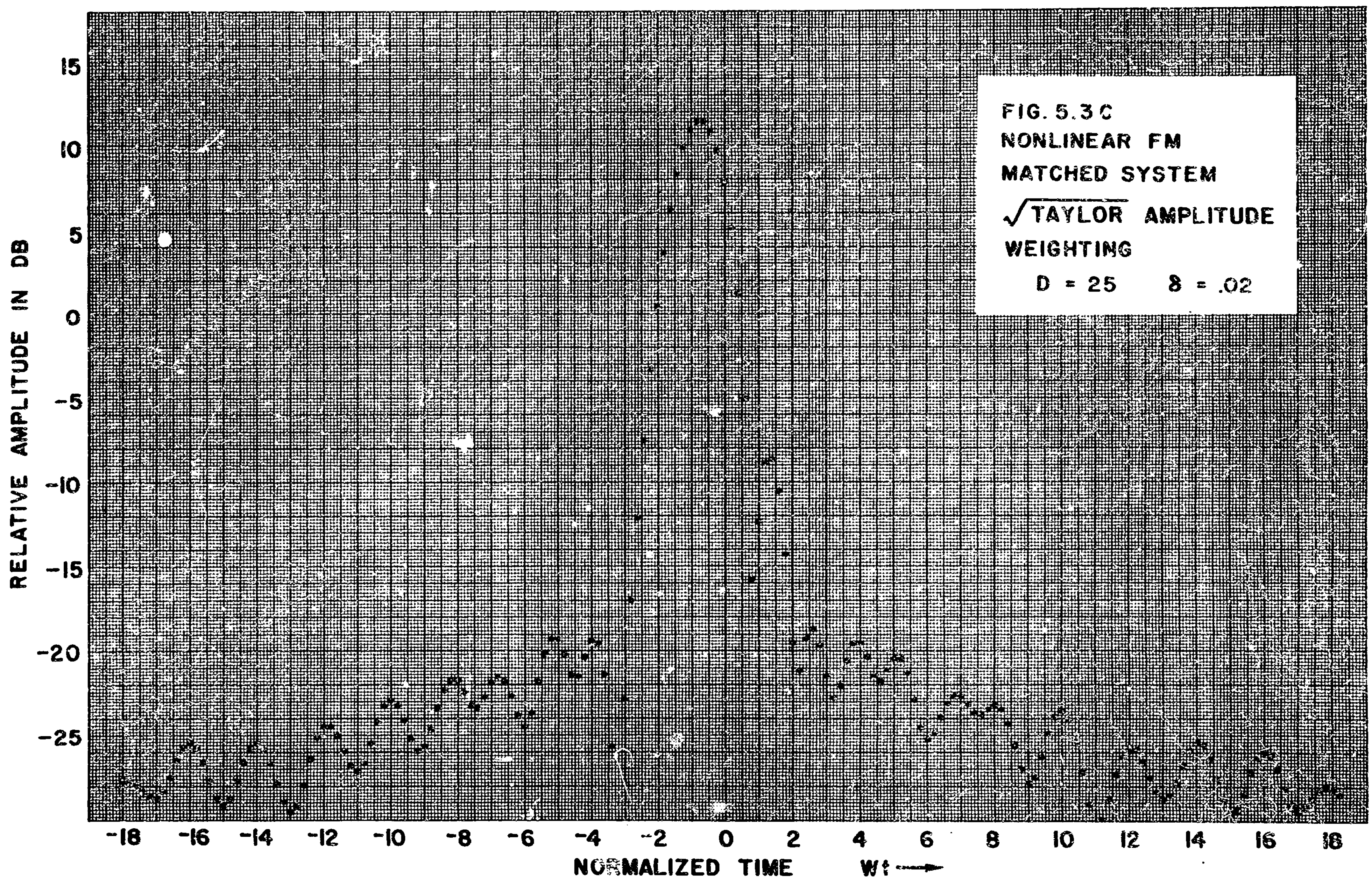


FIG. 5.3C
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{TAYLOR}}$ AMPLITUDE
WEIGHTING
D = 25 δ = .02
RELATIVE AMPLITUDE IN DB
15
10
5
0
-5
-10
-15
-20
-25
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME Wt →

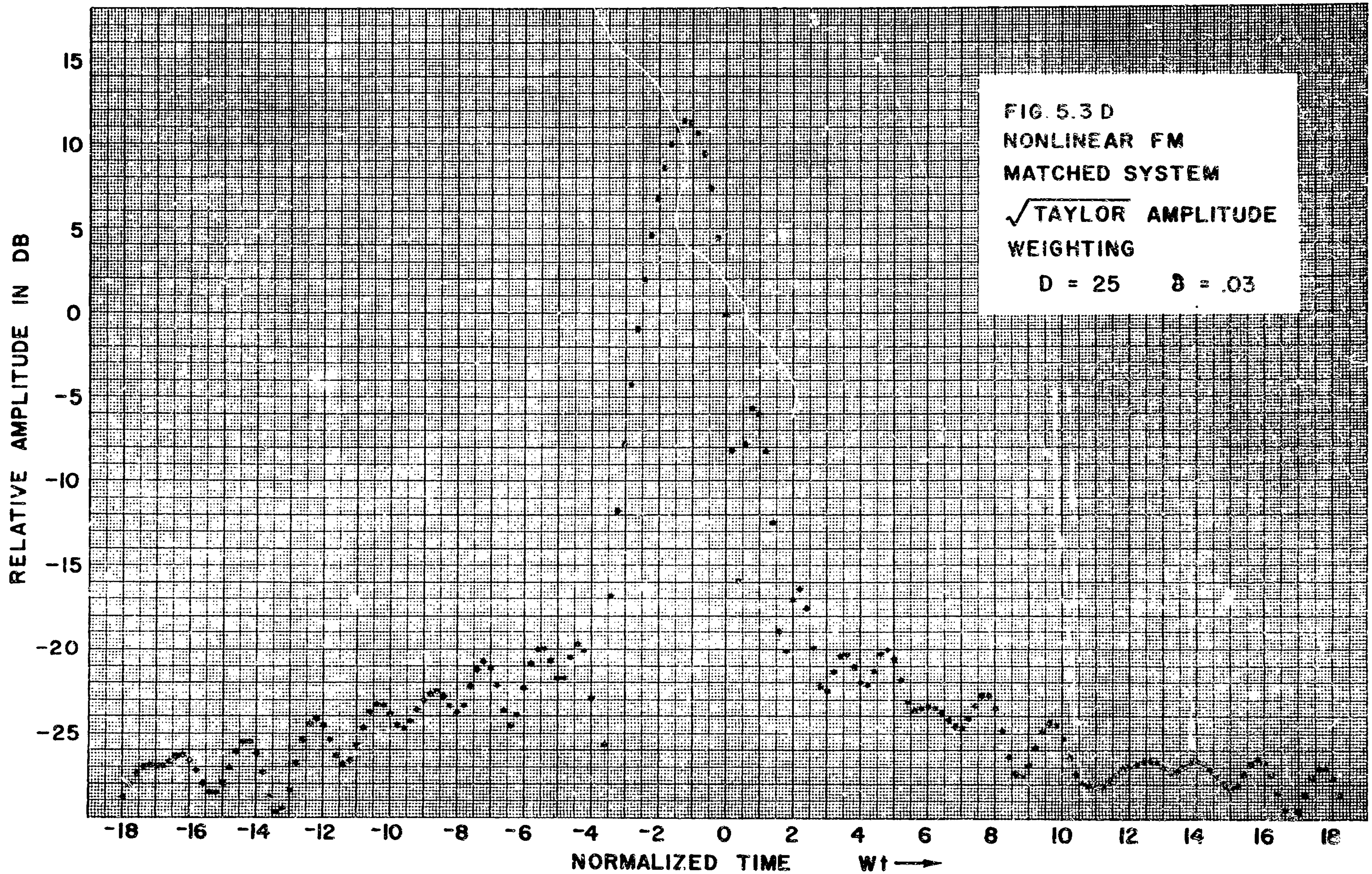


FIG. 5.3 D
NONLINEAR FM
MATCHED SYSTEM
√TAYLOR AMPLITUDE
WEIGHTING
D = 25   δ = .03
RELATIVE AMPLITUDE IN DB
15
10
5
0
-5
-10
-15
-20
-25
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

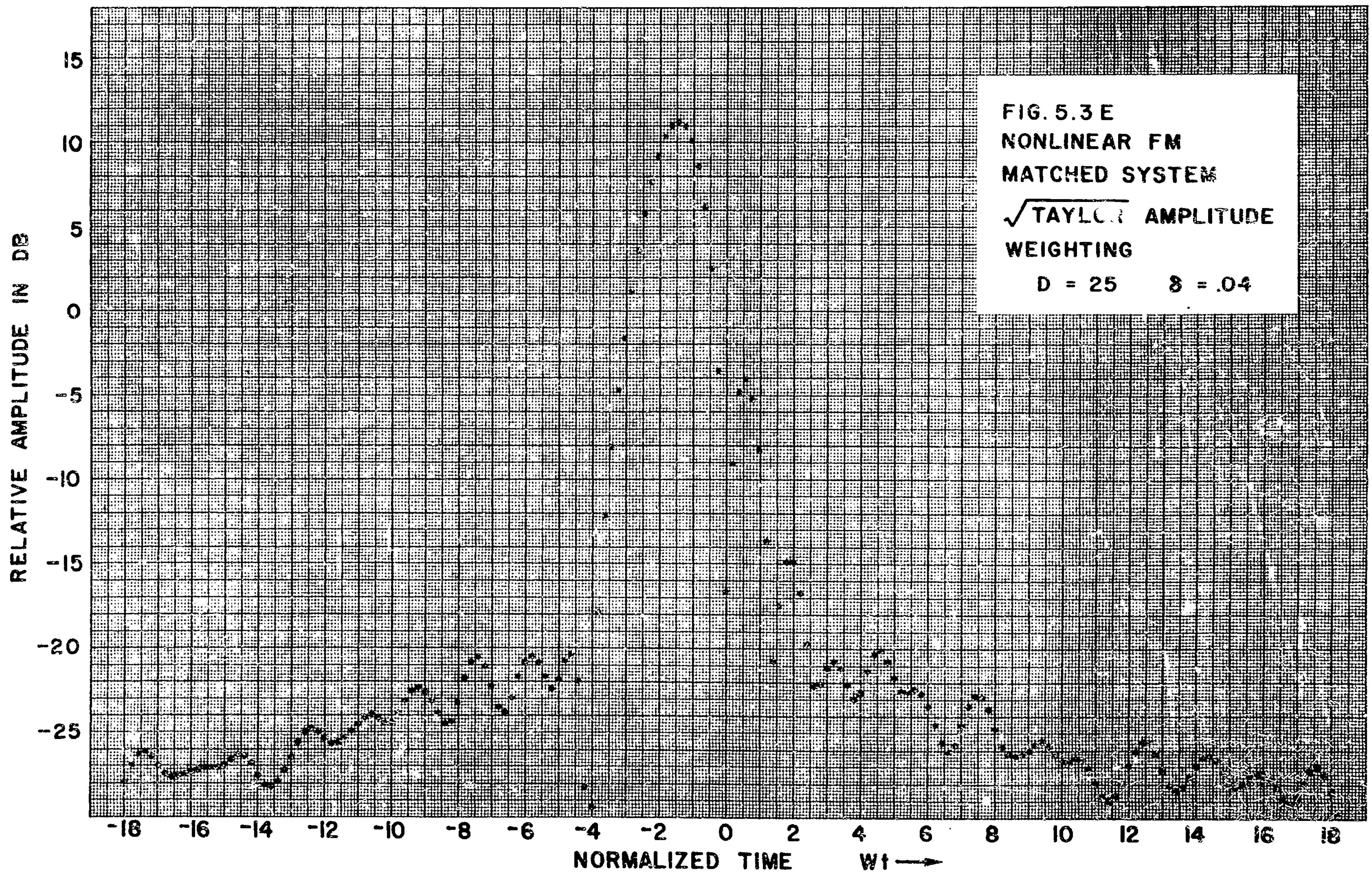


FIG. 5.3 E
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{TAYLOR}}$ AMPLITUDE
WEIGHTING
D = 25 δ = .04
RELATIVE AMPLITUDE IN DB
15
10
5
0
-5
-10
-15
-20
-25
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME Wt →

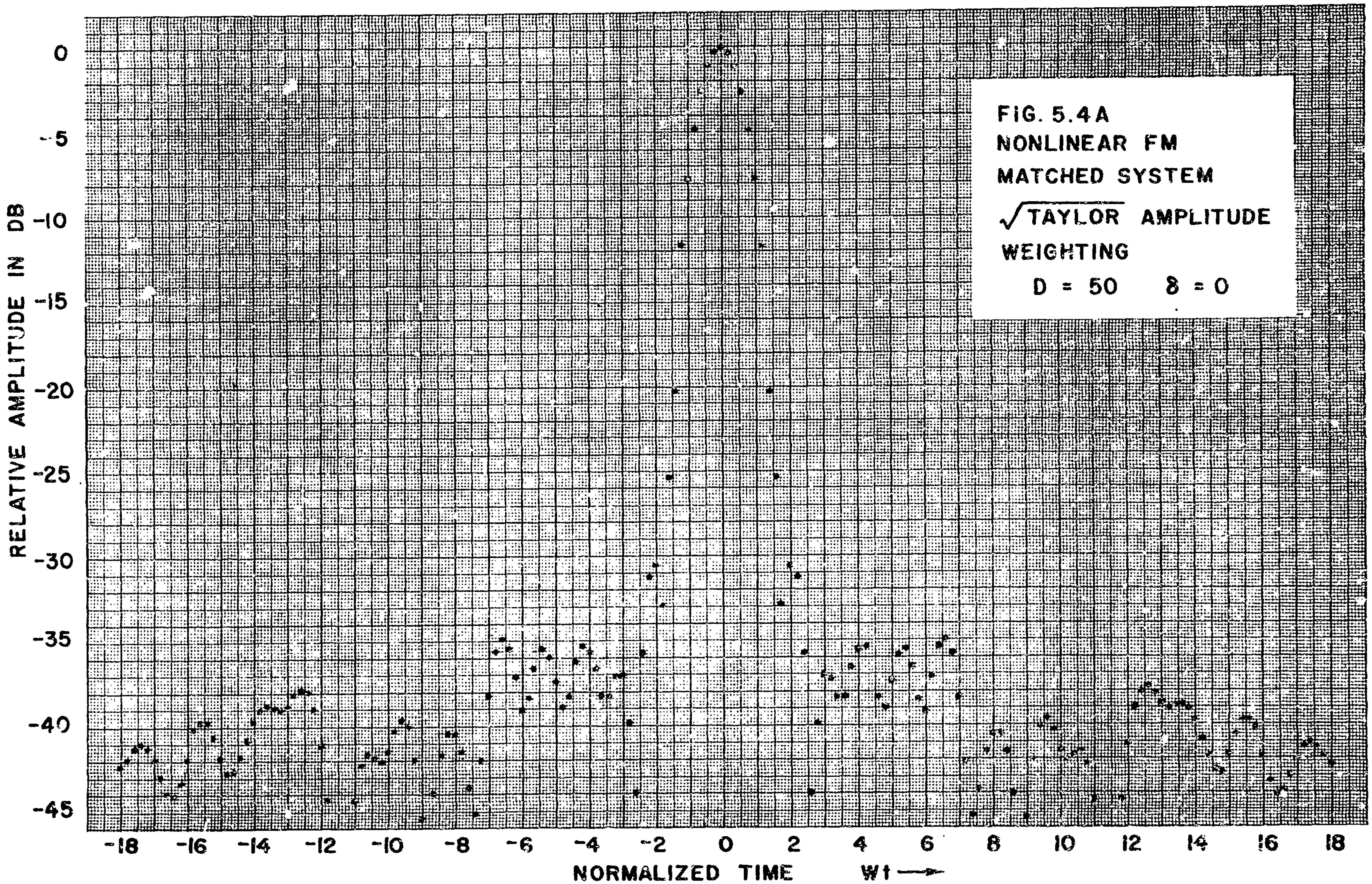


FIG. 5.4A
NONLINEAR FM
MATCHED SYSTEM
√TAYLOR AMPLITUDE
WEIGHTING
D = 50 δ = 0
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME Wt →

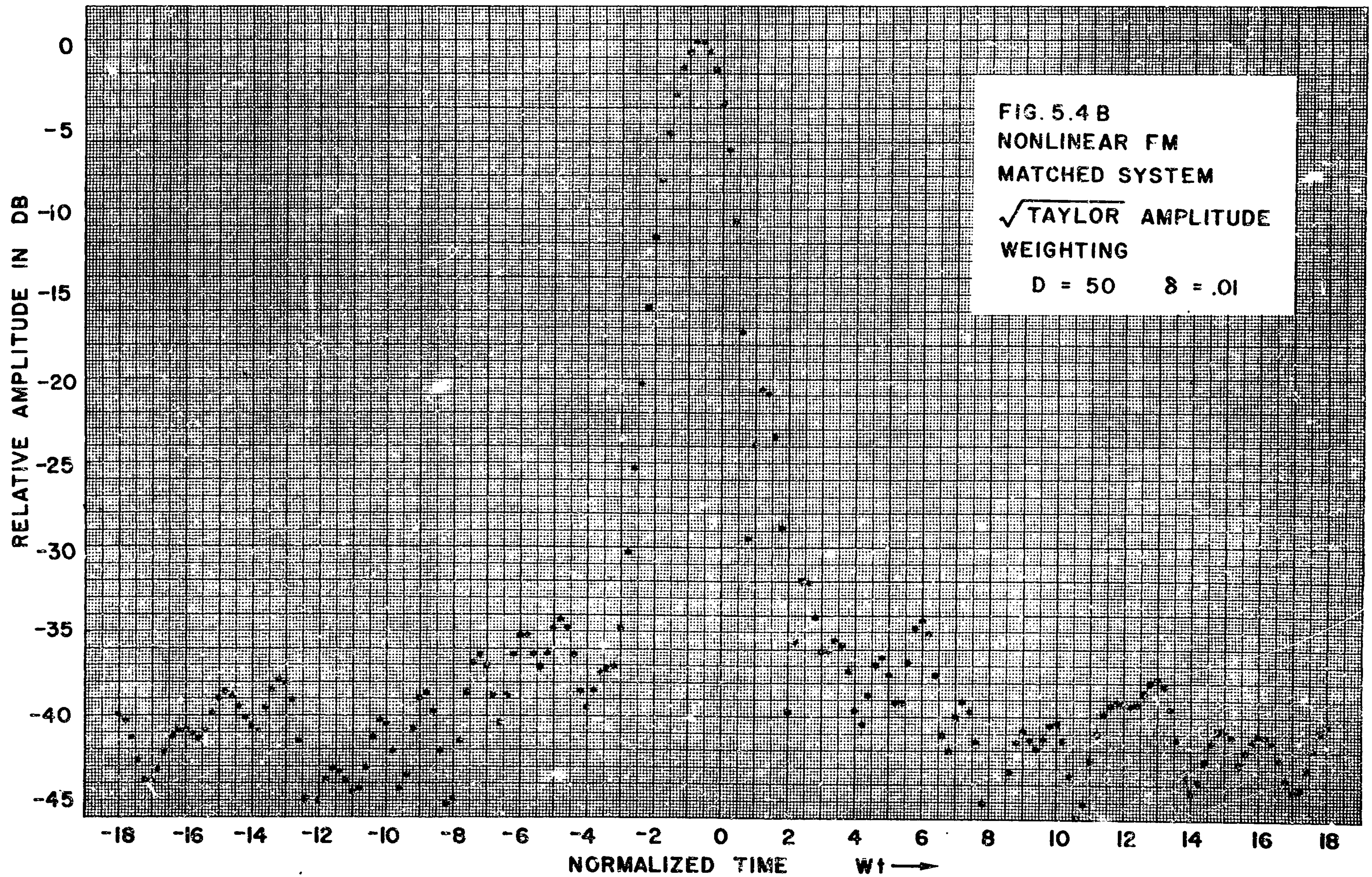


FIG. 5.4 B
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{TAYLOR}}$ AMPLITUDE
WEIGHTING
D = 50 δ = .01
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME Wt

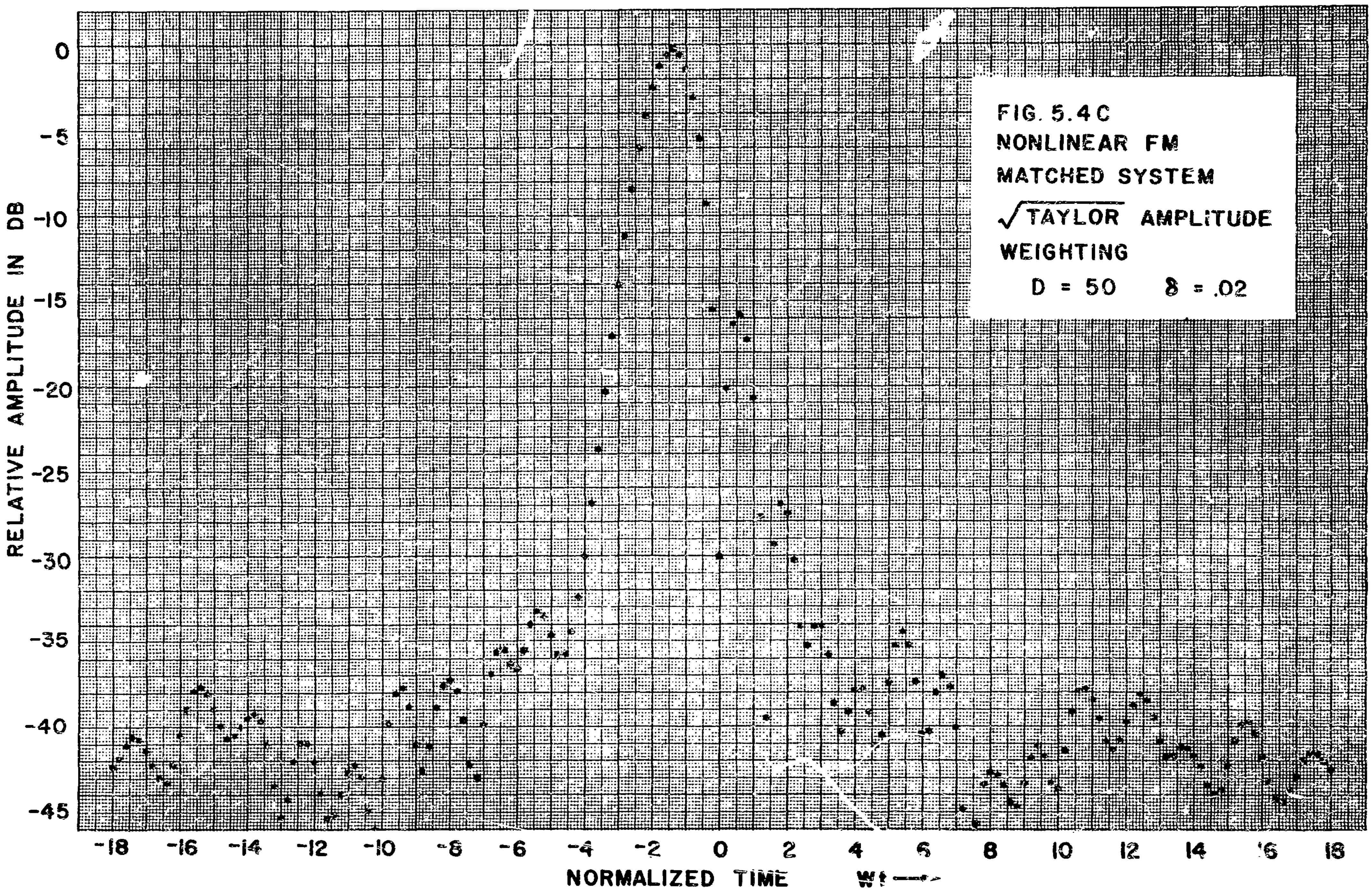


FIG. 5.4C
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{TAYLOR}}$ AMPLITUDE
WEIGHTING
D = 50 δ = .02

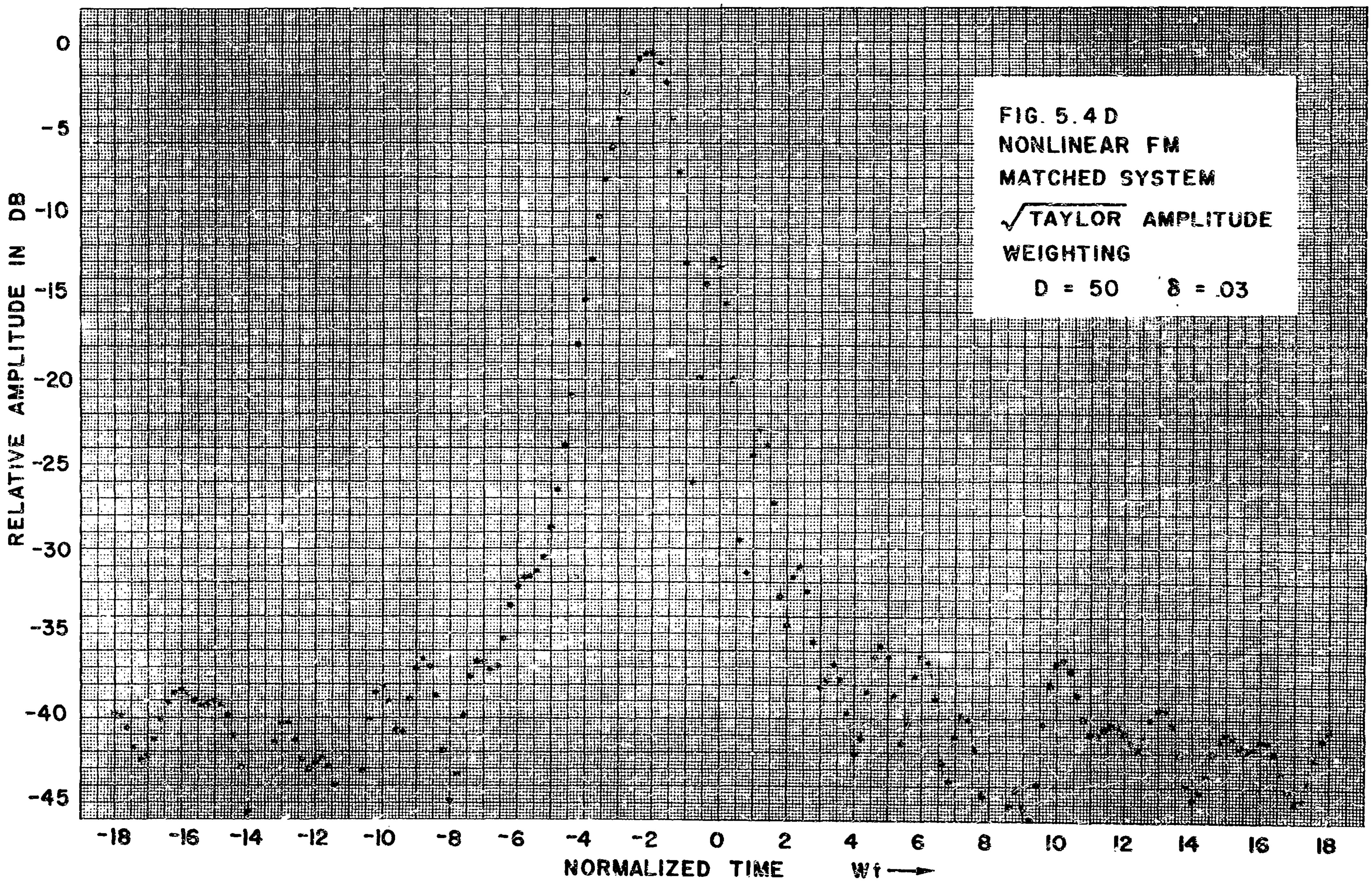


FIG. 5.4 D
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{TAYLOR}}$ AMPLITUDE
WEIGHTING
D = 50   δ = .03
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt ⟶

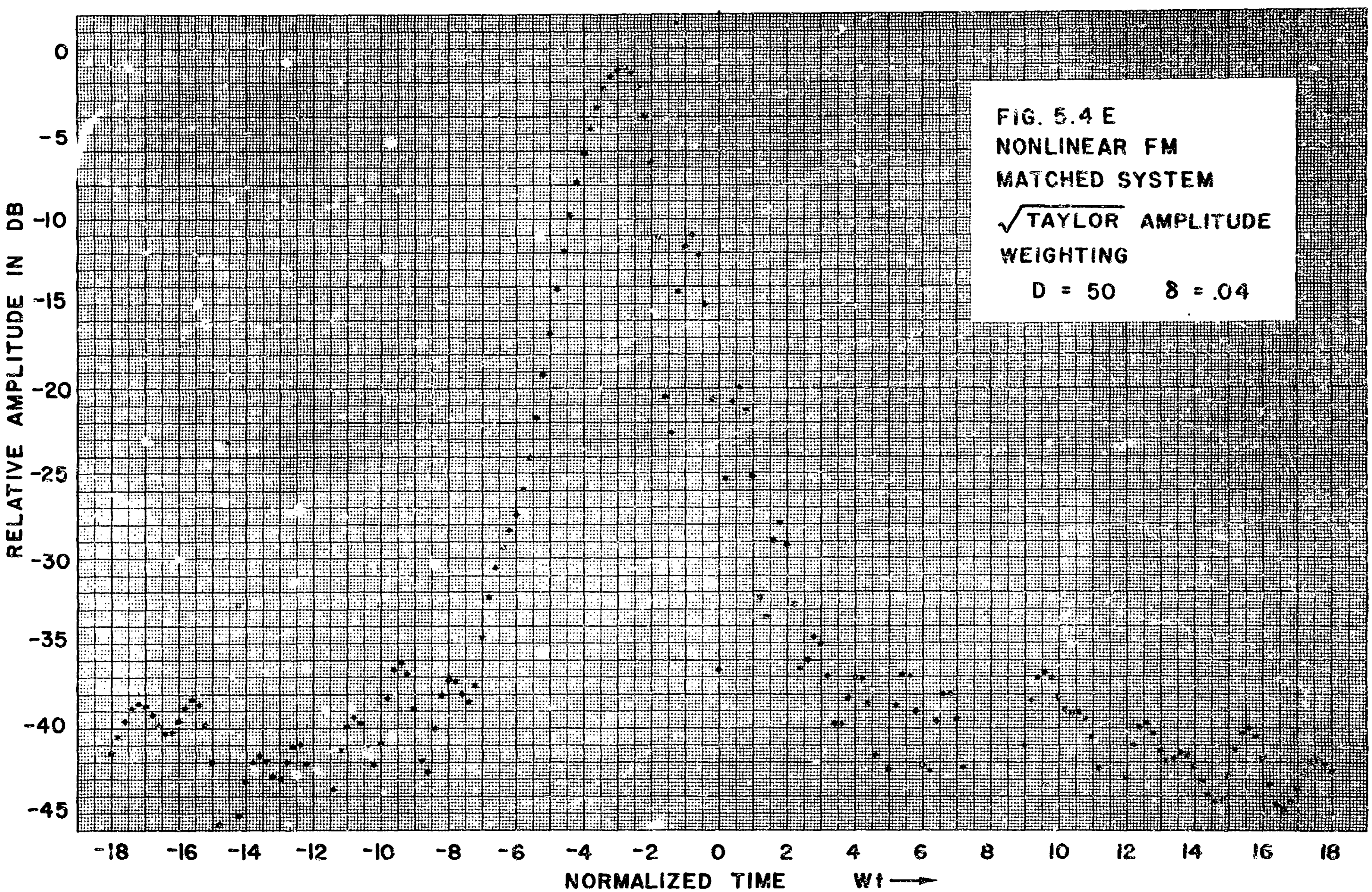


FIG. 5.4 E
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{TAYLOR}}$ AMPLITUDE
WEIGHTING
D = 50 δ = .04

PAGES FIG. 6.1, 6.2, 7.1, 7.2 ARE MISSING IN ORIGINAL DOCUMENT

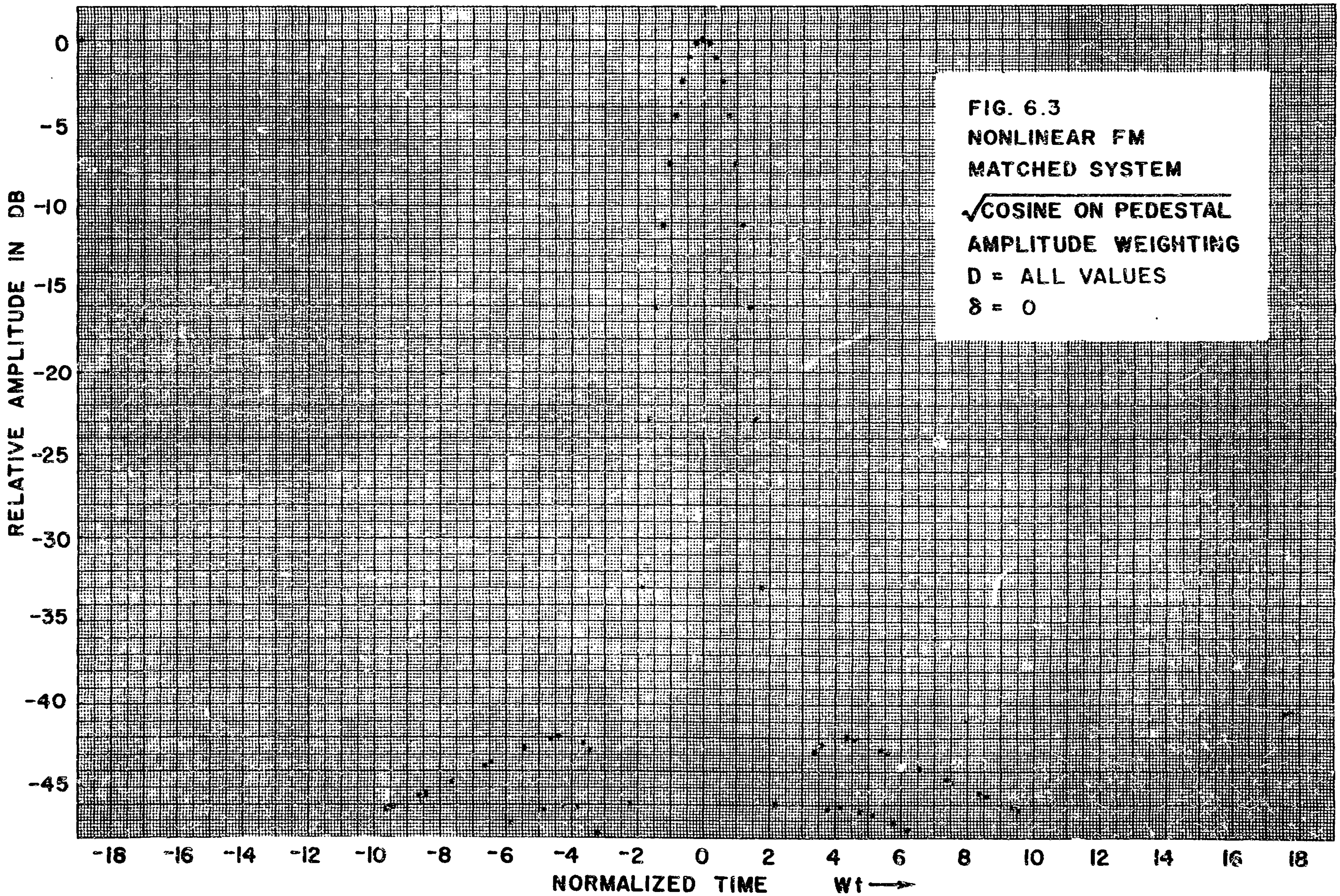


FIG. 6.3
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{COSINE ON PEDESTAL}}$
AMPLITUDE WEIGHTING
D = ALL VALUES
δ = 0

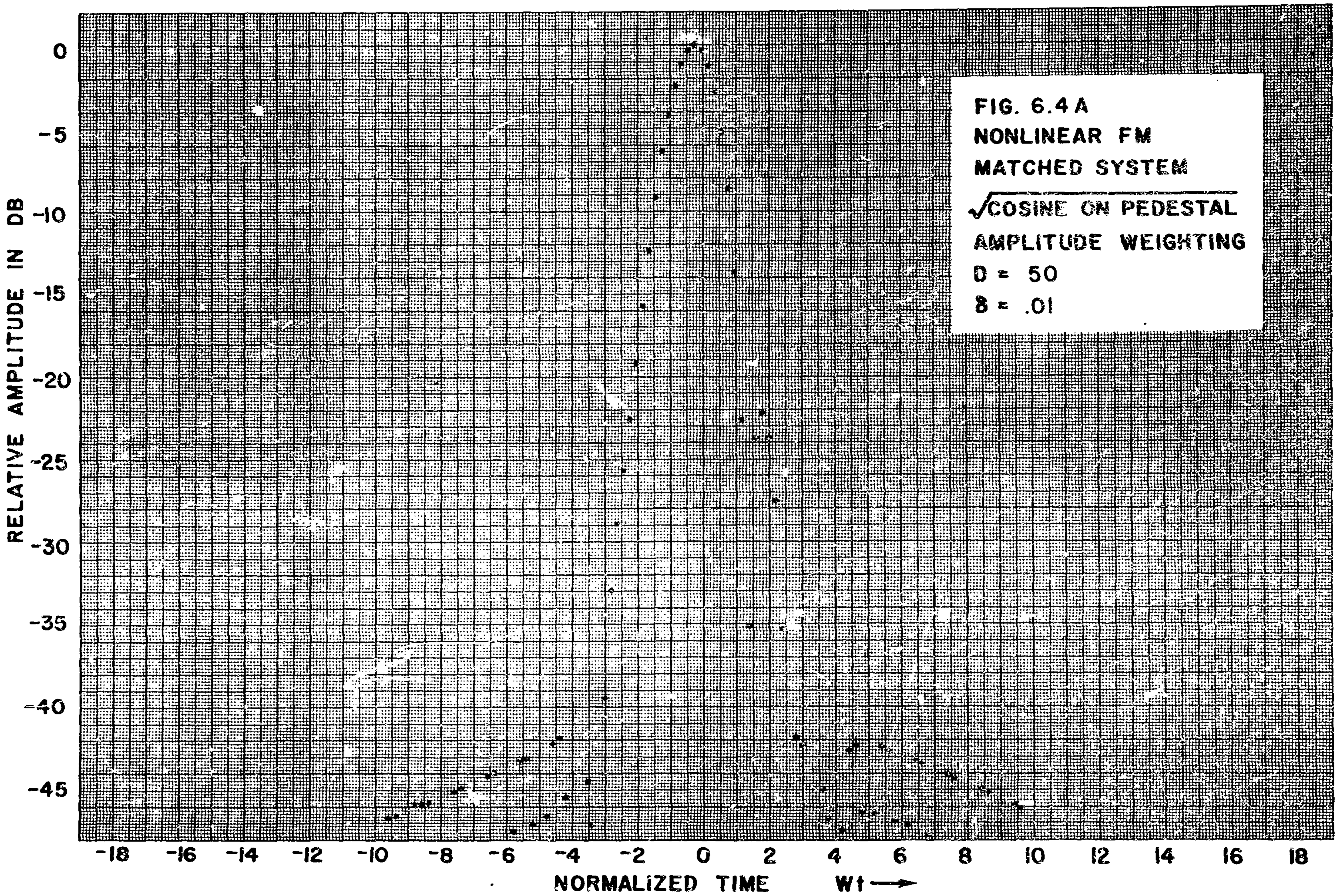


FIG. 6.4 A
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{COSINE ON PEDESTAL}}$
AMPLITUDE WEIGHTING
D = 50
δ = .01
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

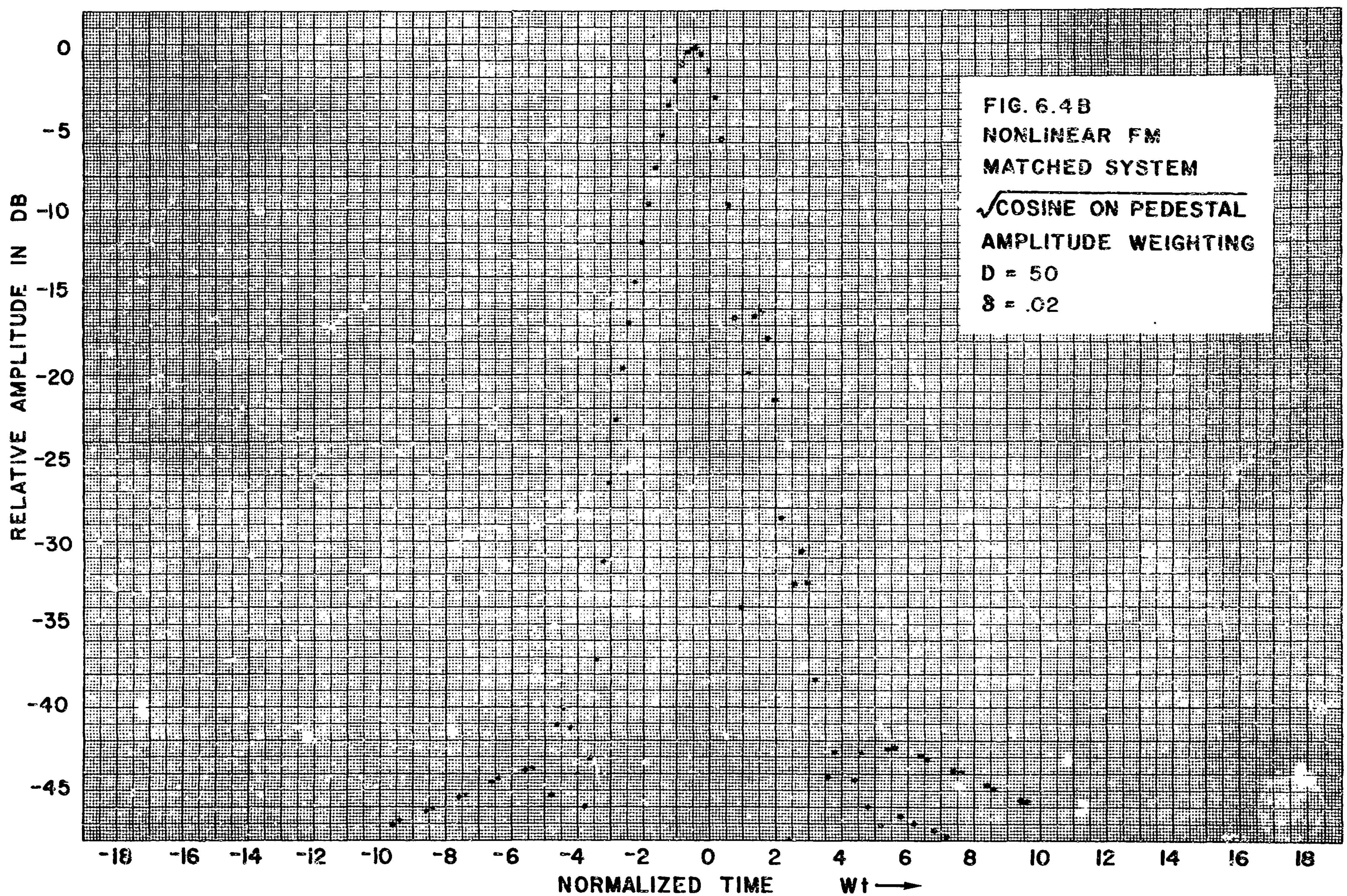


FIG. 6.4B
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{COSINE ON PEDESTAL}}$
AMPLITUDE WEIGHTING
D = 50
δ = .02
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt →

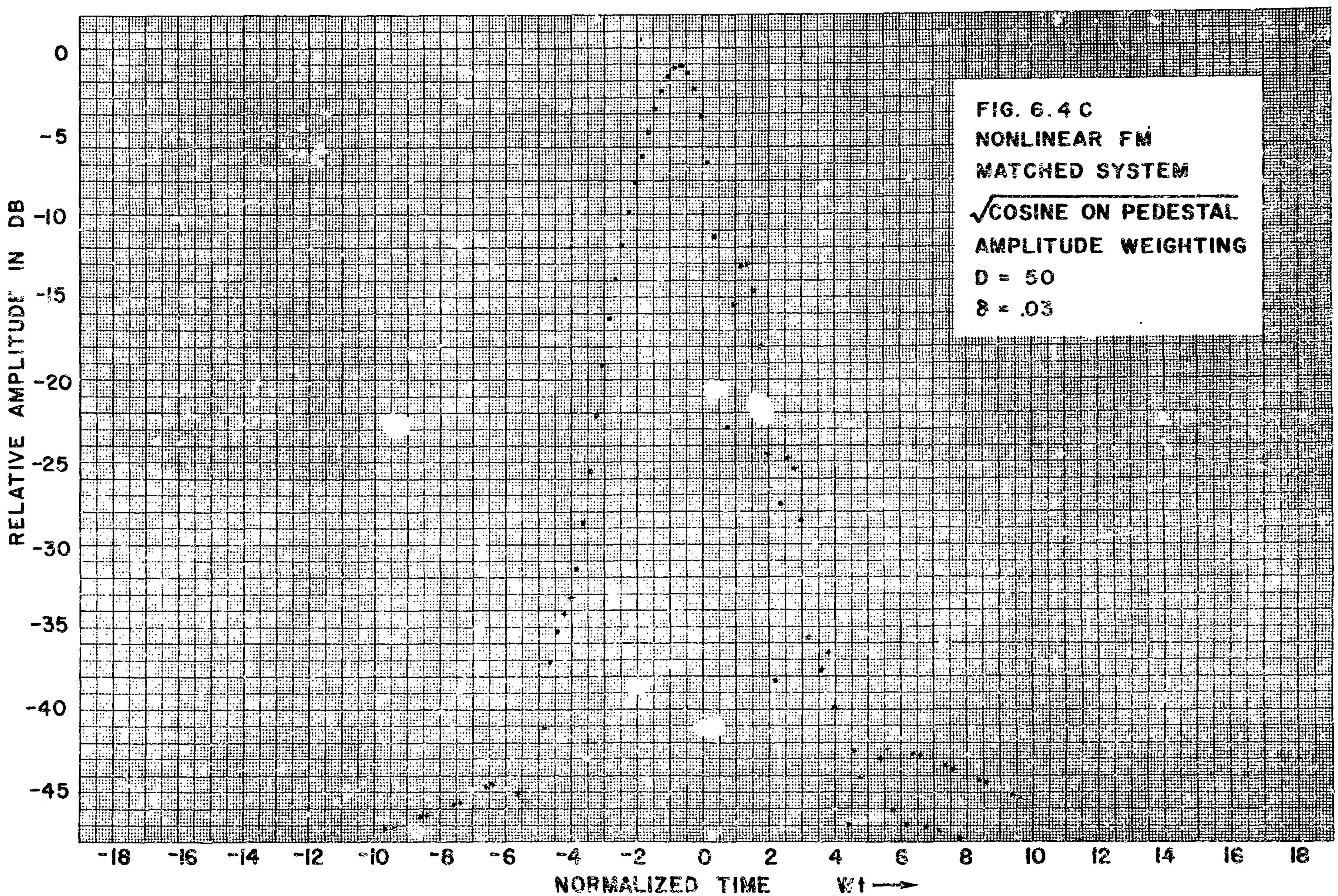


FIG. 6.4 C
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 50
δ = .03
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt →

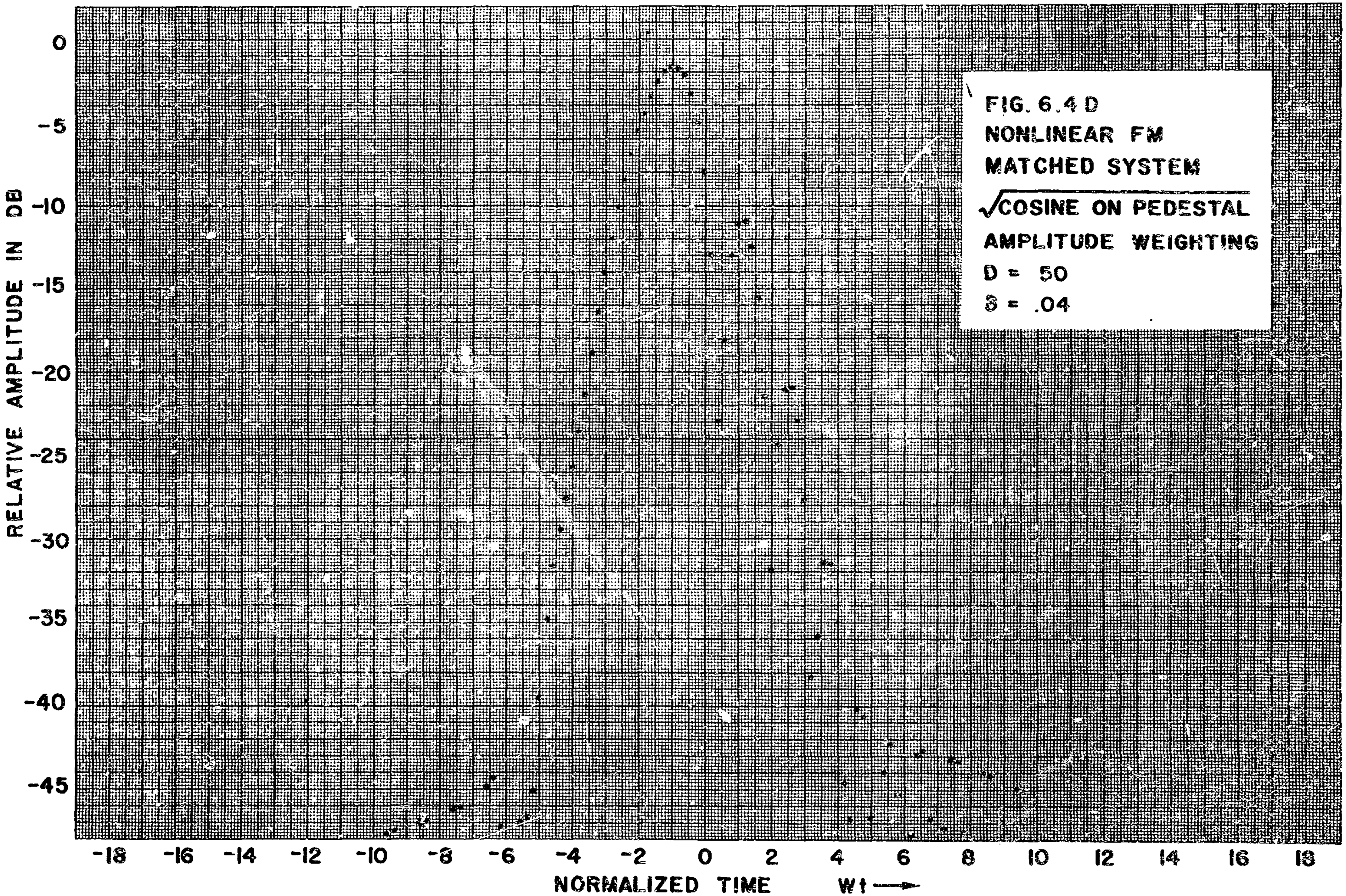


FIG. 6.4 D
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 50
δ = .04
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

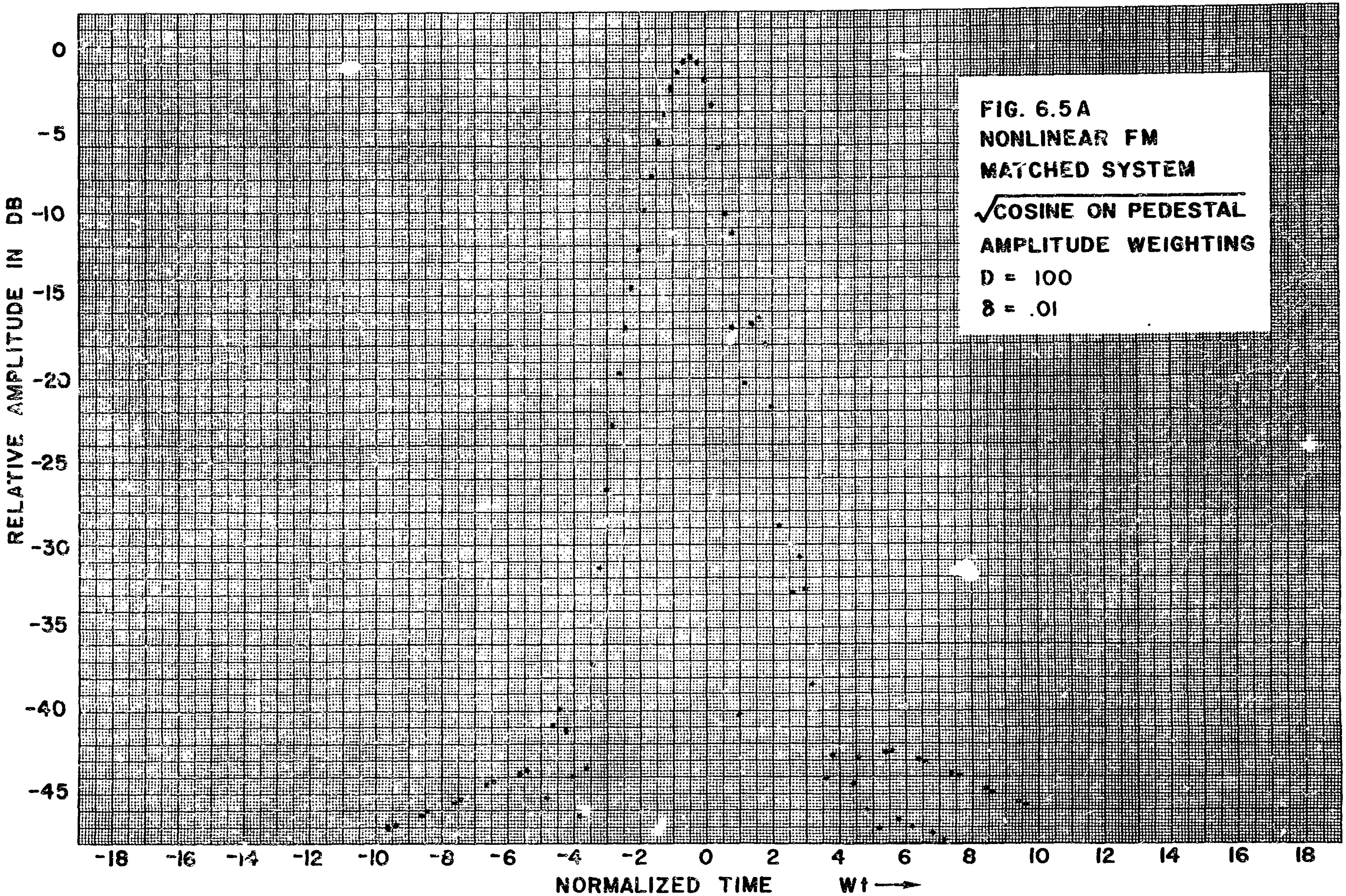


FIG. 6.5 A
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 100
δ = .01
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME Wt

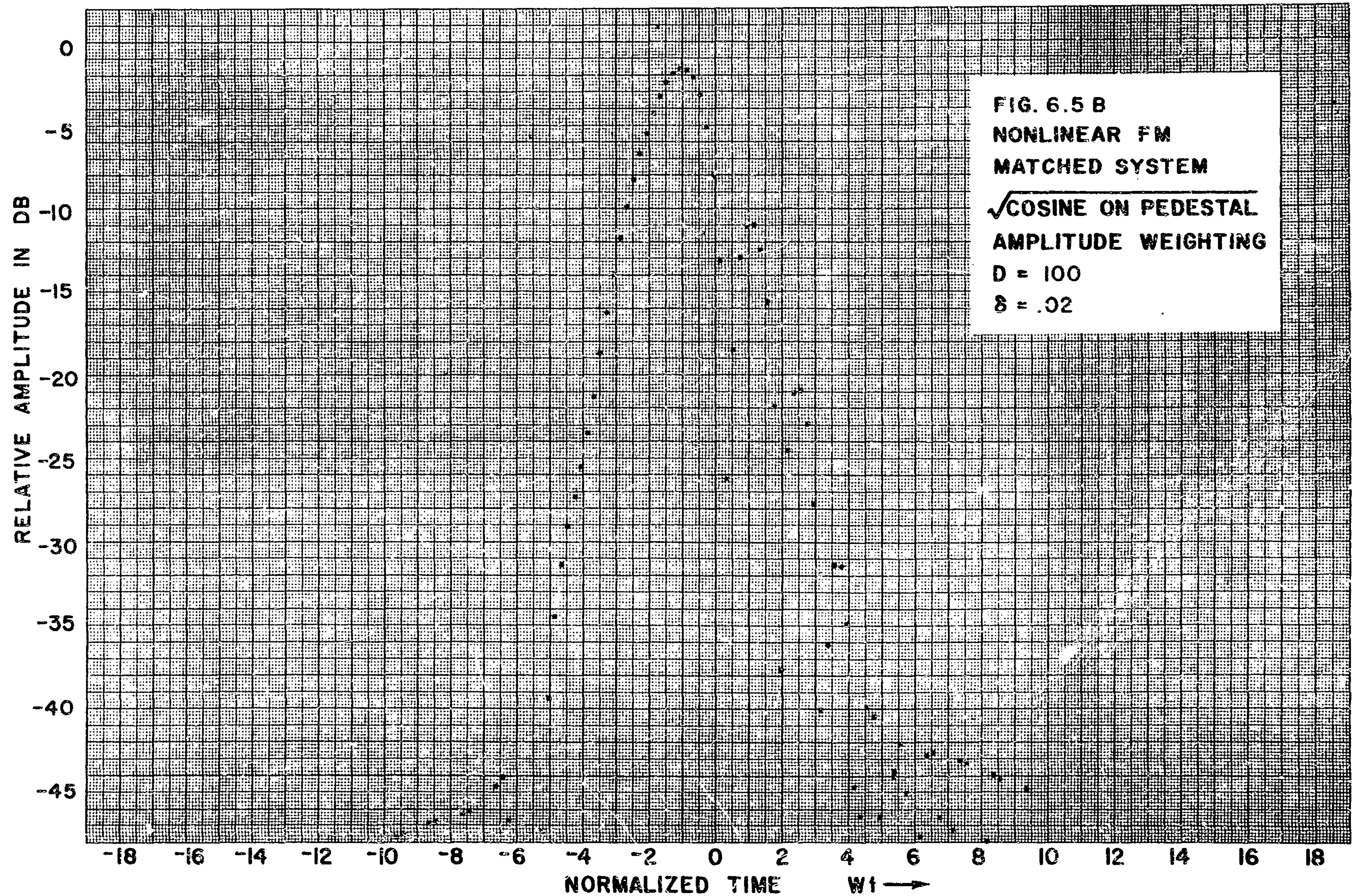


FIG. 6.5 B
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{COSINE ON PEDESTAL}}$
AMPLITUDE WEIGHTING
D = 100
δ = .02
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt →

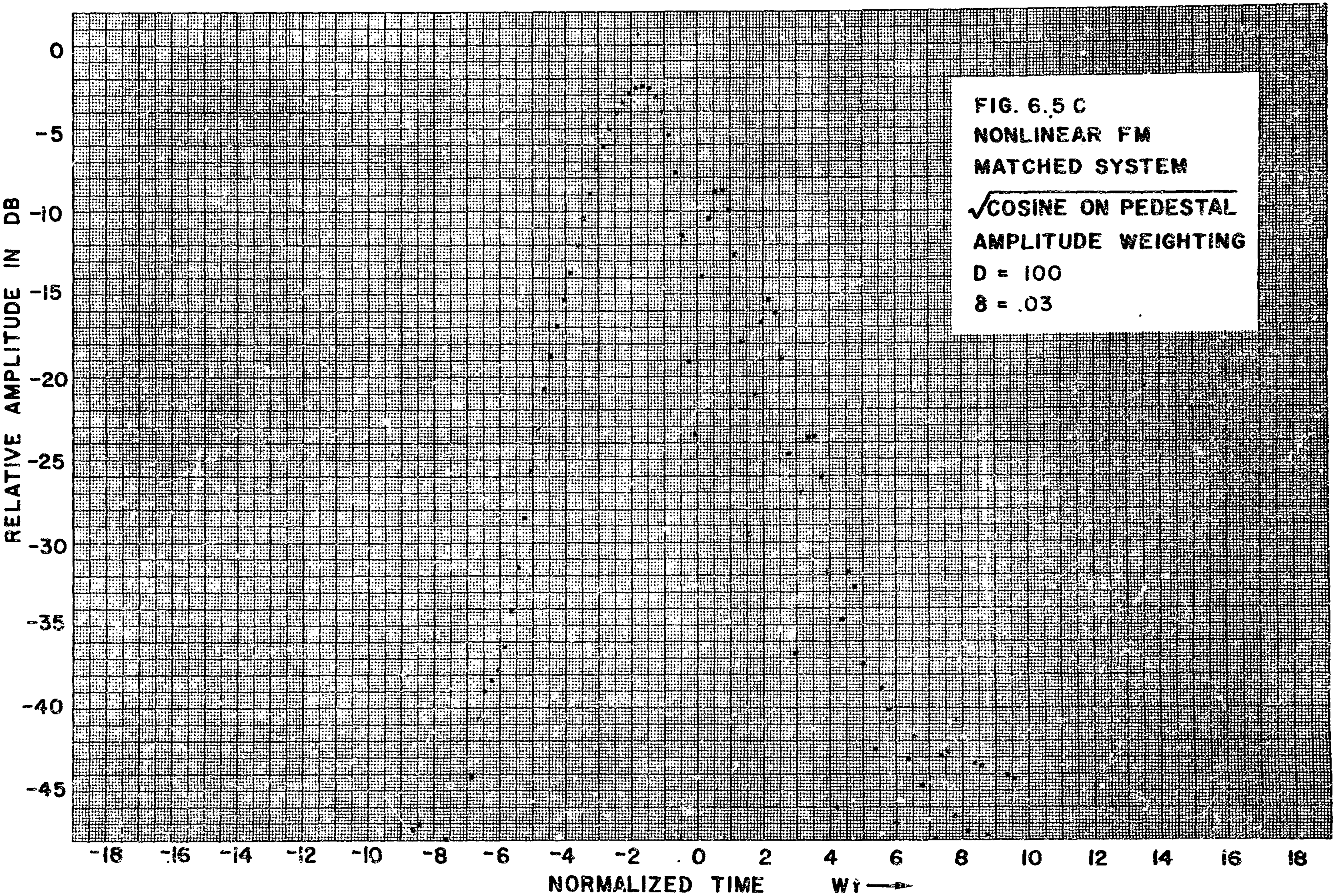


FIG. 6.5 C
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 100
8 = .03
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME
Wt

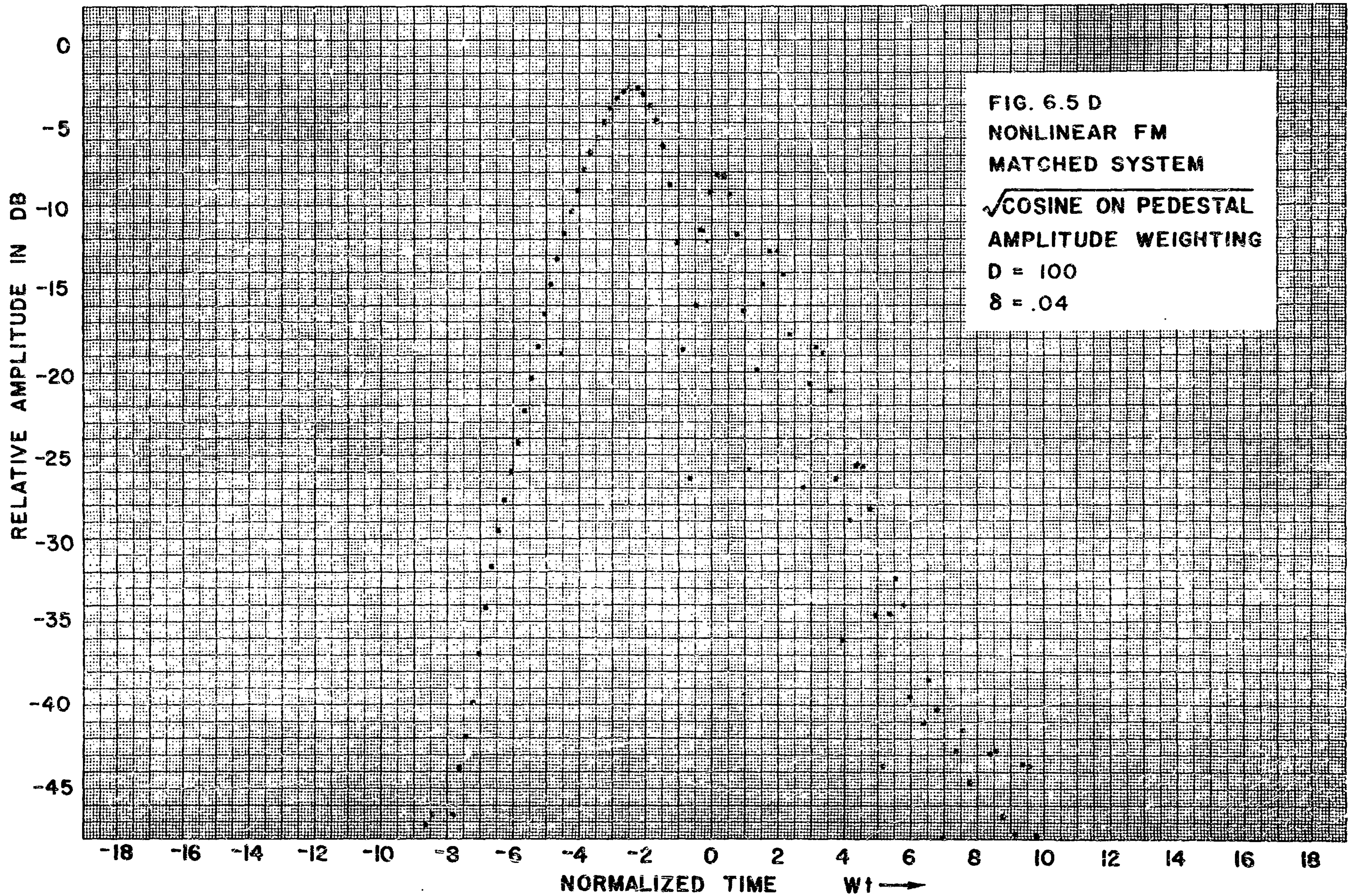


FIG. 6.5 D
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{COSINE ON PEDESTAL}}$
AMPLITUDE WEIGHTING
D = 100
δ = .04
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

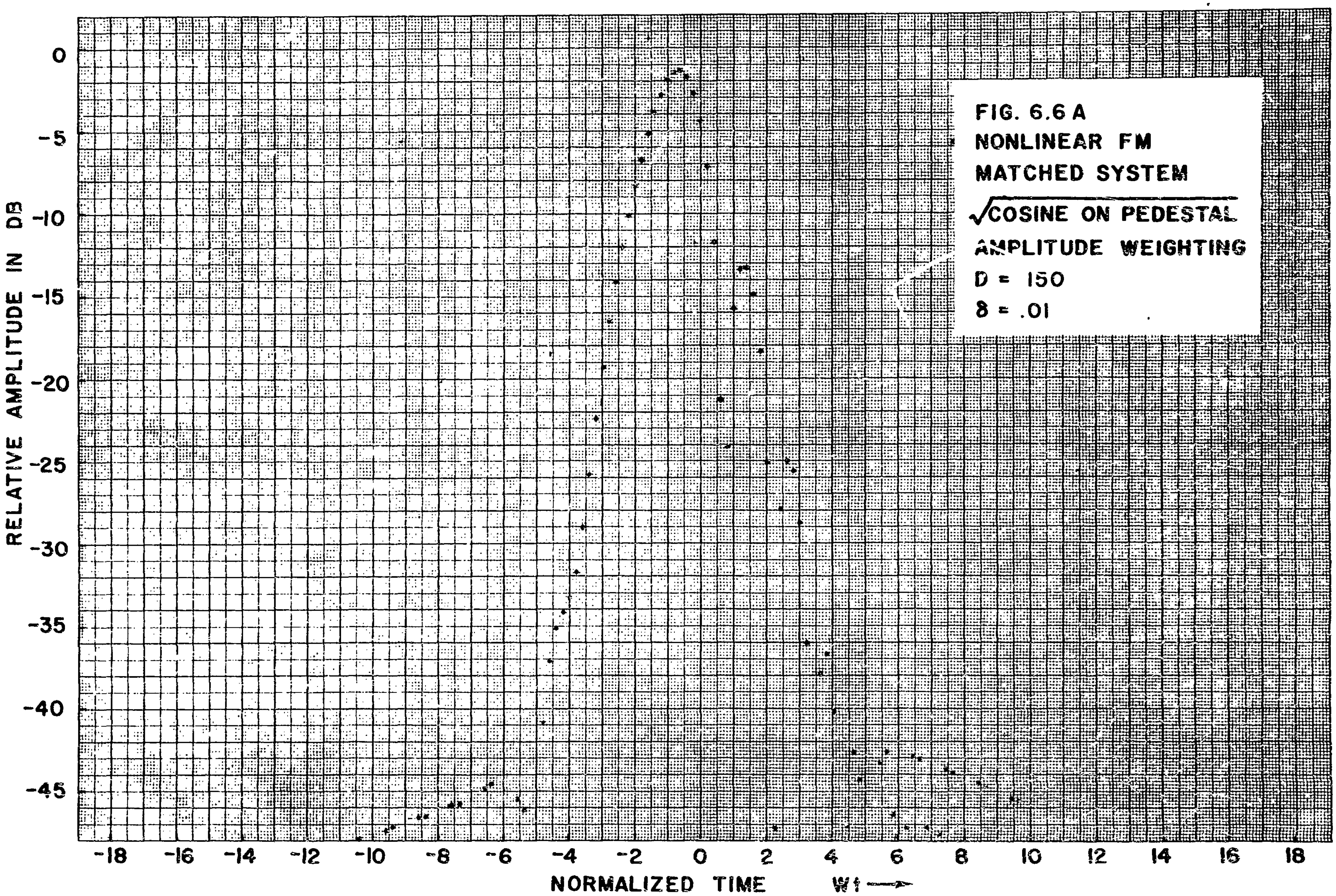


FIG. 6.6 A
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{COSINE ON PEDESTAL}}$
AMPLITUDE WEIGHTING
D = 150
δ = .01
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt →

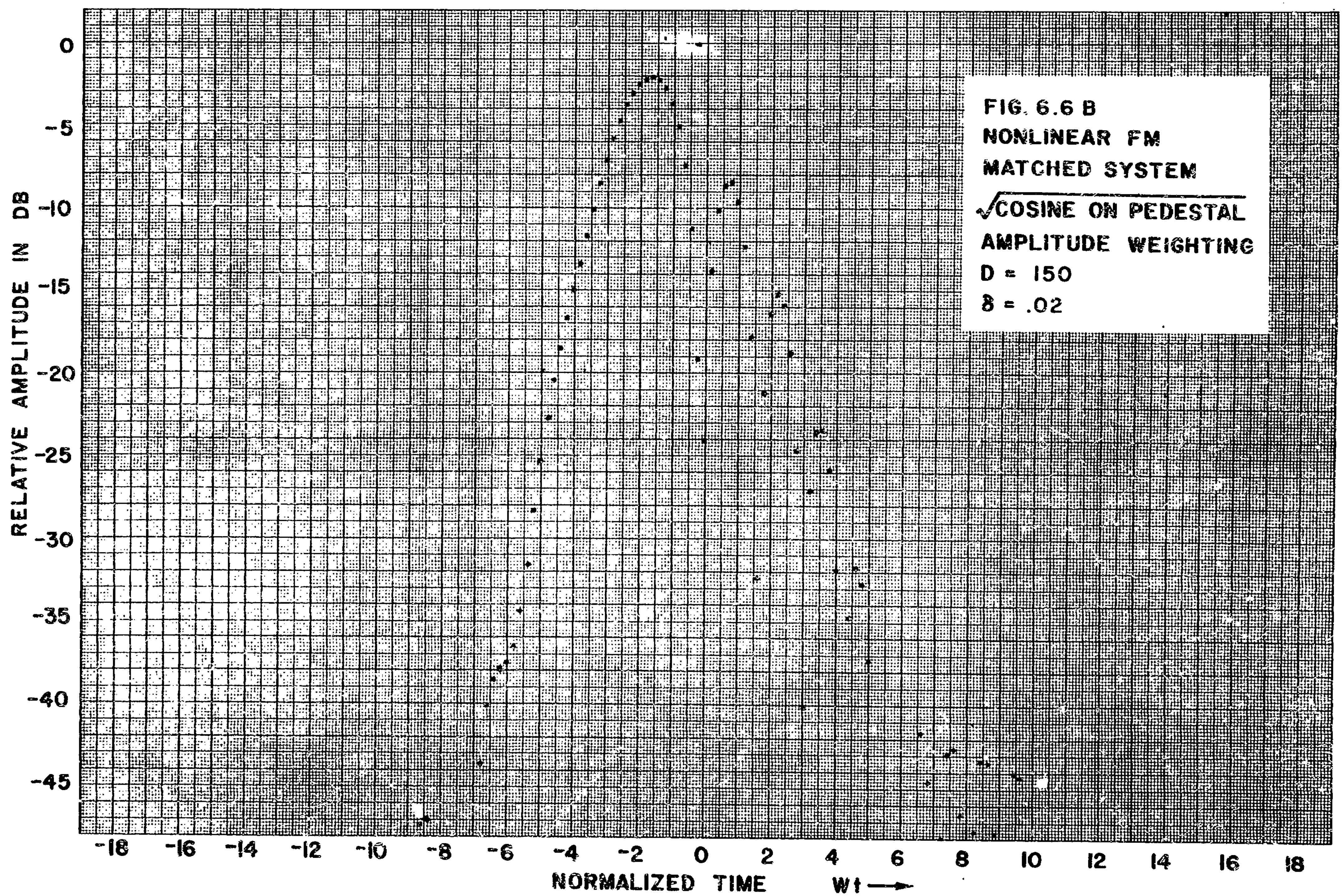


FIG. 6.6 B
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 150
δ = .02
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME Wt

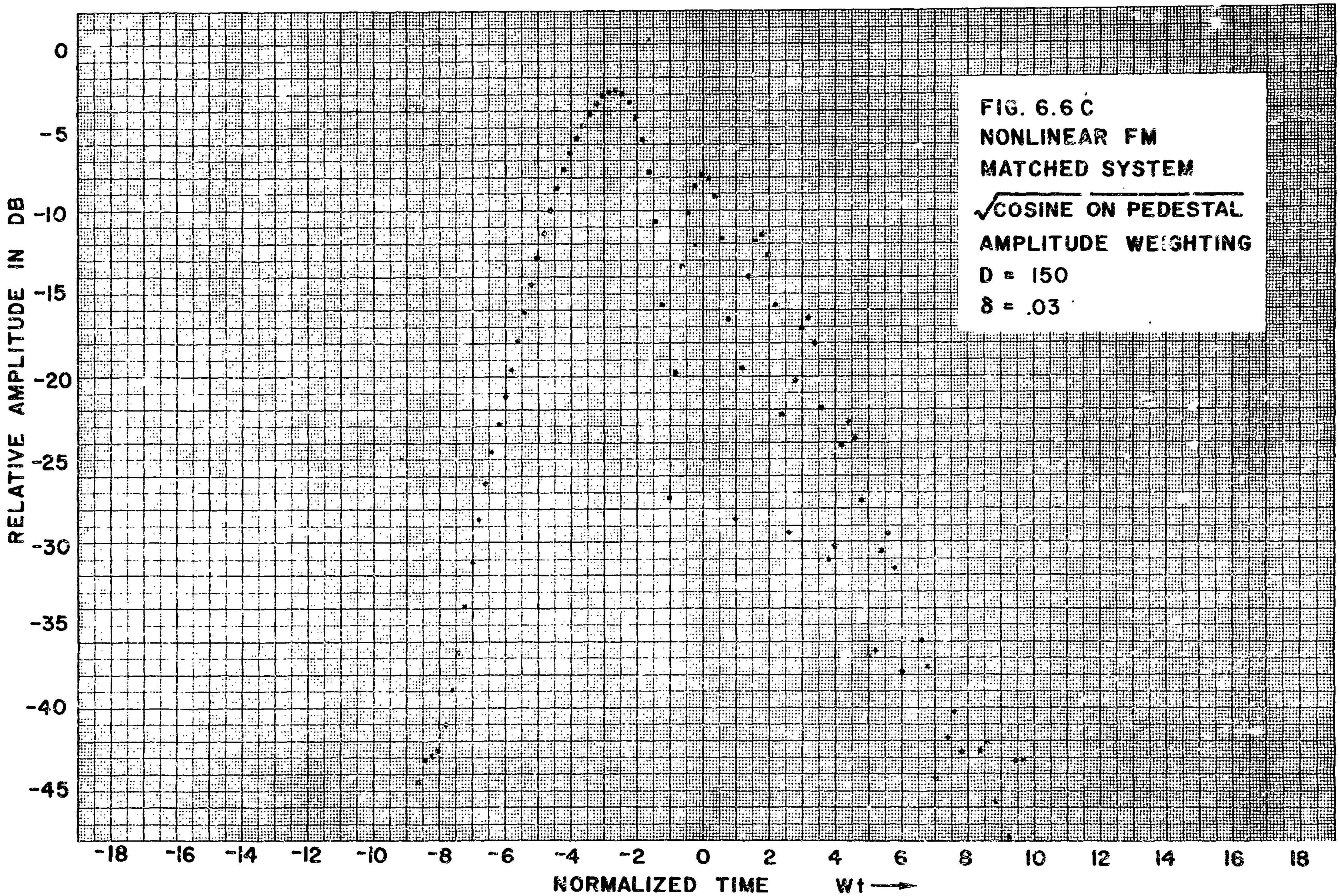


FIG. 6.6 C
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 150
δ = .03
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME Wt

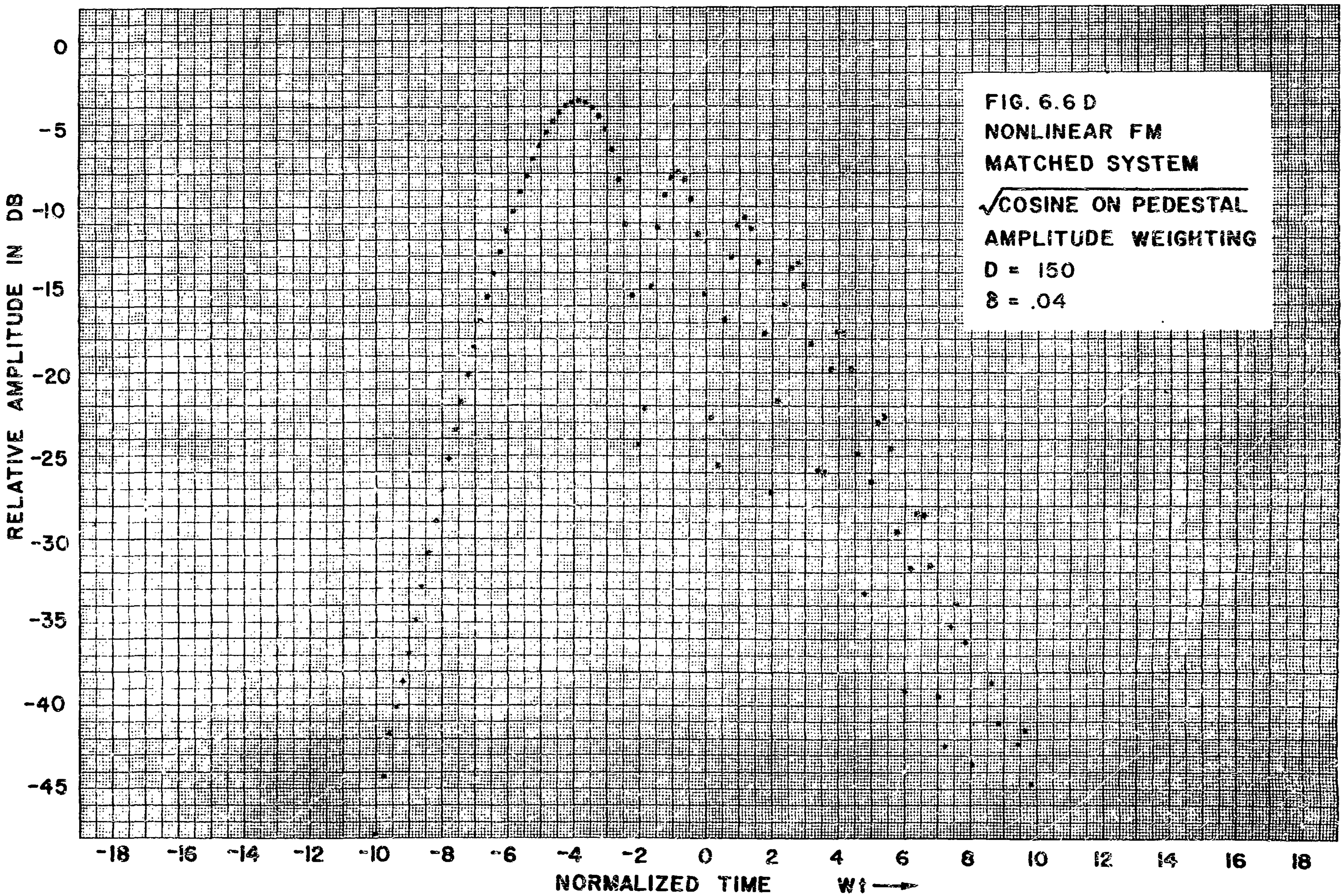


FIG. 6.6 D
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{COSINE ON PEDESTAL}}$
AMPLITUDE WEIGHTING
D = 150
δ = .04
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt

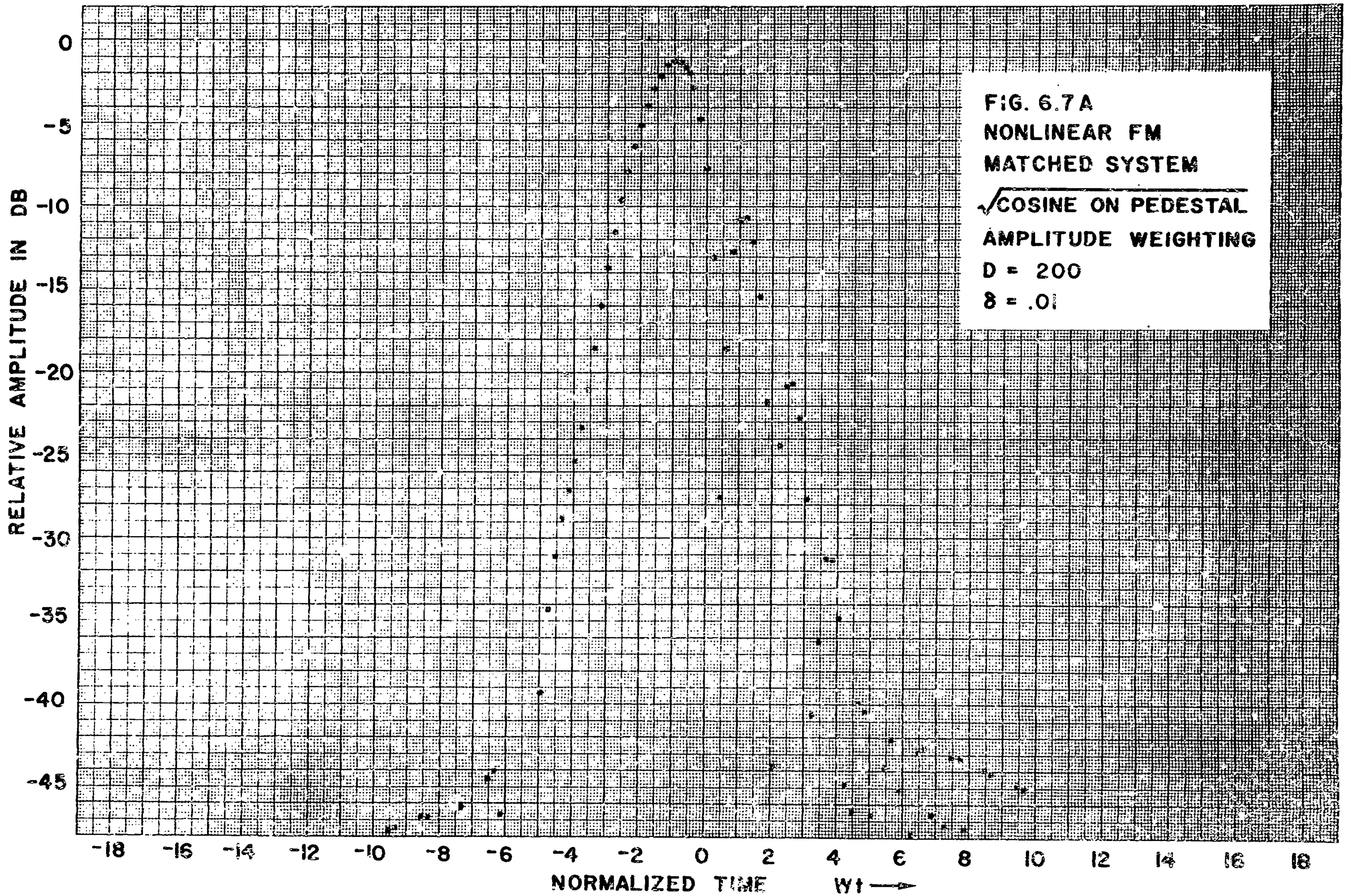


FIG. 6.7A
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 200
δ = .01
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

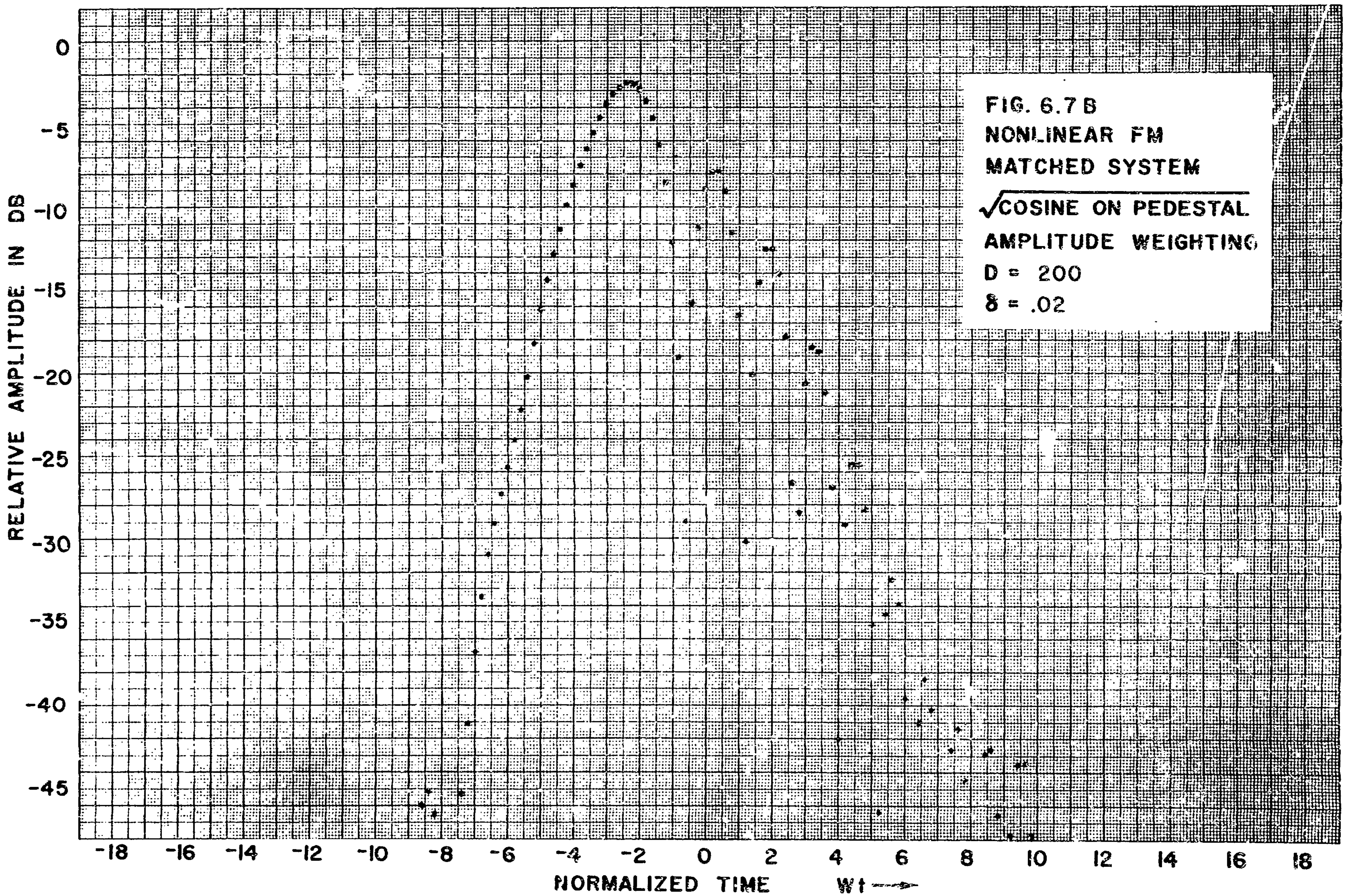


FIG. 6.7 B
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 200
δ = .02
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

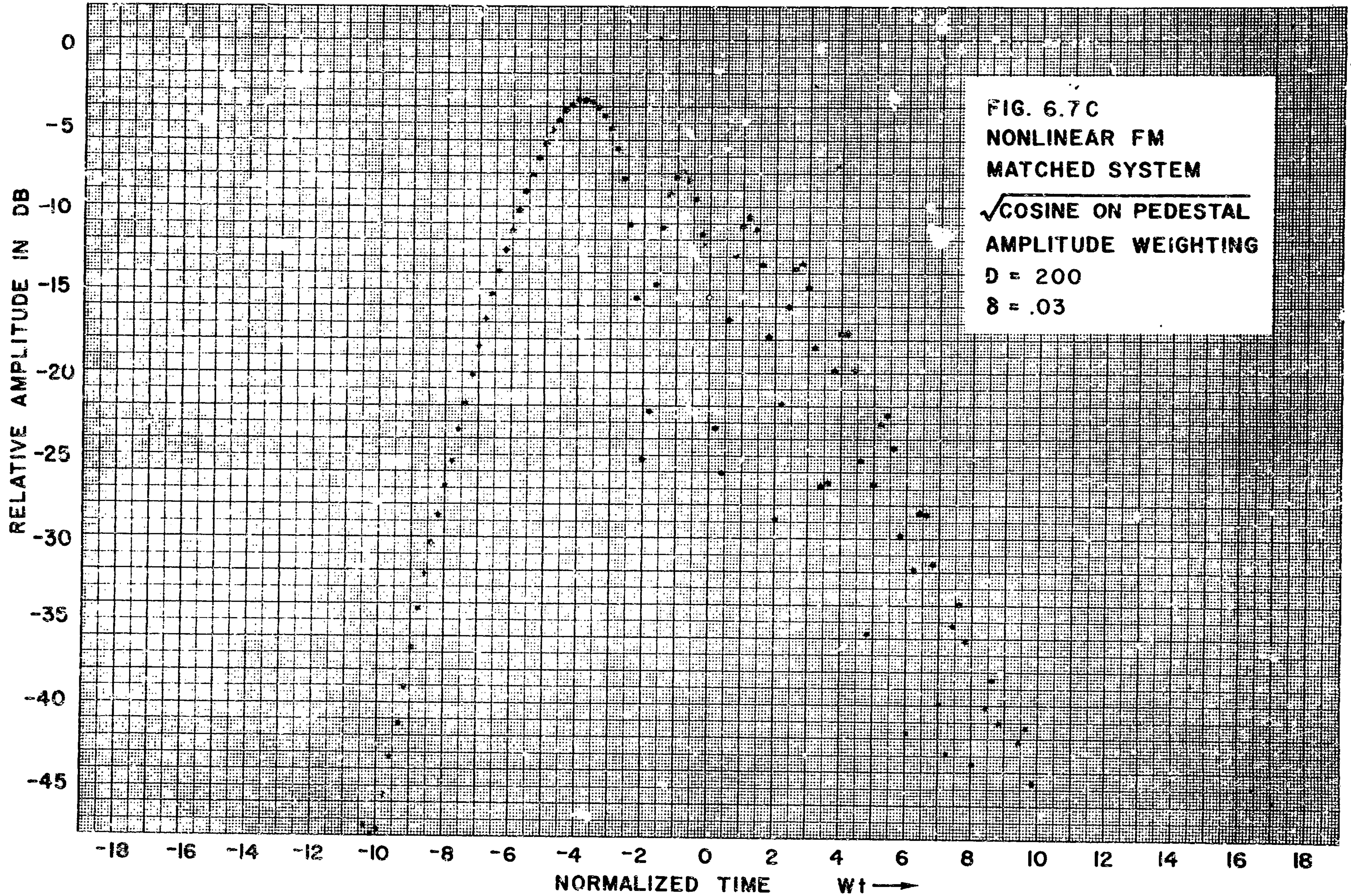


FIG. 6.7C
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 200
δ = .03
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt

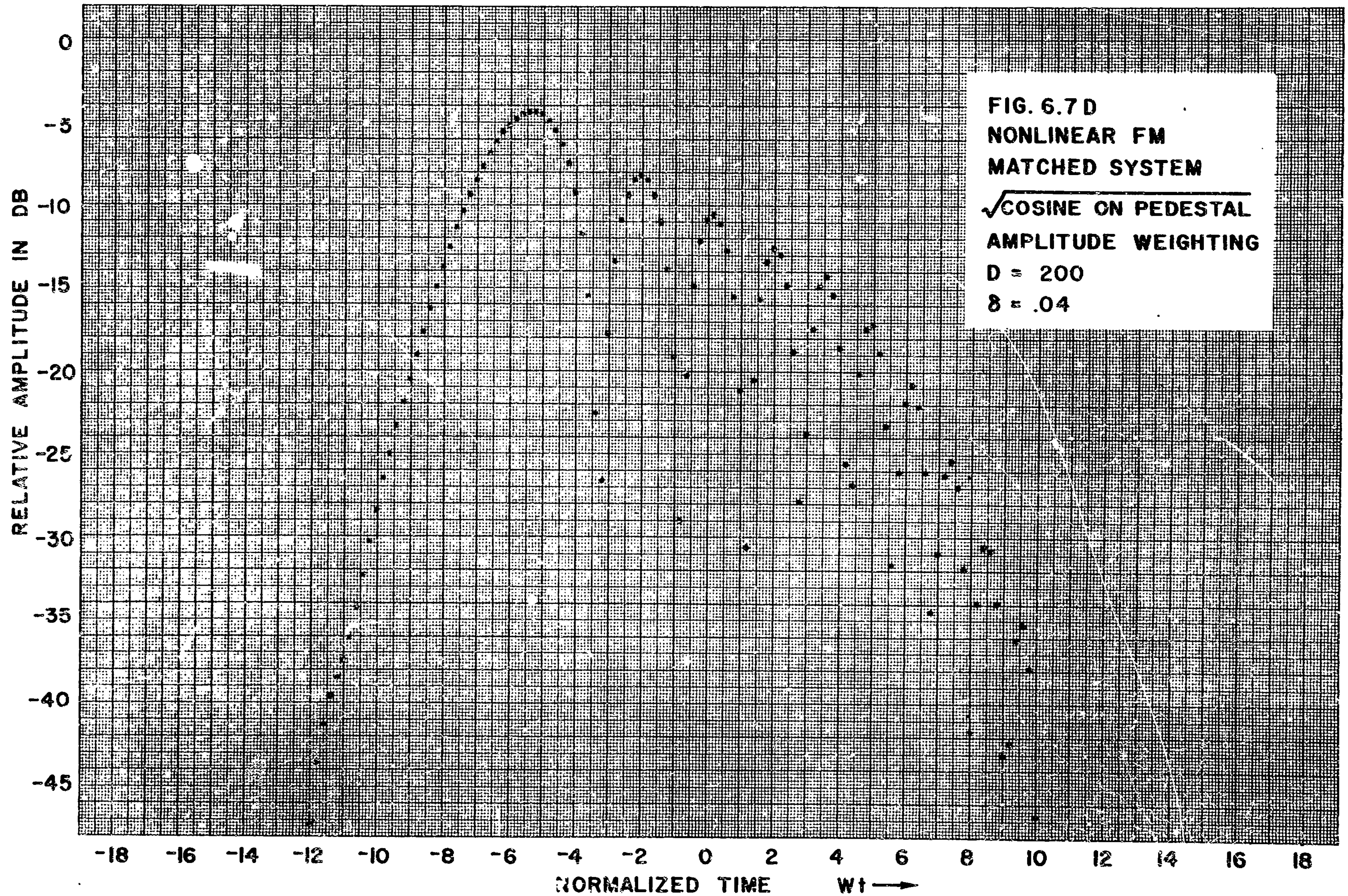


FIG. 6.7 D
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{COSINE ON PEDESTAL}}$
AMPLITUDE WEIGHTING
D = 200
δ = .04
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt

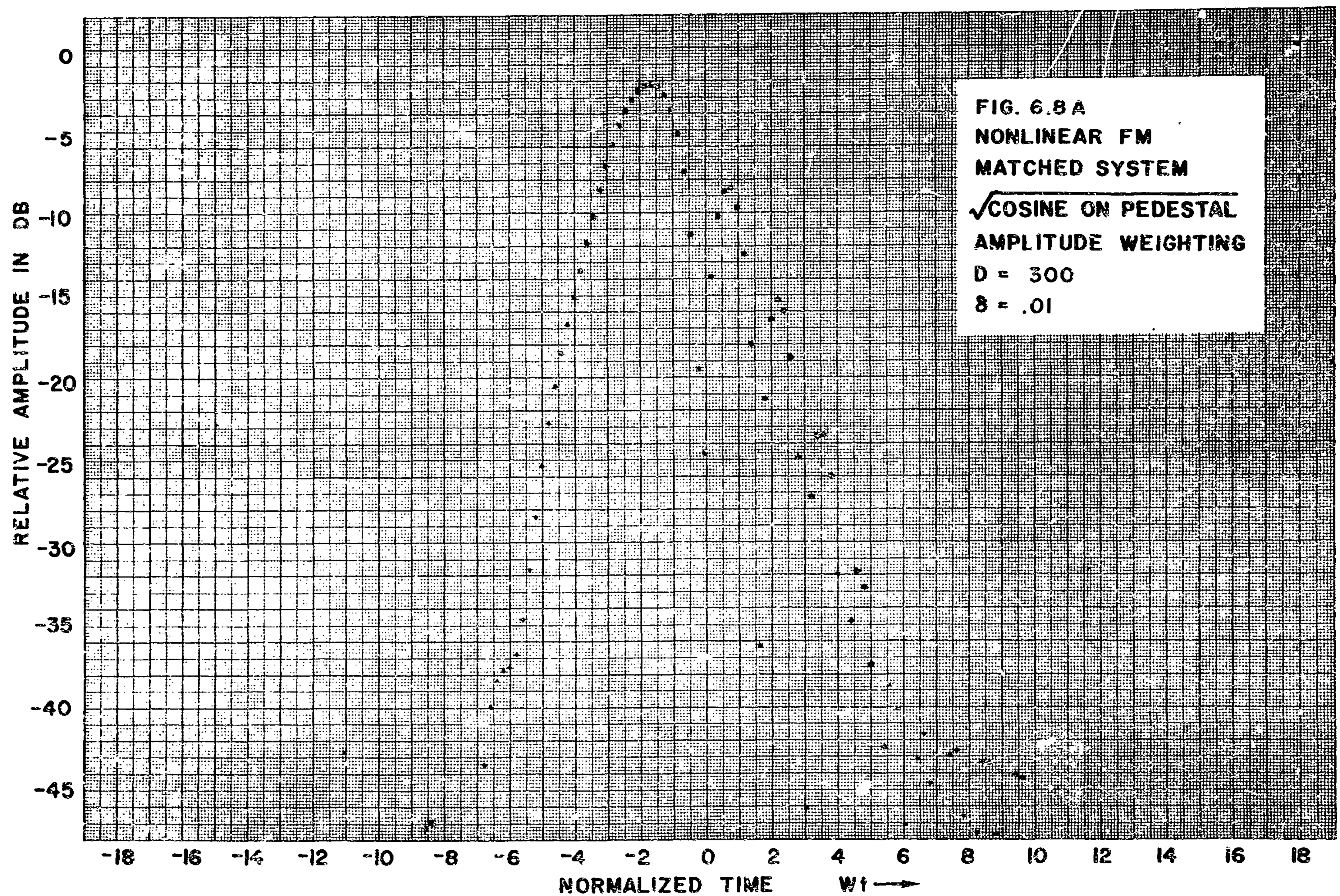


FIG. 6.8 A
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 300
8 = .01
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME Wt →

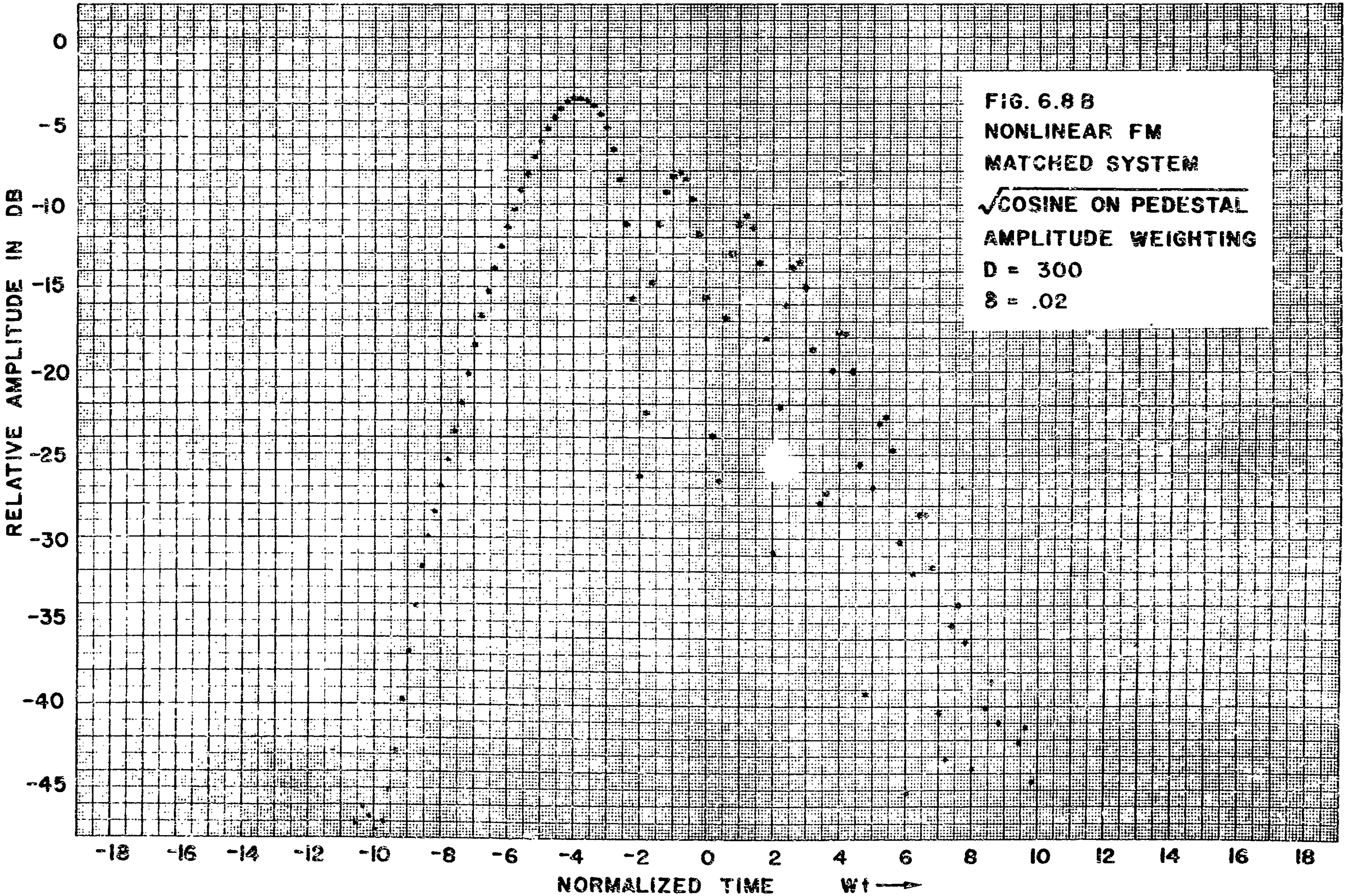


FIG. 6.8 B
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{COSINE ON PEDESTAL}}$
AMPLITUDE WEIGHTING
D = 300
δ = .02
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt

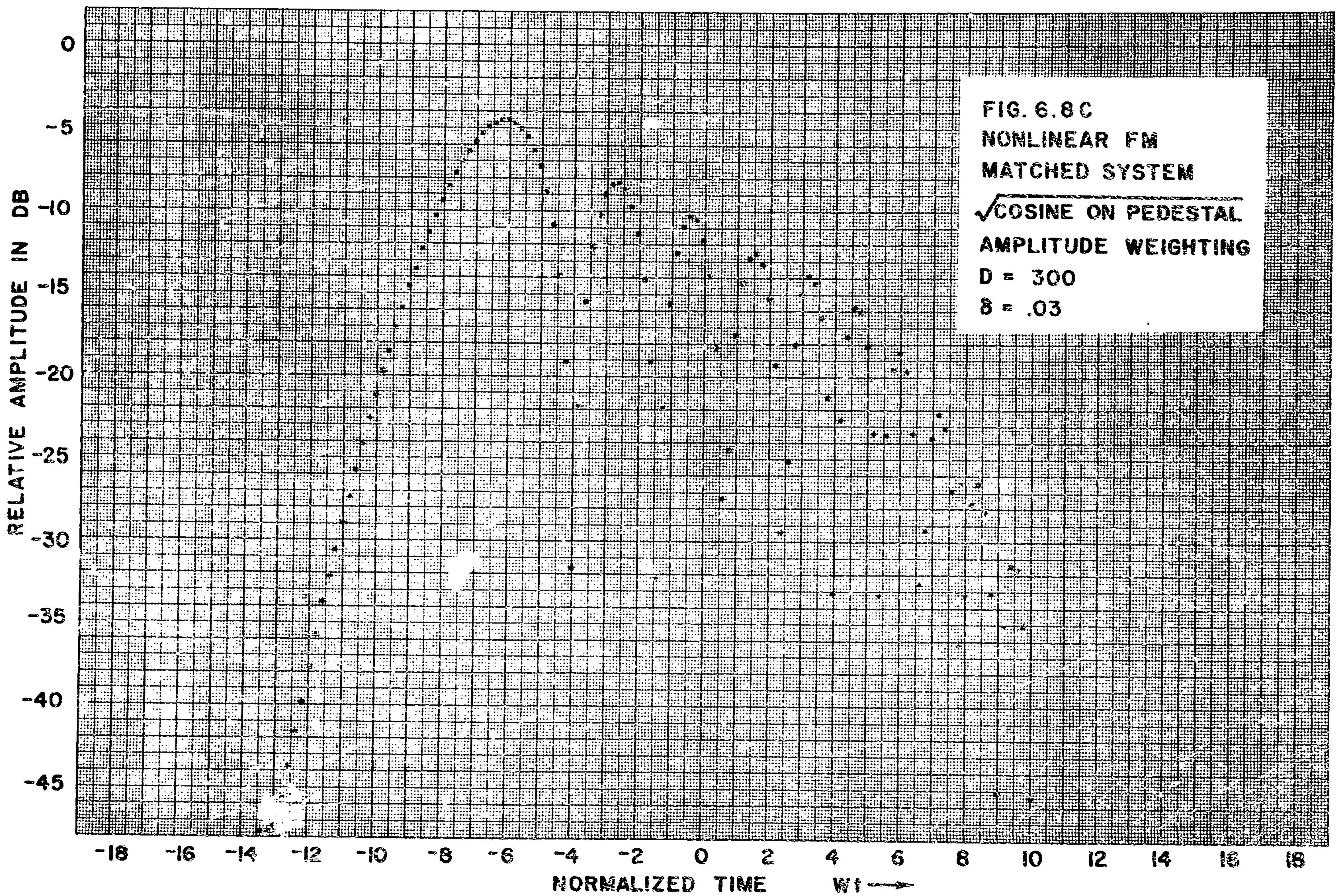


FIG. 6.8C
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 300
δ = .03
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt →

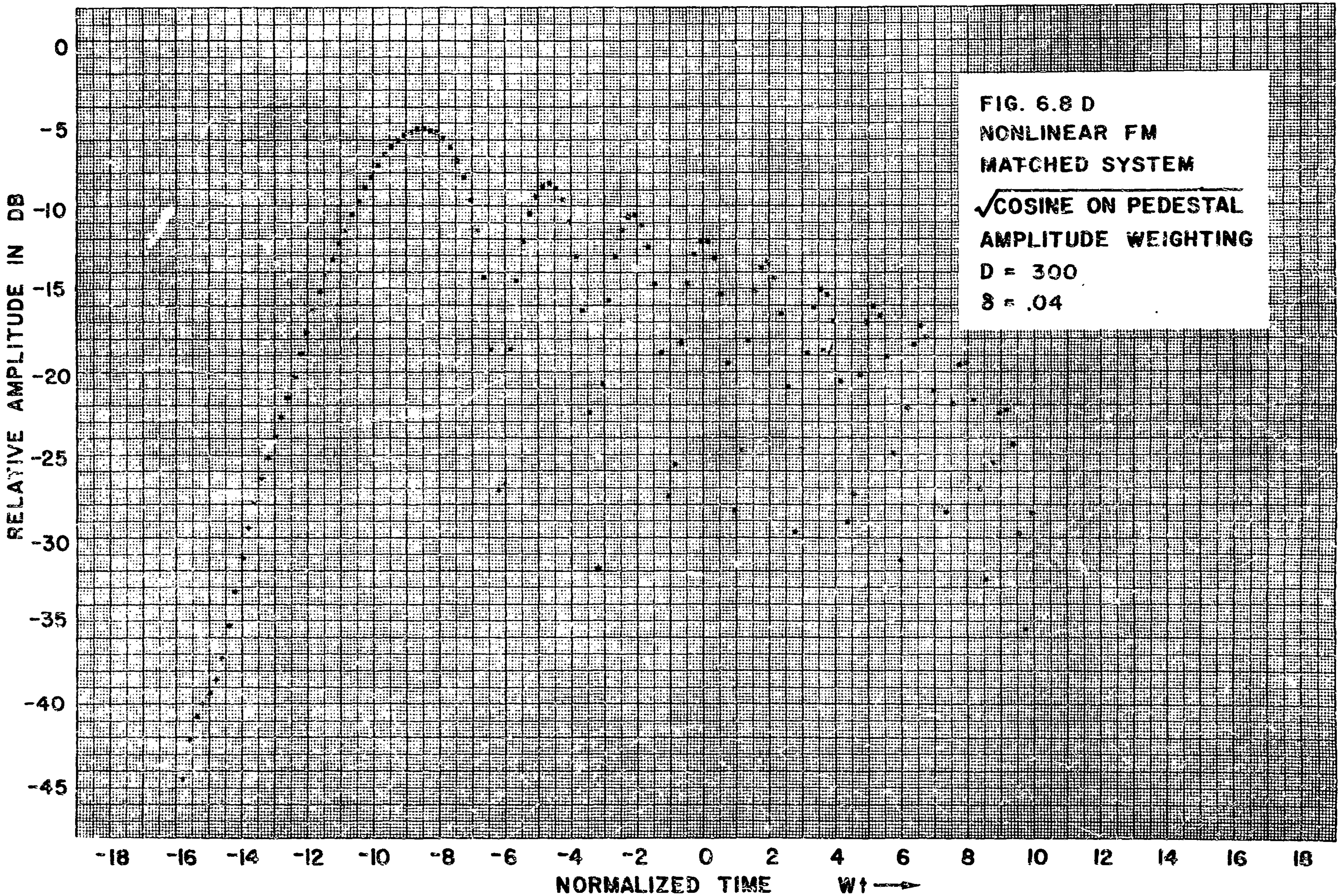


FIG. 6.8 D
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 300
δ = .04
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

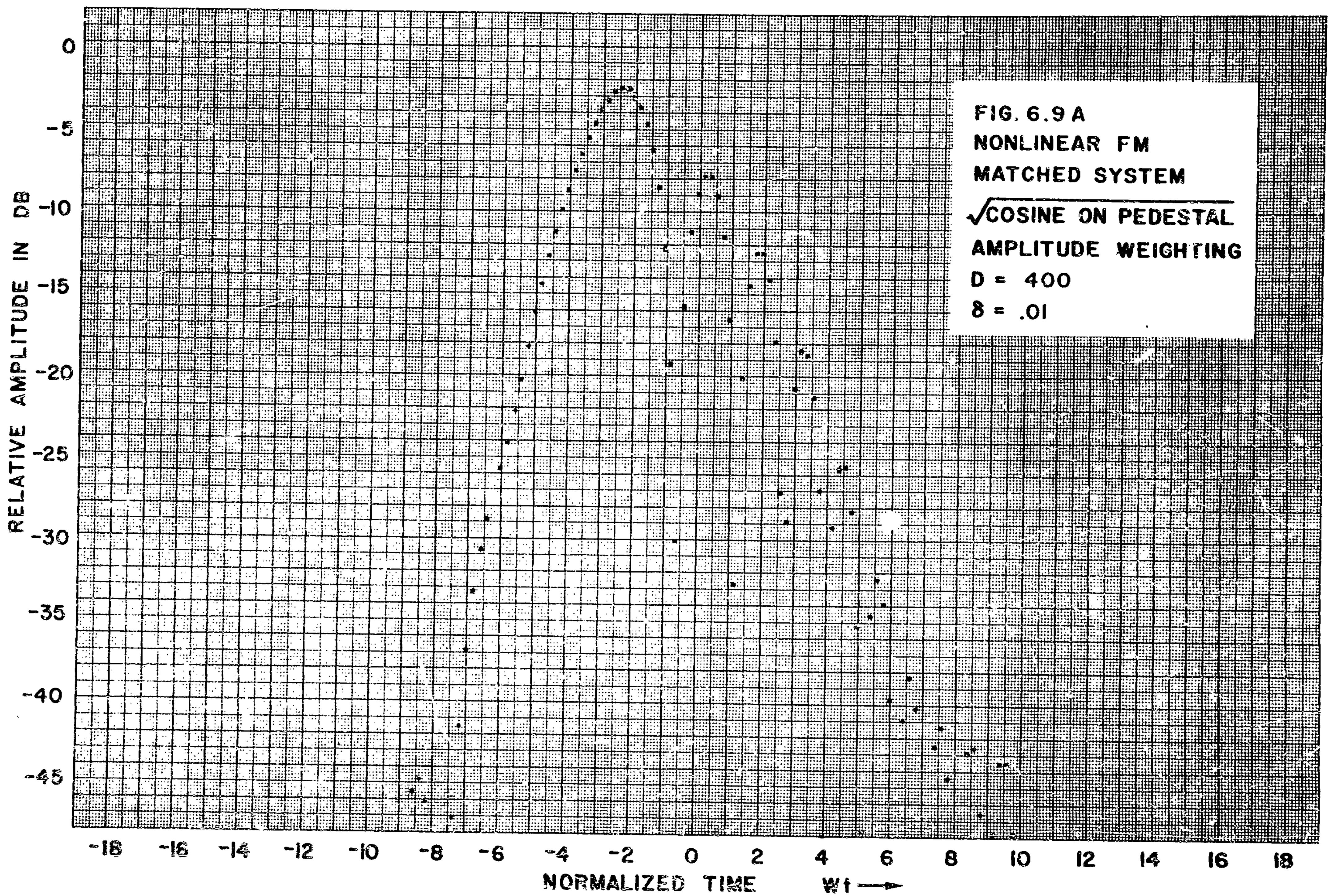


FIG. 6.9 A
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 400
8 = .01
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

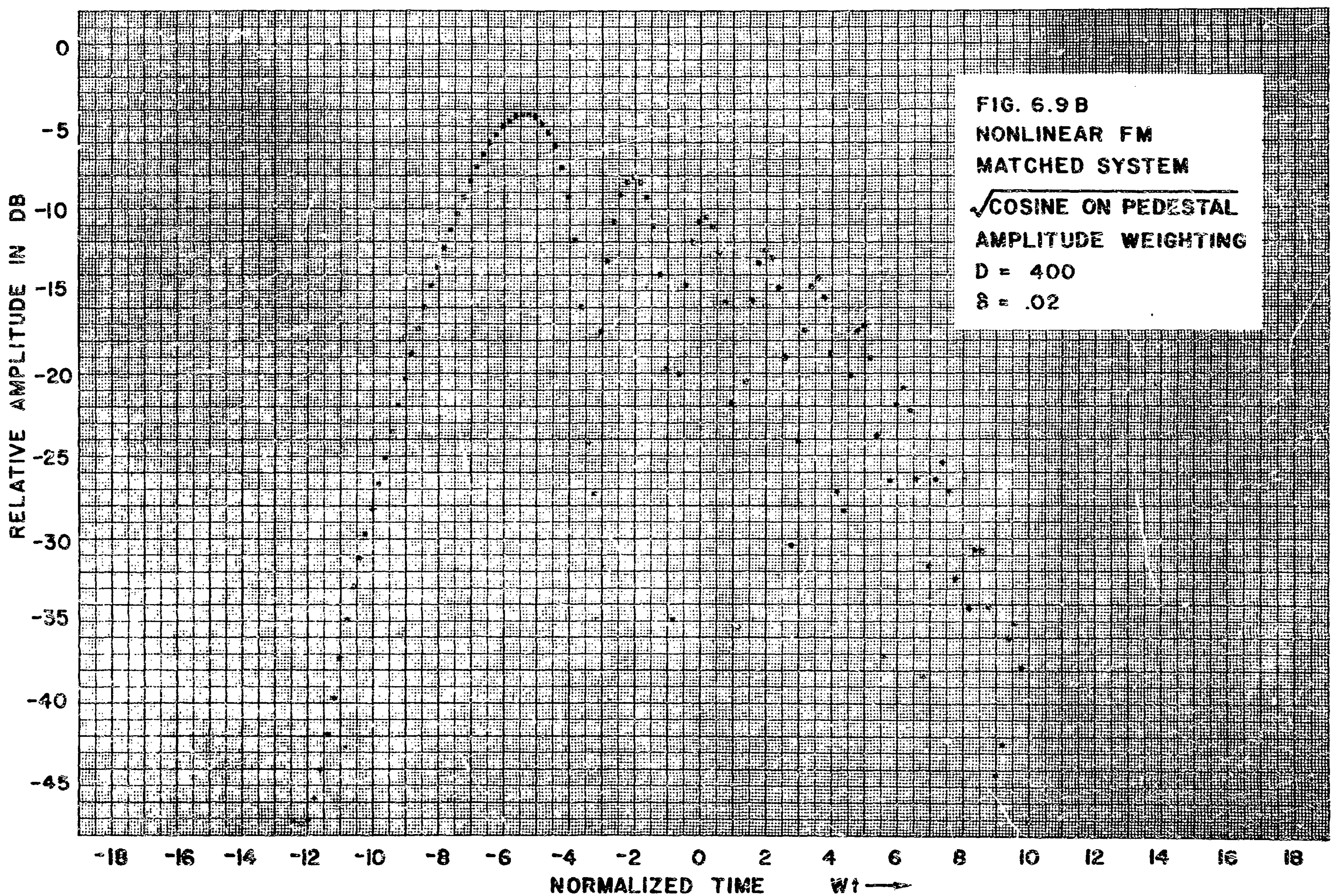


FIG. 6.9 B
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 400
δ = .02
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME Wt →

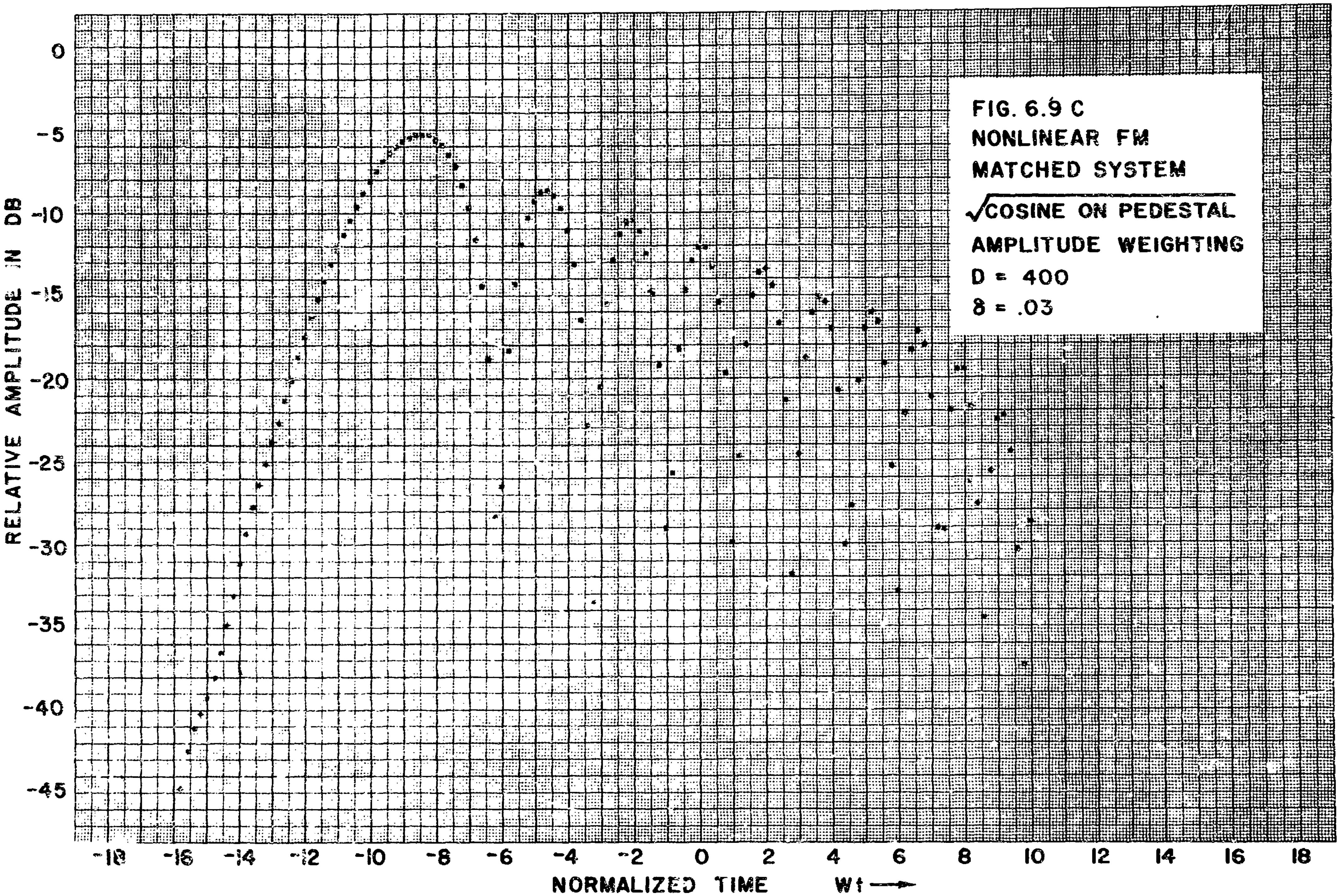


FIG. 6.9 C
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 400
8 = .03
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

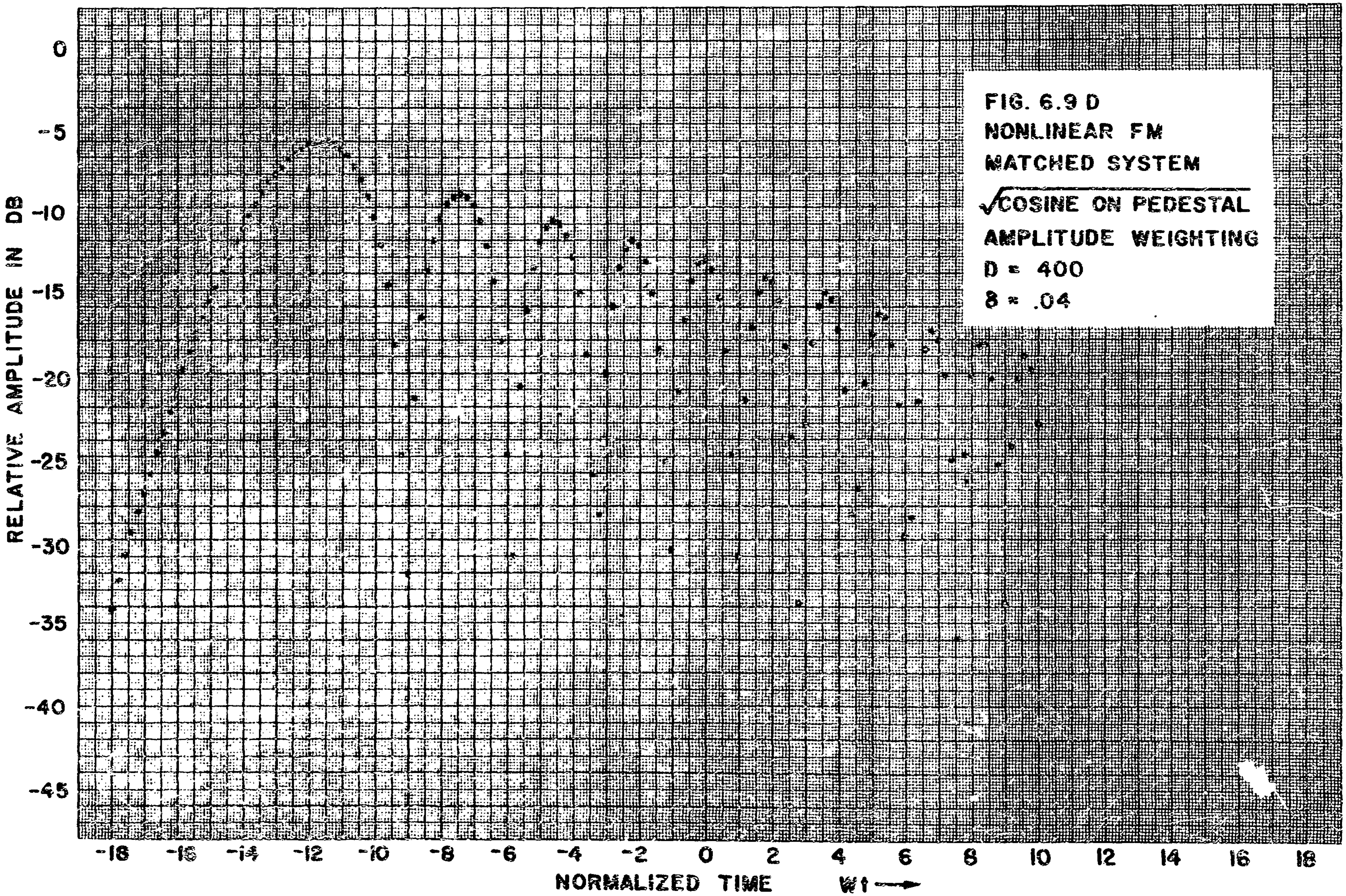


FIG. 6.9 D
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 400
8 = .04
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16 18
NORMALIZED TIME Wt

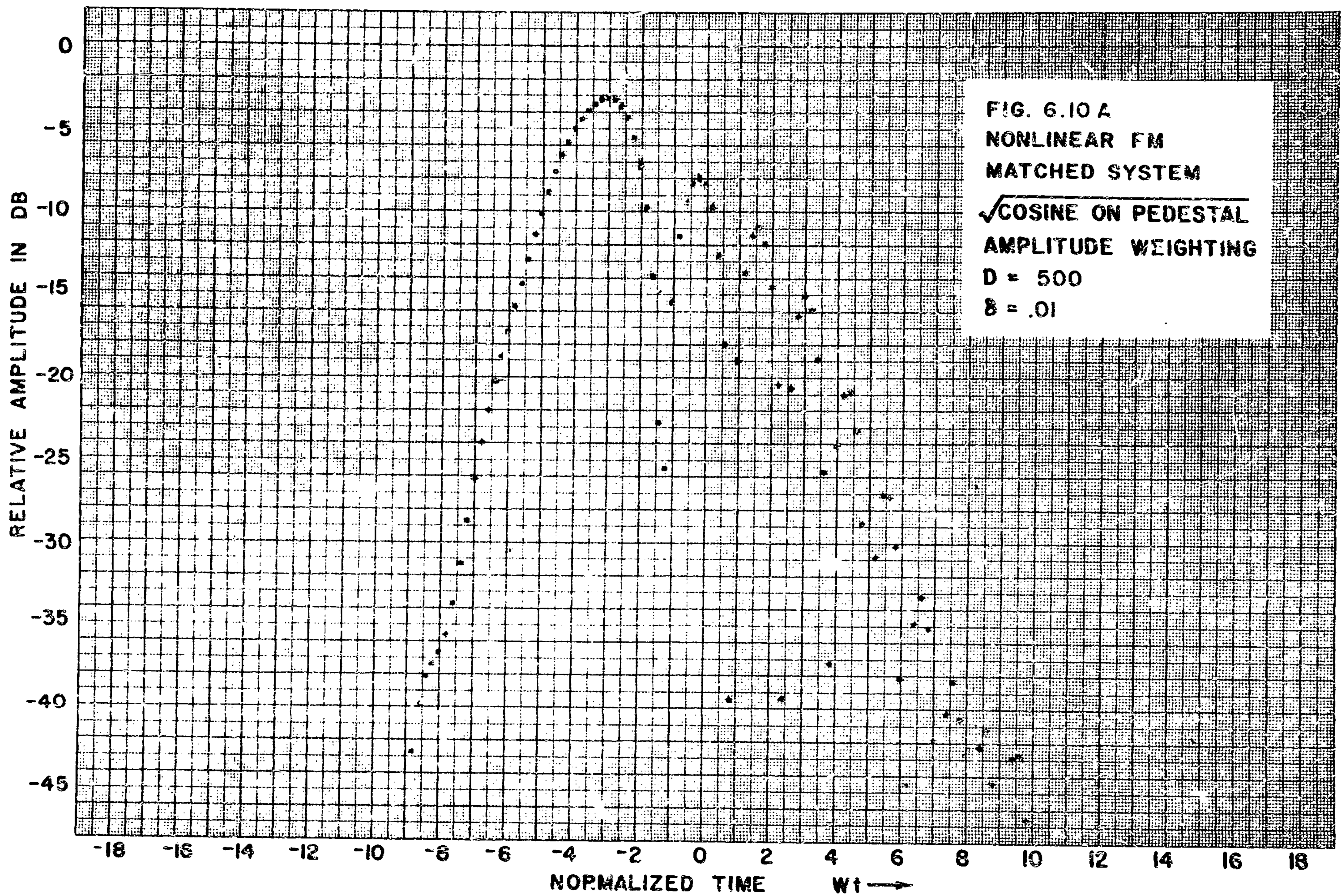


FIG. 6.10 A
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 500
δ = .01
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt →

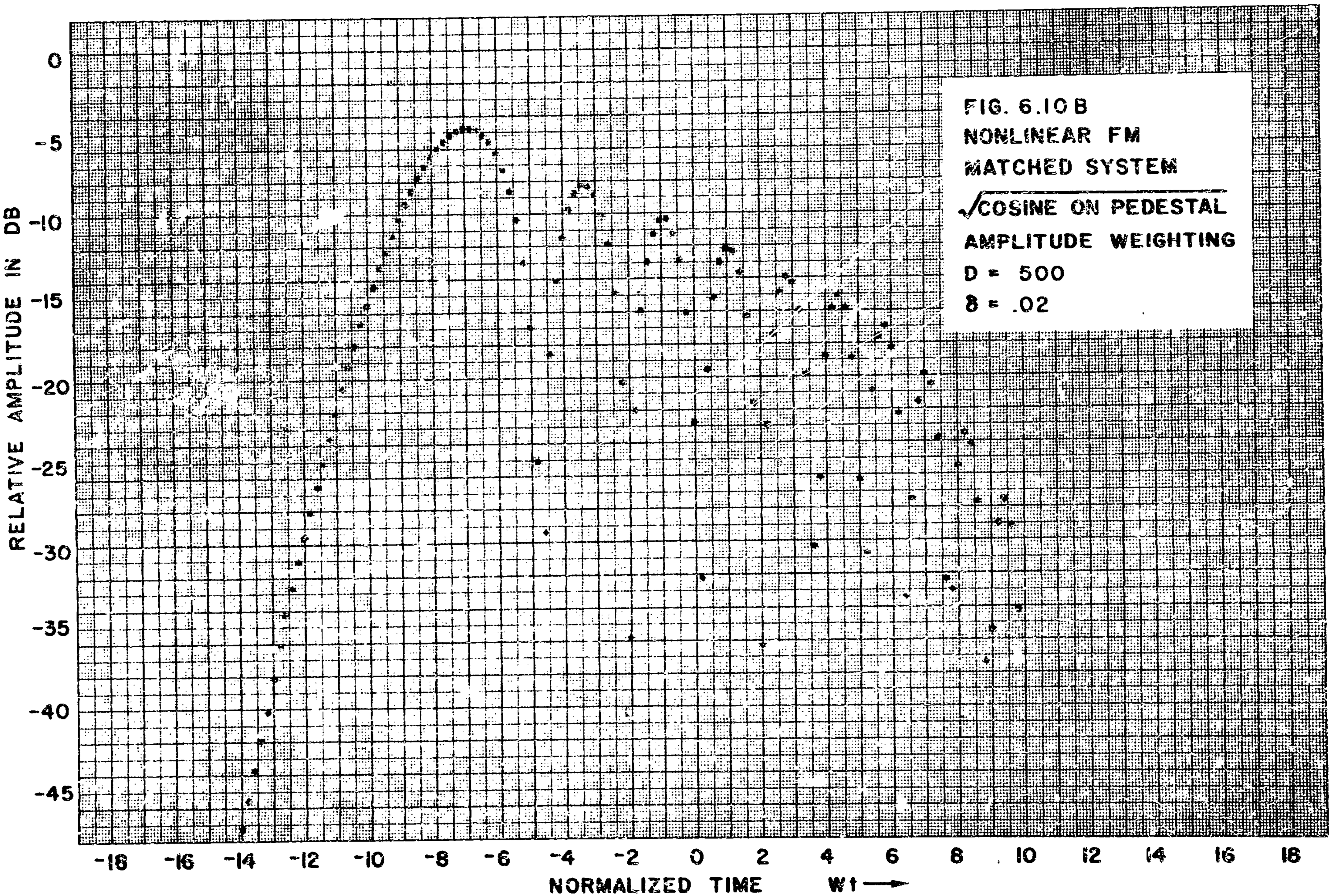


FIG. 6.10 B
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{COSINE ON PEDESTAL}}$
AMPLITUDE WEIGHTING
D = 500
δ = .02
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

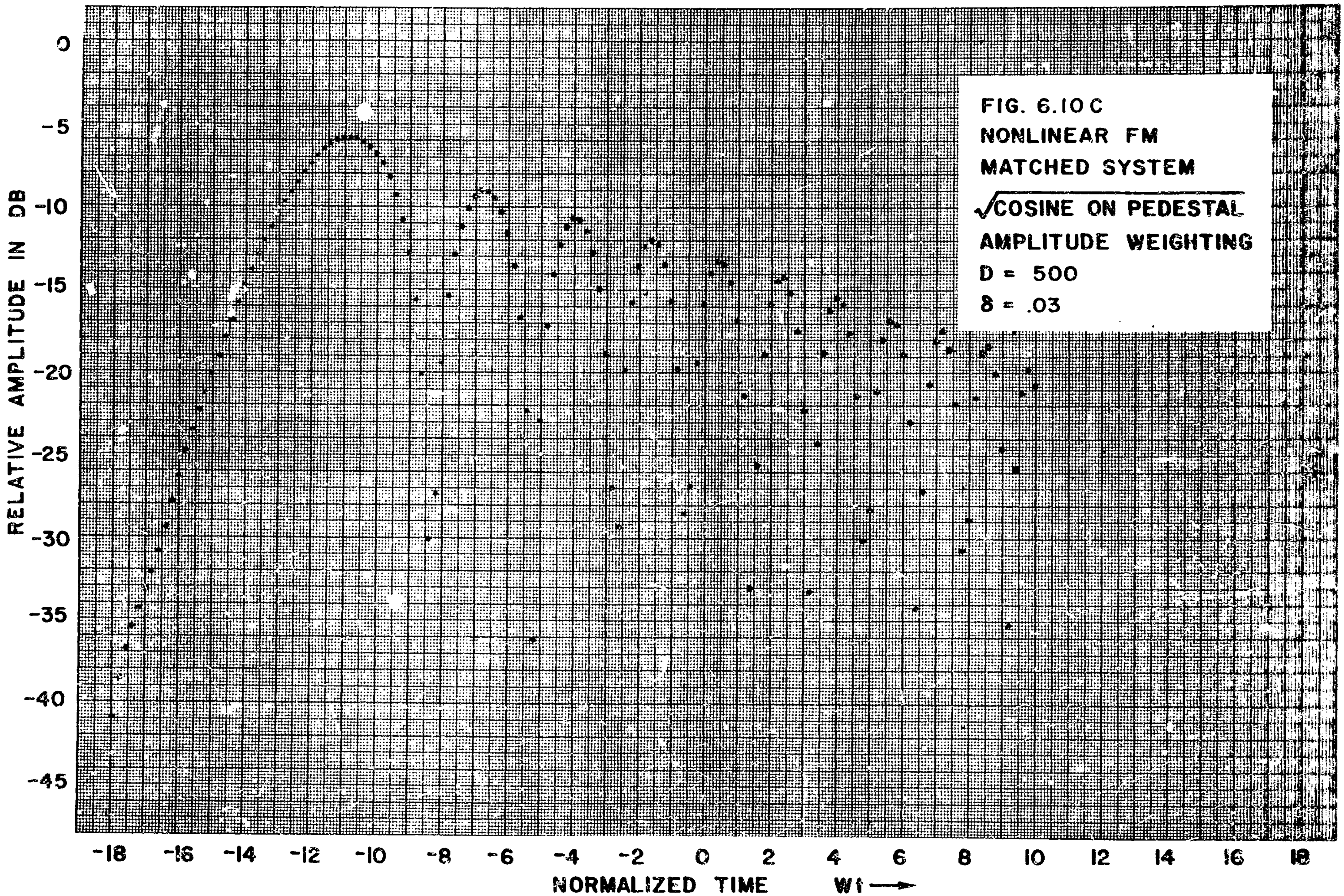


FIG. 6.10C
NONLINEAR FM
MATCHED SYSTEM
√COSINE ON PEDESTAL
AMPLITUDE WEIGHTING
D = 500
δ = .03
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME
Wt →

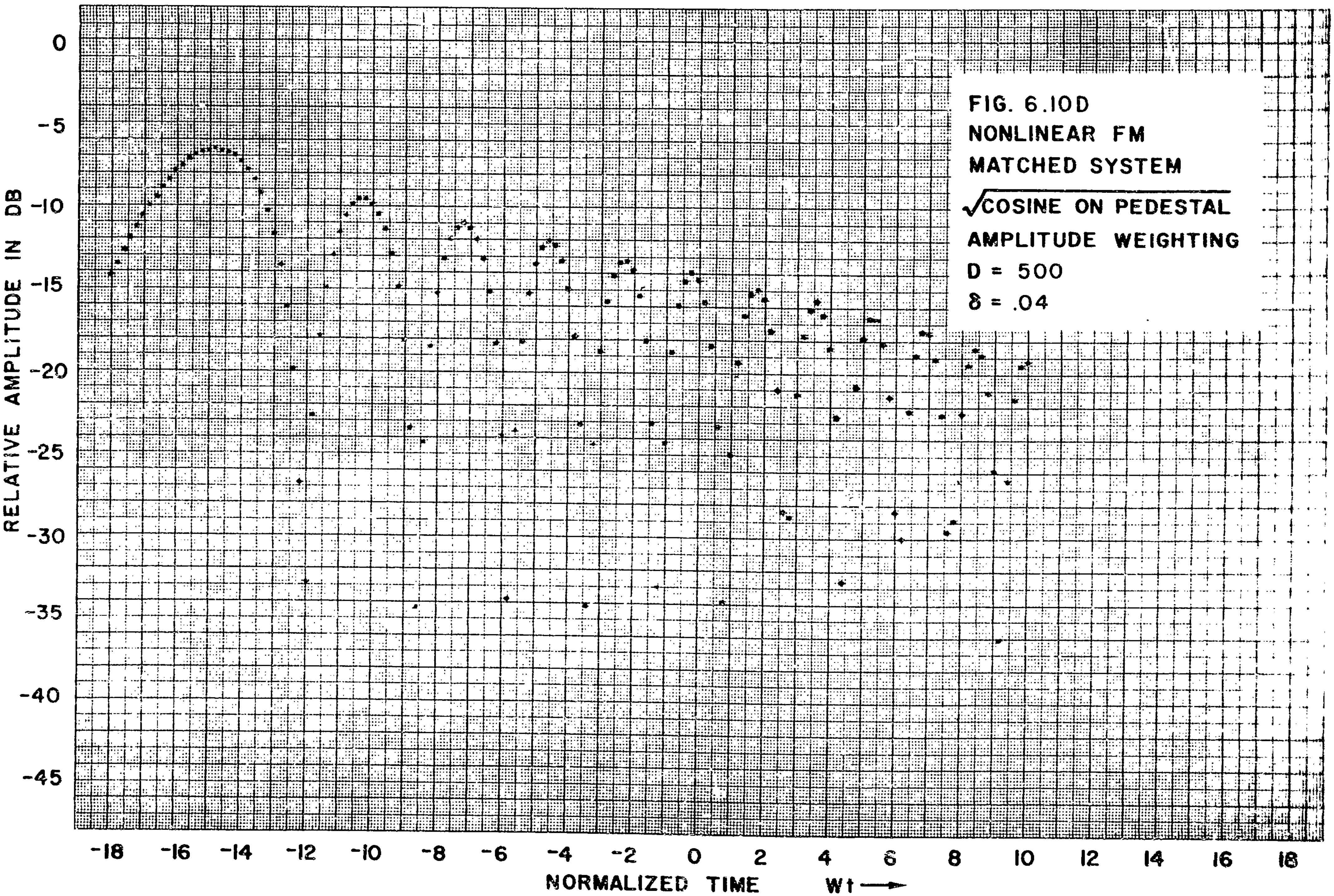


FIG. 6.10D
NONLINEAR FM
MATCHED SYSTEM
$\sqrt{\text{COSINE ON PEDESTAL}}$
AMPLITUDE WEIGHTING
D = 500
δ = .04
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
NORMALIZED TIME Wt

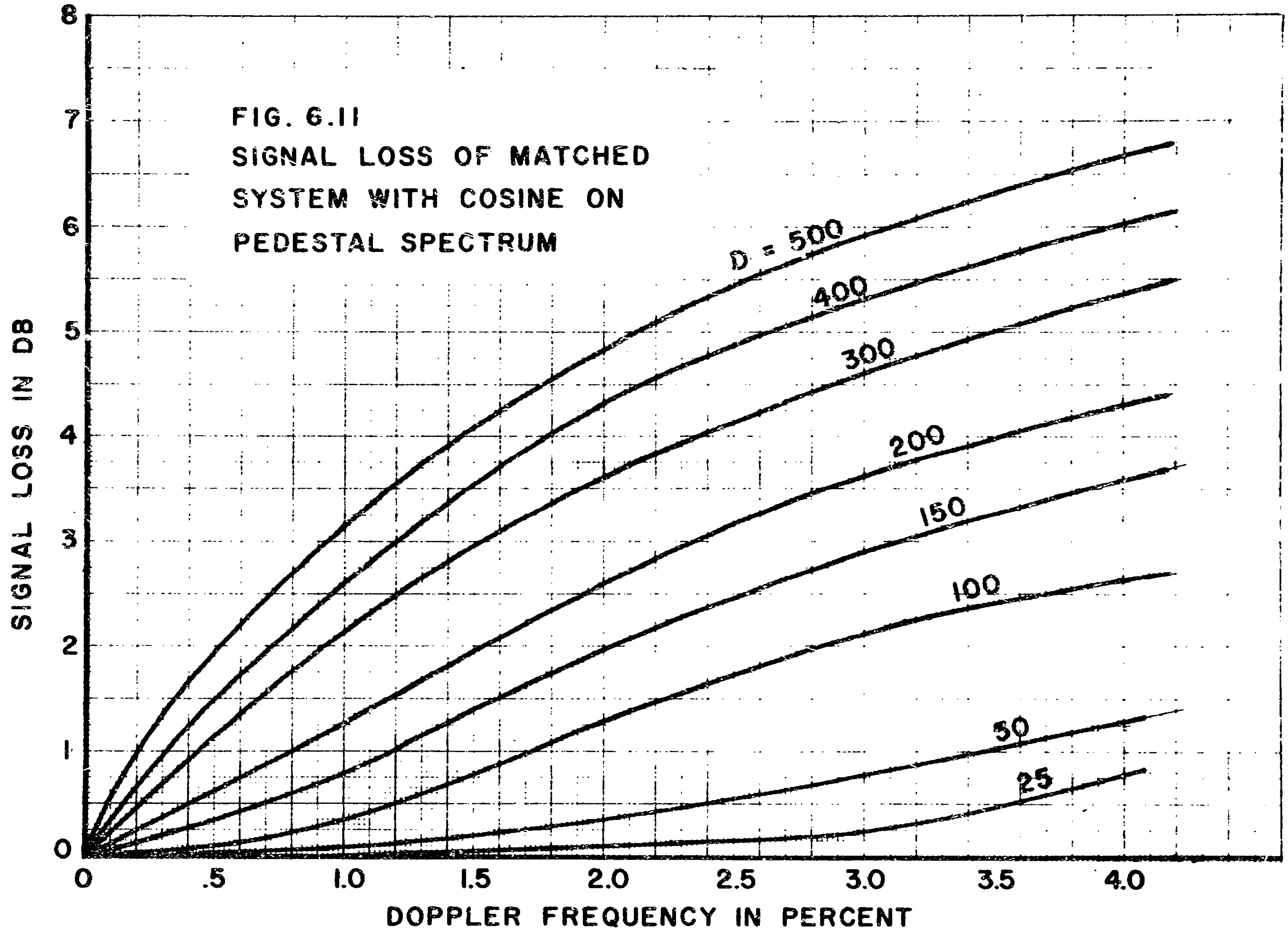


FIG. 6.11
SIGNAL LOSS OF MATCHED SYSTEM WITH COSINE ON PEDESTAL SPECTRUM

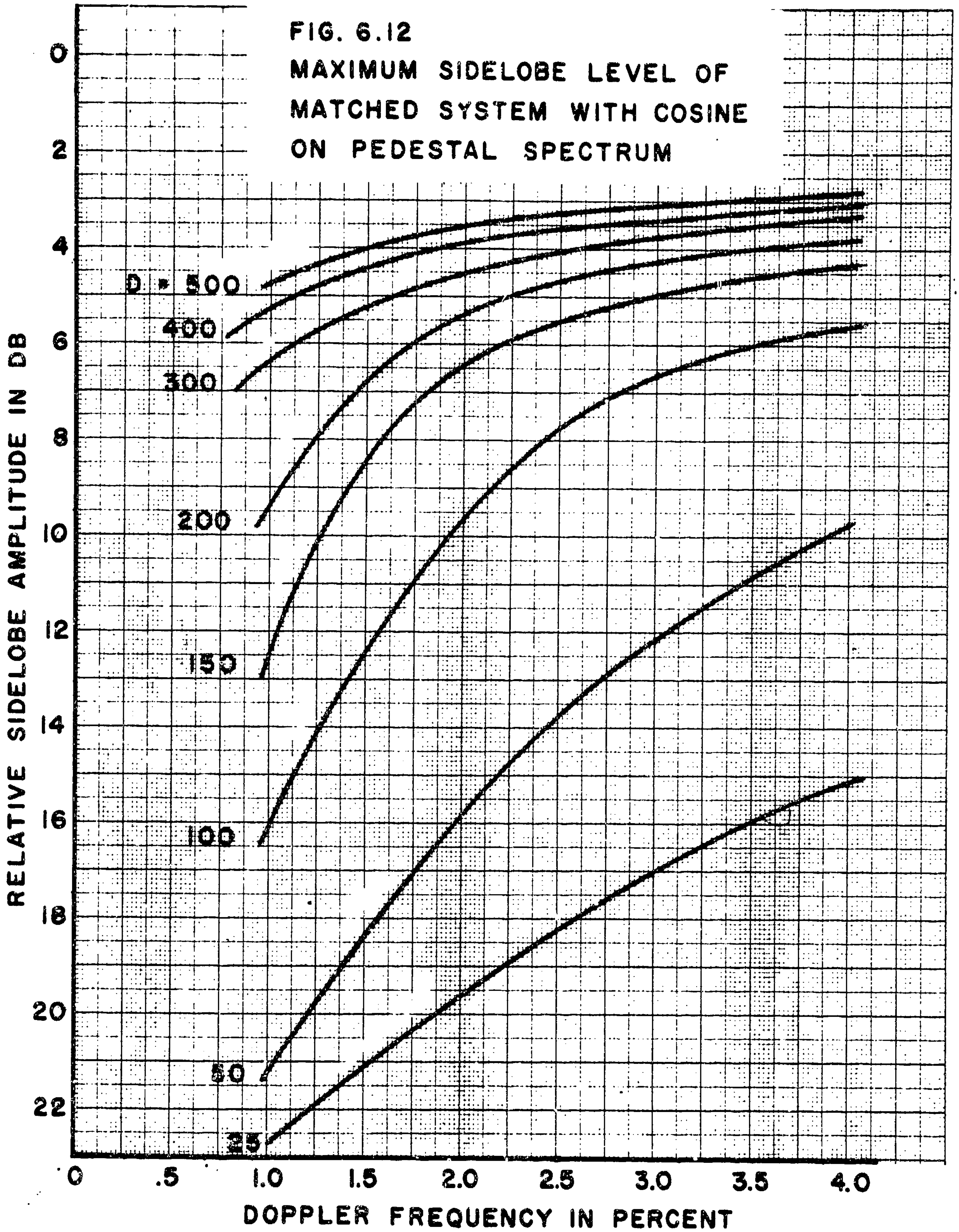


FIG. 6.12
MAXIMUM SIDELOBE LEVEL OF MATCHED SYSTEM WITH COSINE ON PEDESTAL SPECTRUM

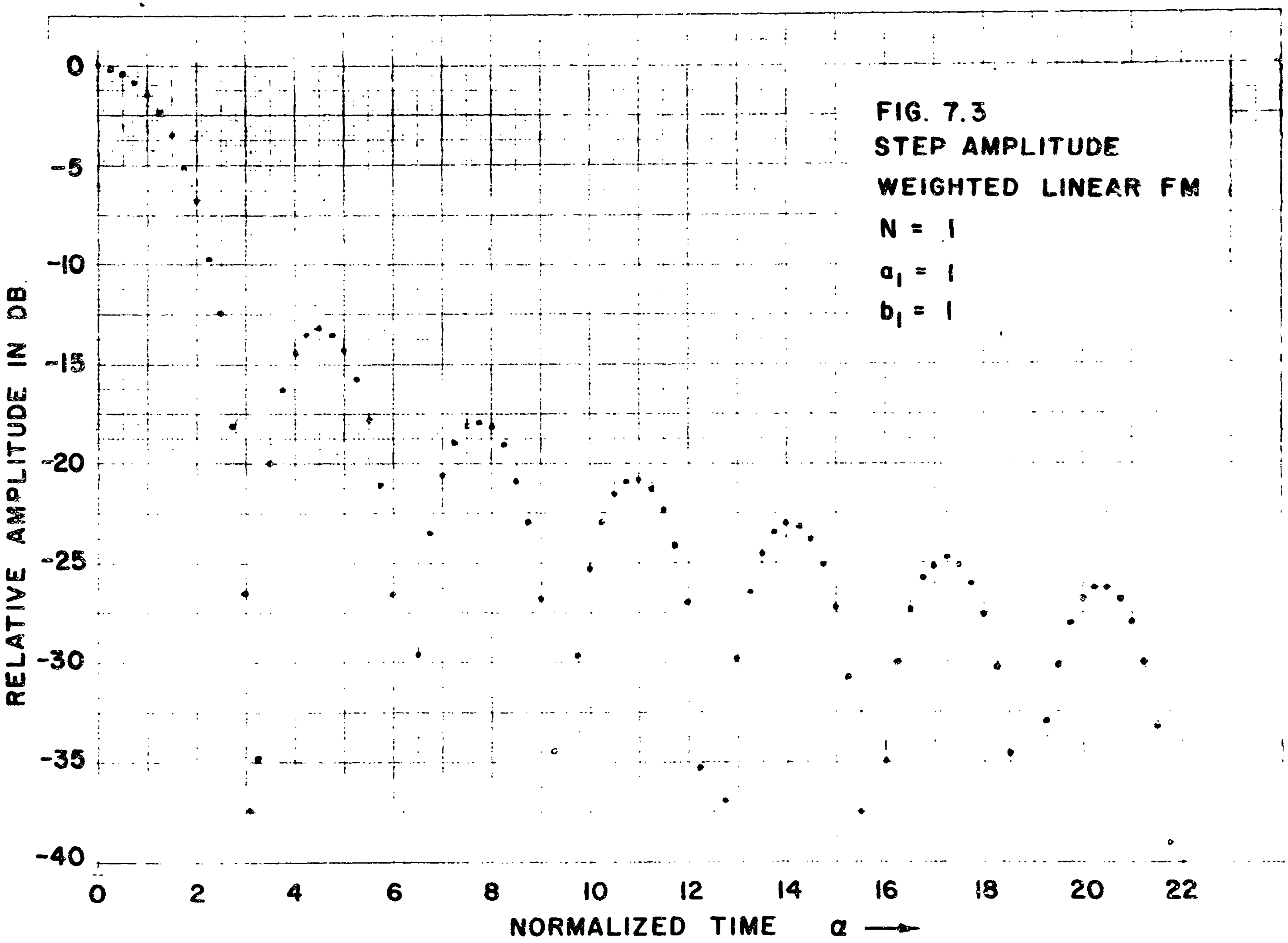


FIG. 7.3
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 1
a_1 = 1
b_1 = 1
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
0
2
4
6
8
10
12
14
16
18
20
22
NORMALIZED TIME α

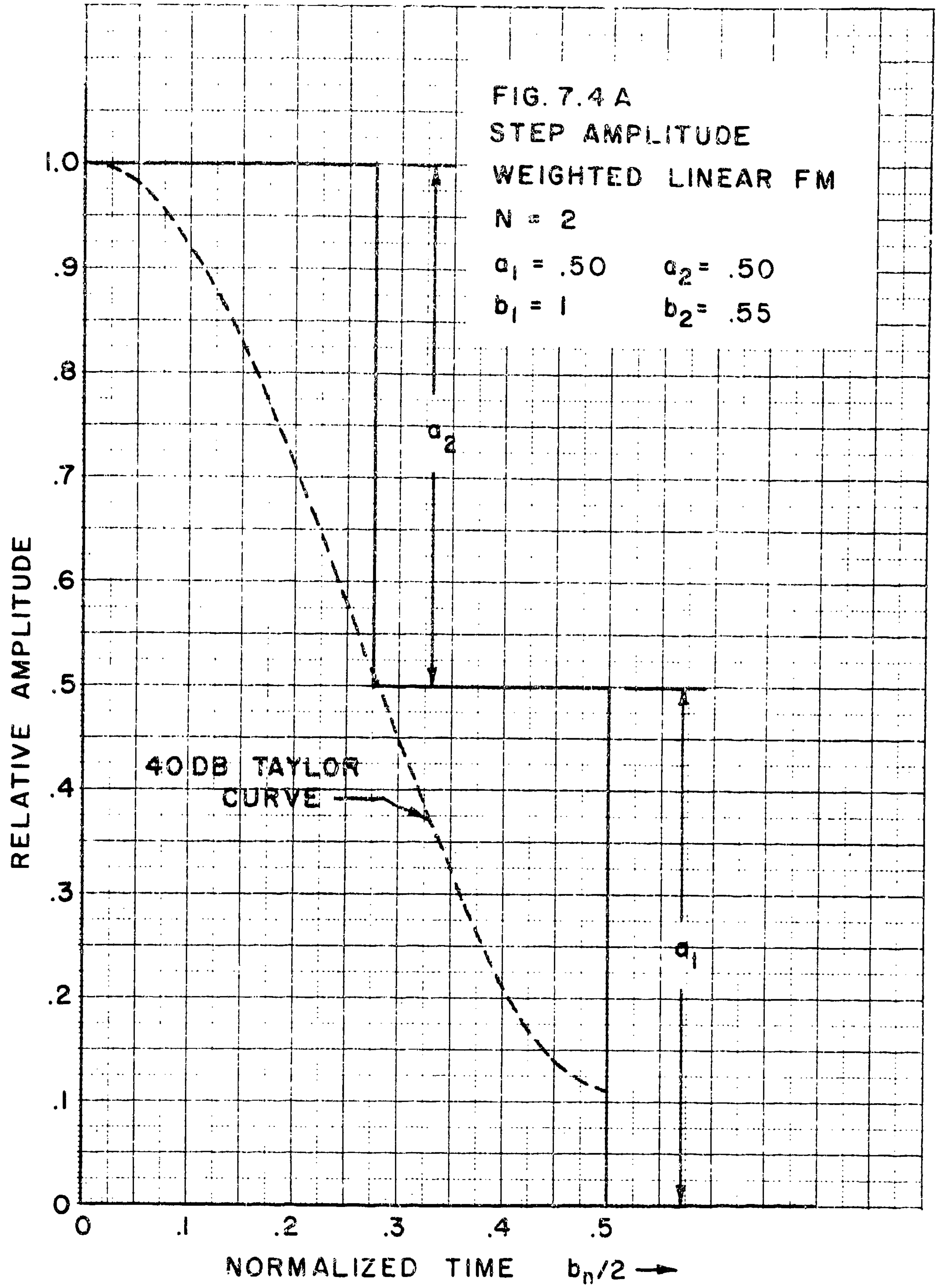


FIG. 7.4 A
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 2
a_1 = .50 a_2 = .50
b_1 = 1 b_2 = .55
RELATIVE AMPLITUDE
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
a_2
a_1
40 DB TAYLOR CURVE
0
.1
.2
.3
.4
.5
NORMALIZED TIME $b_n/2$

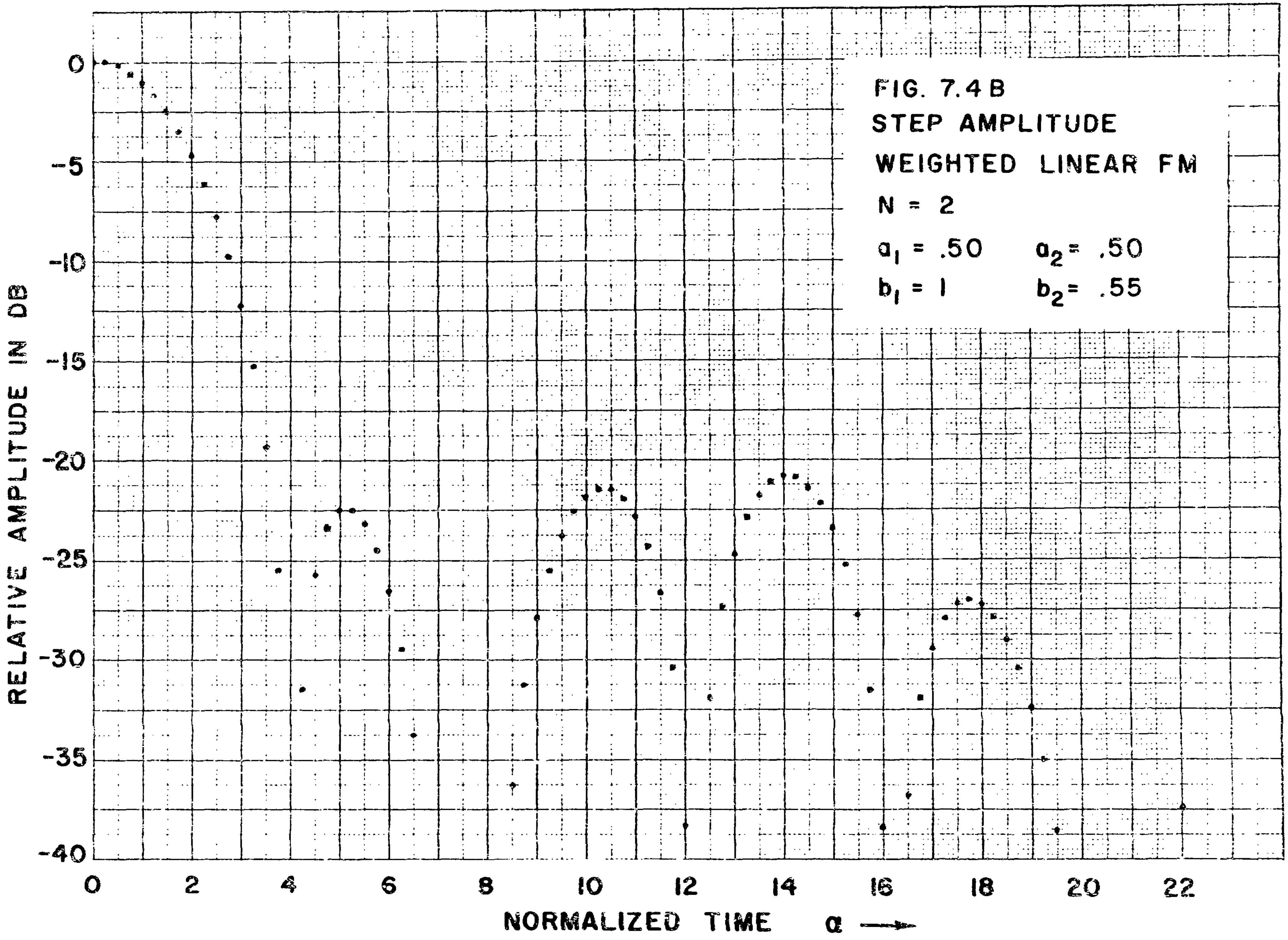


FIG. 7.4 B
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 2
a1 = .50   a2 = .50
b1 = 1   b2 = .55
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
0 2 4 6 8 10 12 14 16 18 20 22
NORMALIZED TIME α

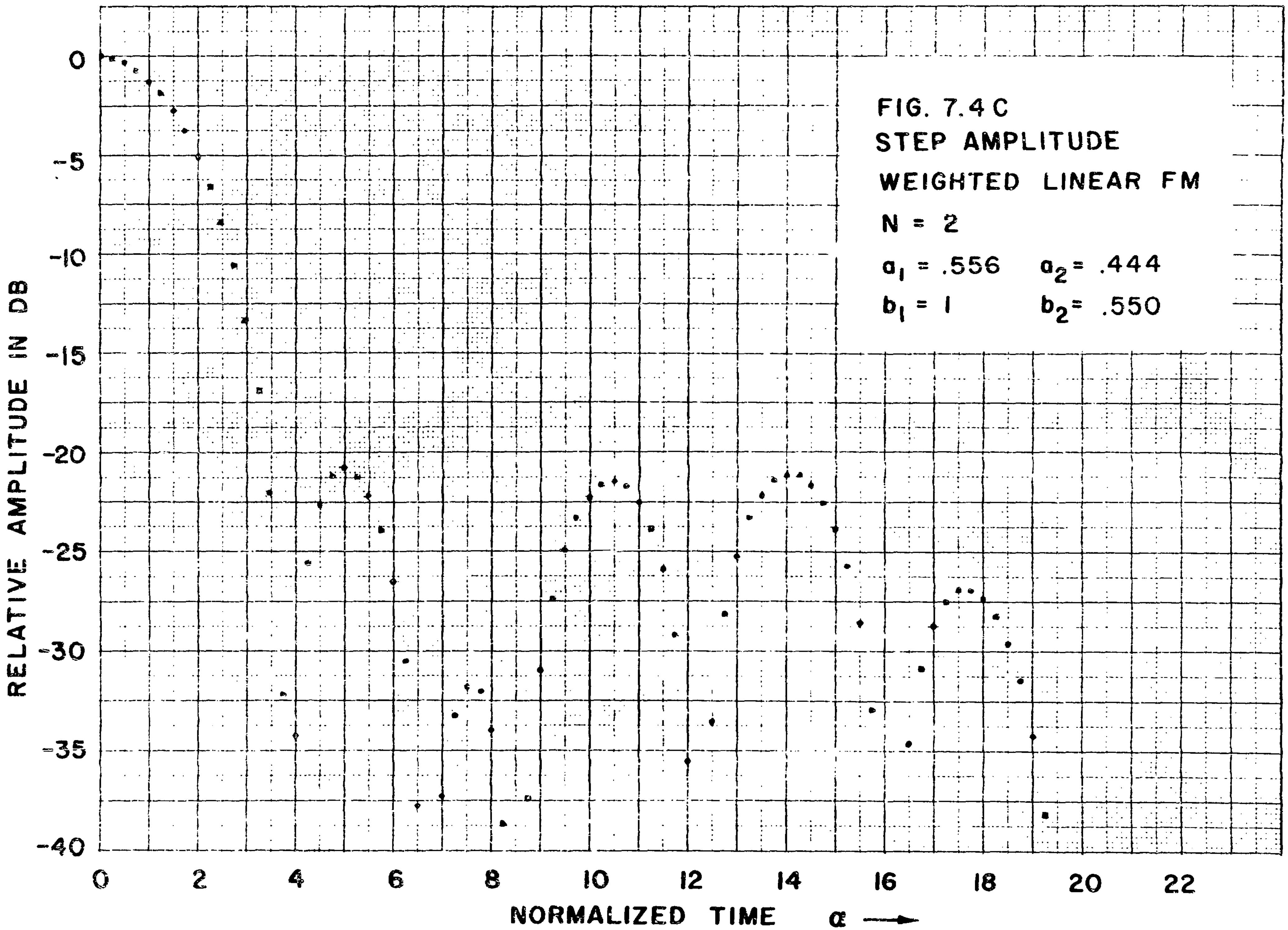


FIG. 7.4 C
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 2
a1 = .556 a2 = .444
b1 = 1 b2 = .550
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
0
2
4
6
8
10
12
14
16
18
20
22
NORMALIZED TIME α

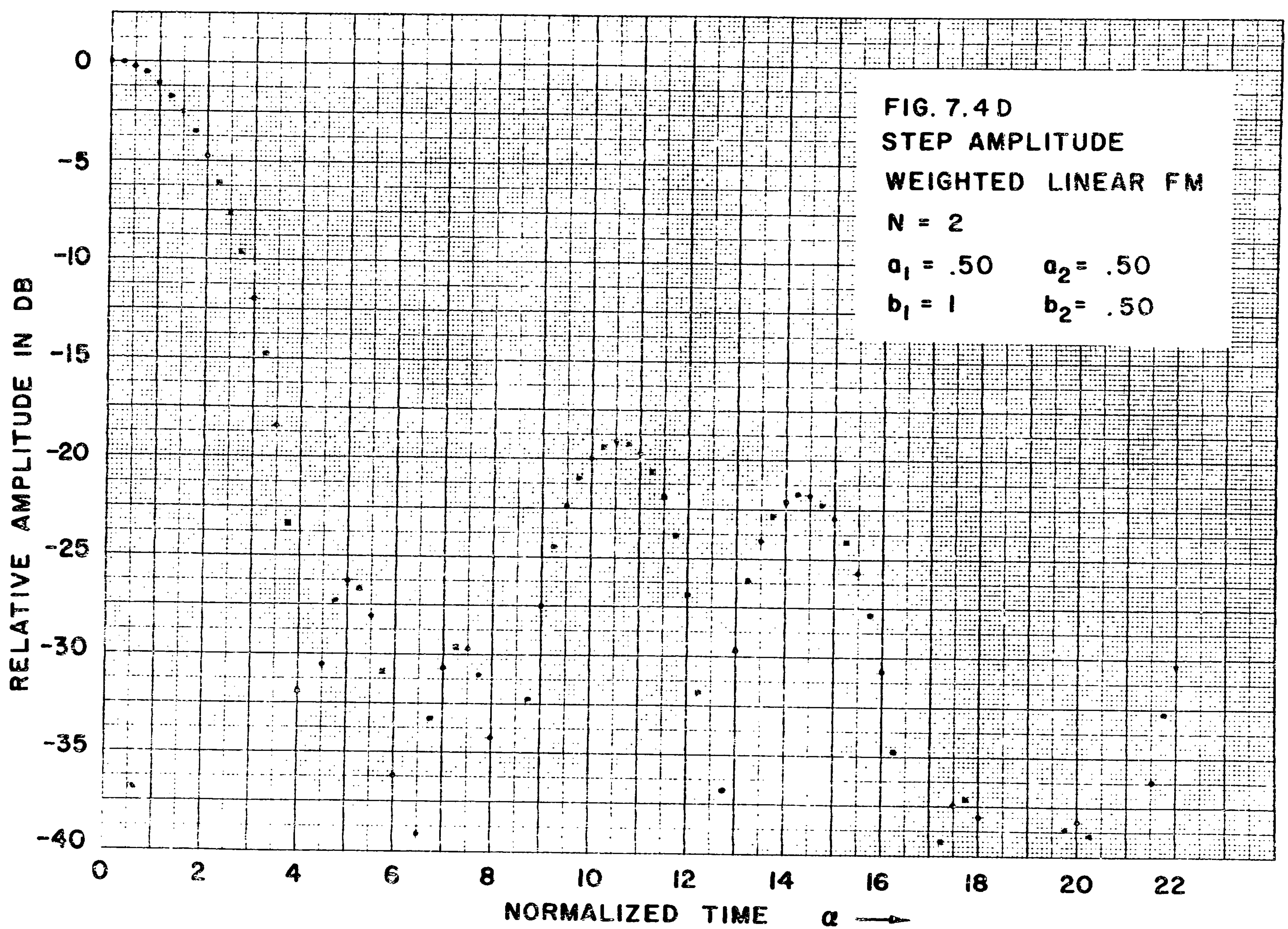


FIG. 7.4 D
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 2
a1 = .50 a2 = .50
b1 = 1 b2 = .50
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
0
2
4
6
8
10
12
14
16
18
20
22
NORMALIZED TIME α

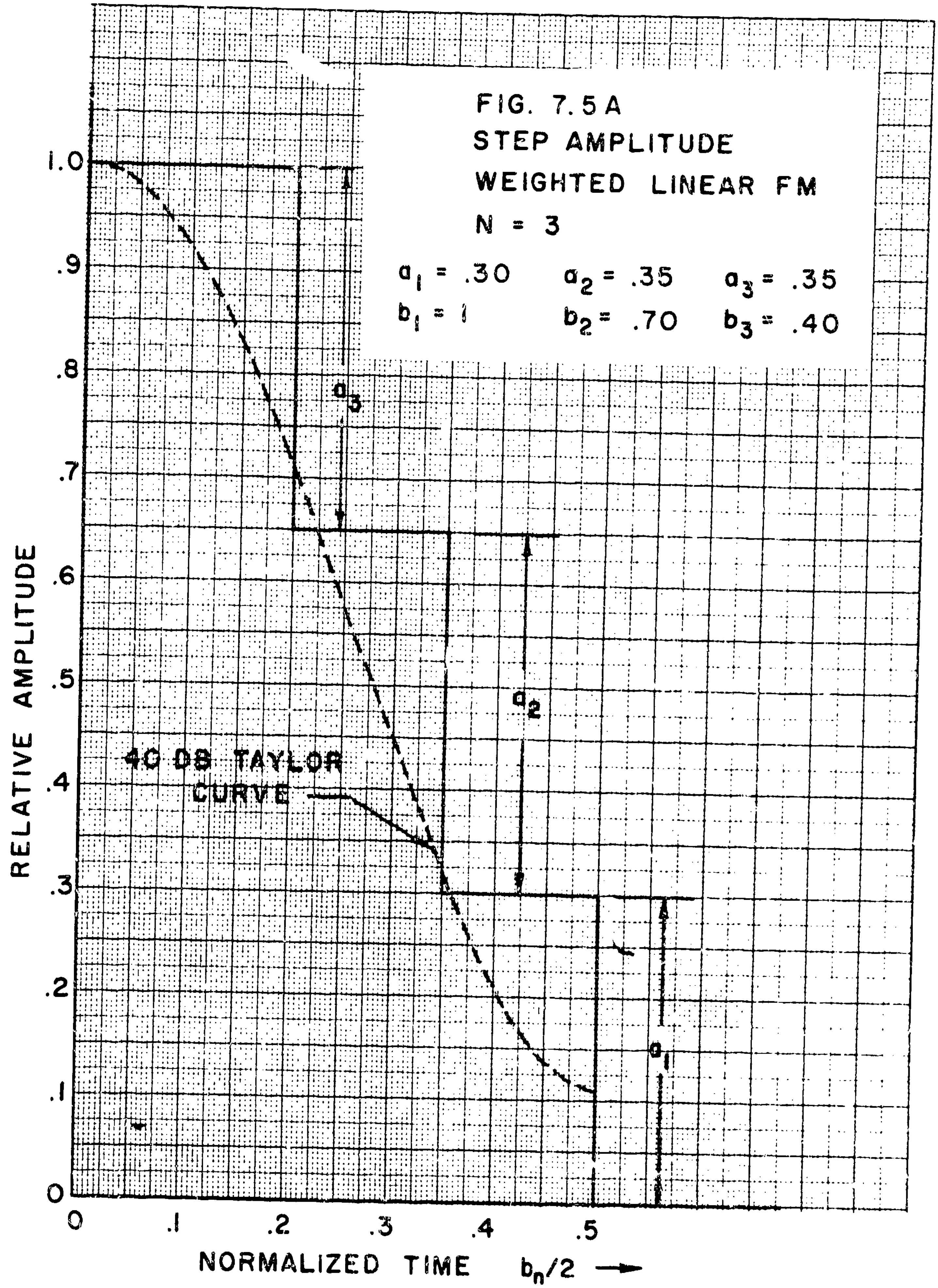


FIG. 7.5A
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 3
a1 = .30 a2 = .35 a3 = .35
b1 = 1 b2 = .70 b3 = .40
RELATIVE AMPLITUDE
NORMALIZED TIME bn/2
40 DB TAYLOR CURVE
a3
a2
a1
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0

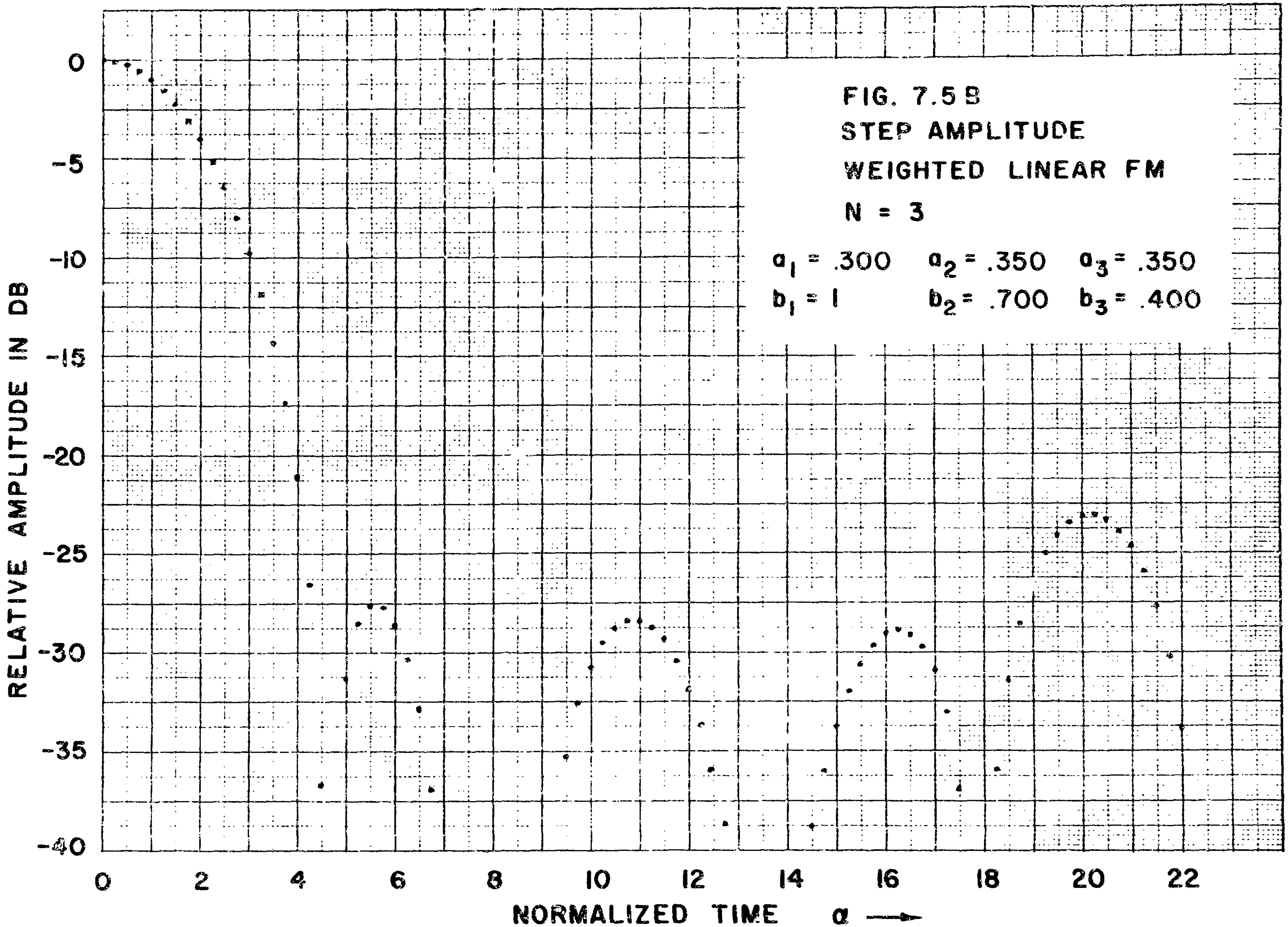


FIG. 7.5 B
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 3
a1 = .300 a2 = .350 a3 = .350
b1 = 1 b2 = .700 b3 = .400
RELATIVE AMPLITUDE IN DB
NORMALIZED TIME α
0
-5
-10
-15
-20
-25
-30
-35
-40
0
2
4
6
8
10
12
14
16
18
20
22

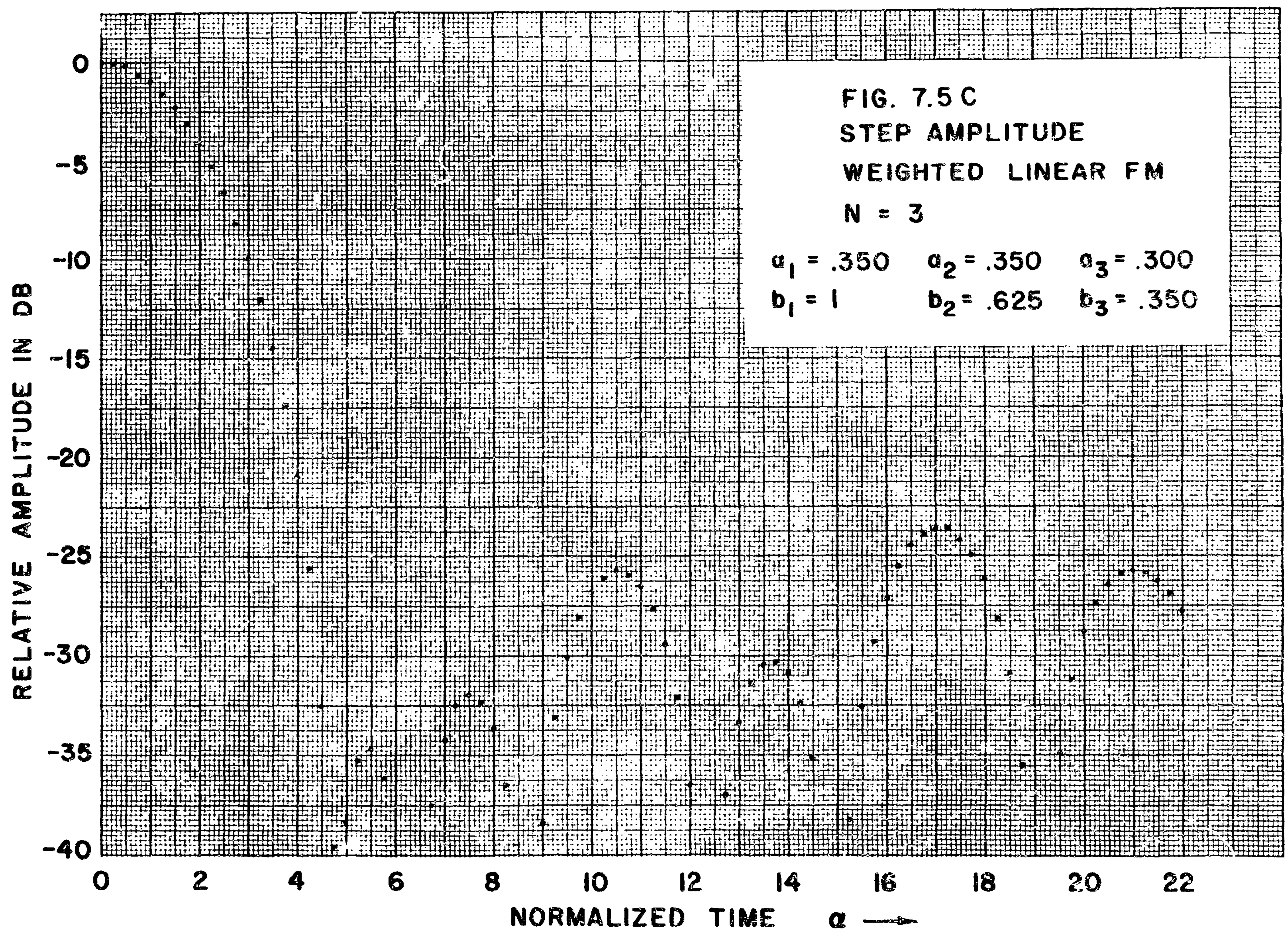


FIG. 7.5 C
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 3
a1 = .350 a2 = .350 a3 = .300
b1 = 1 b2 = .625 b3 = .350
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
0 2 4 6 8 10 12 14 16 18 20 22
NORMALIZED TIME α

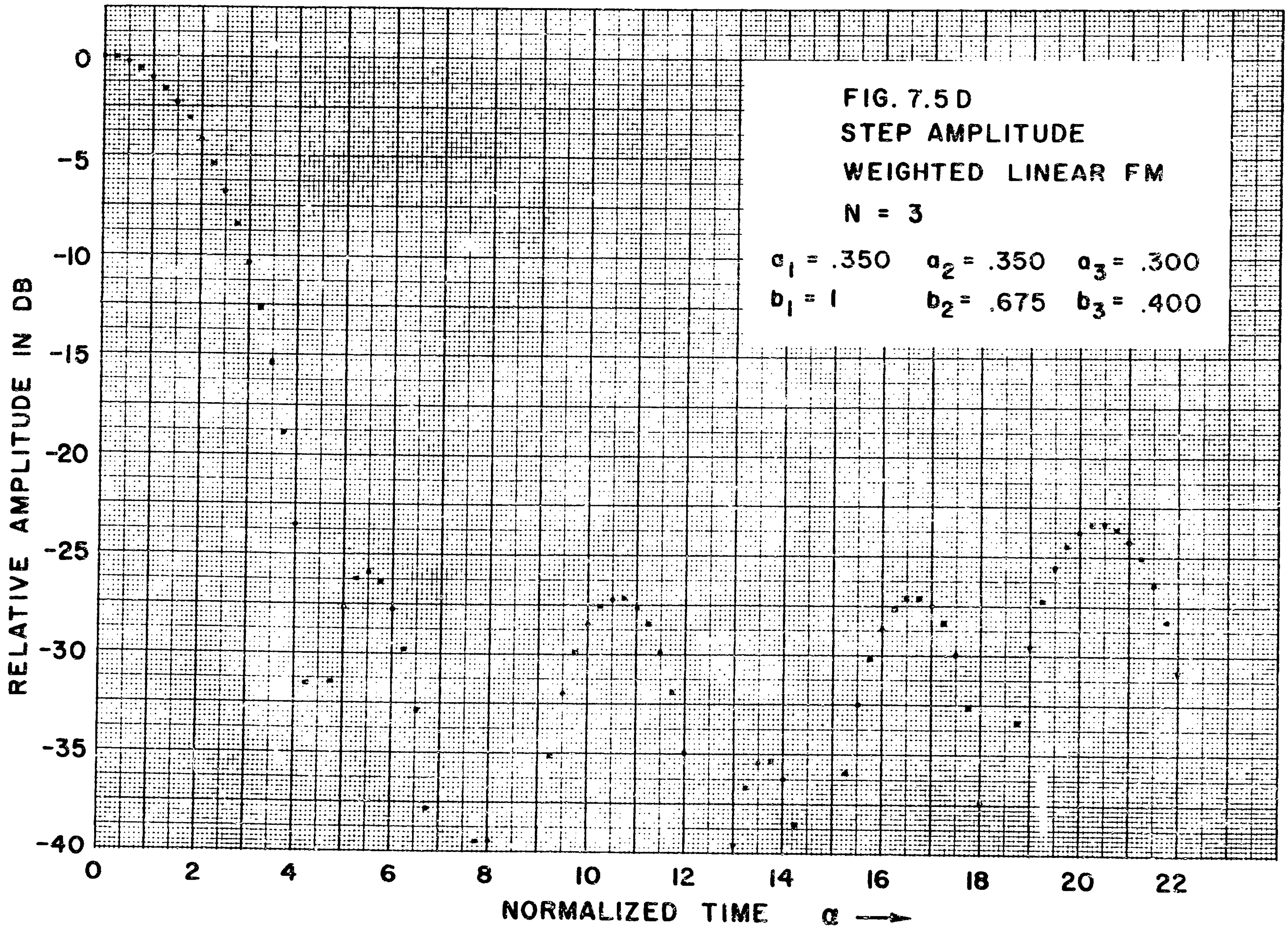


FIG. 7.5D
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 3
a1 = .350 a2 = .350 a3 = .300
b1 = 1 b2 = .675 b3 = .400
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
0 2 4 6 8 10 12 14 16 18 20 22
NORMALIZED TIME α

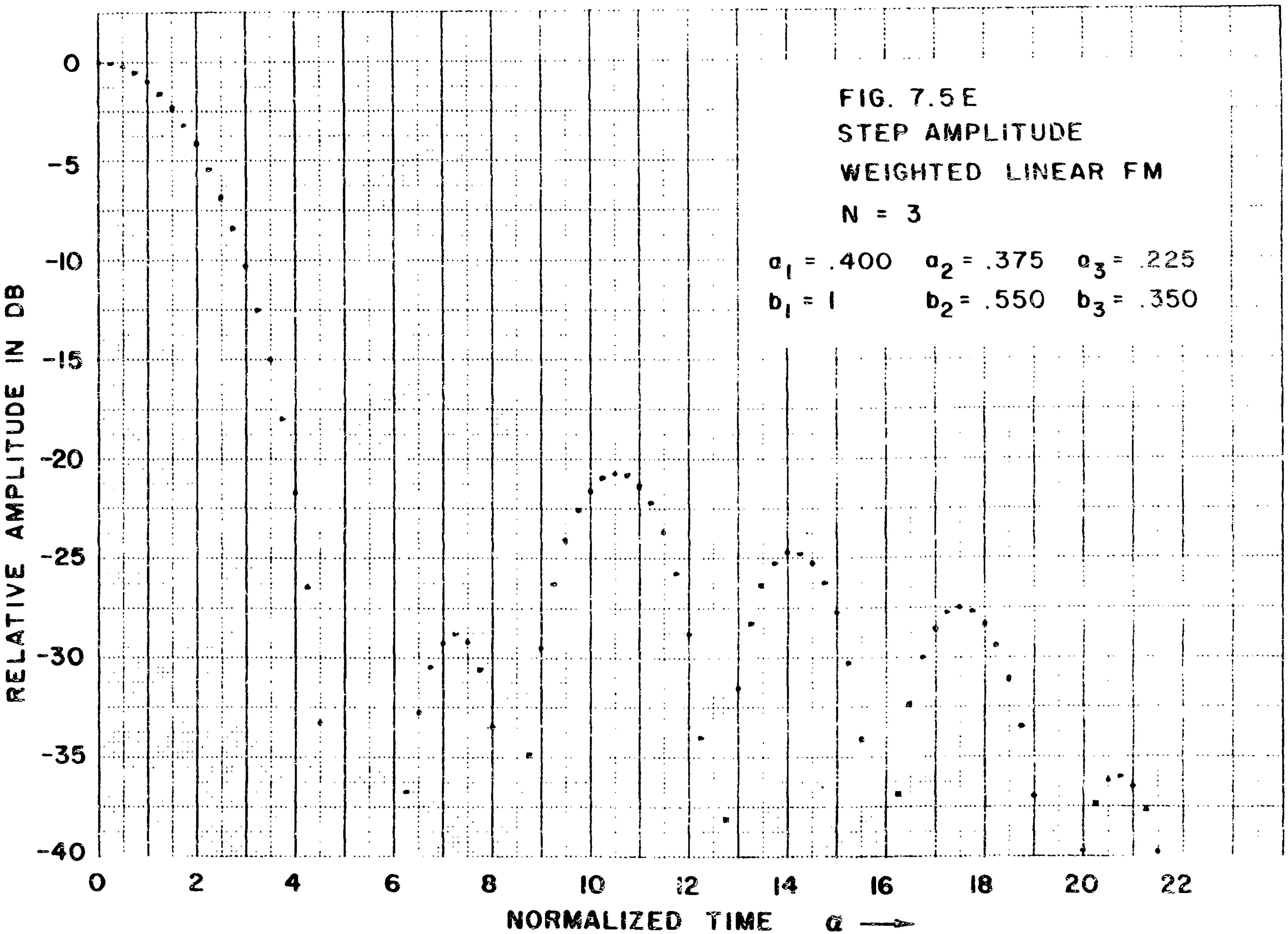


FIG. 7.5E
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 3
a1 = .400 a2 = .375 a3 = .225
b1 = 1 b2 = .550 b3 = .350
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
0
2
4
6
8
10
12
14
16
18
20
22
NORMALIZED TIME α →

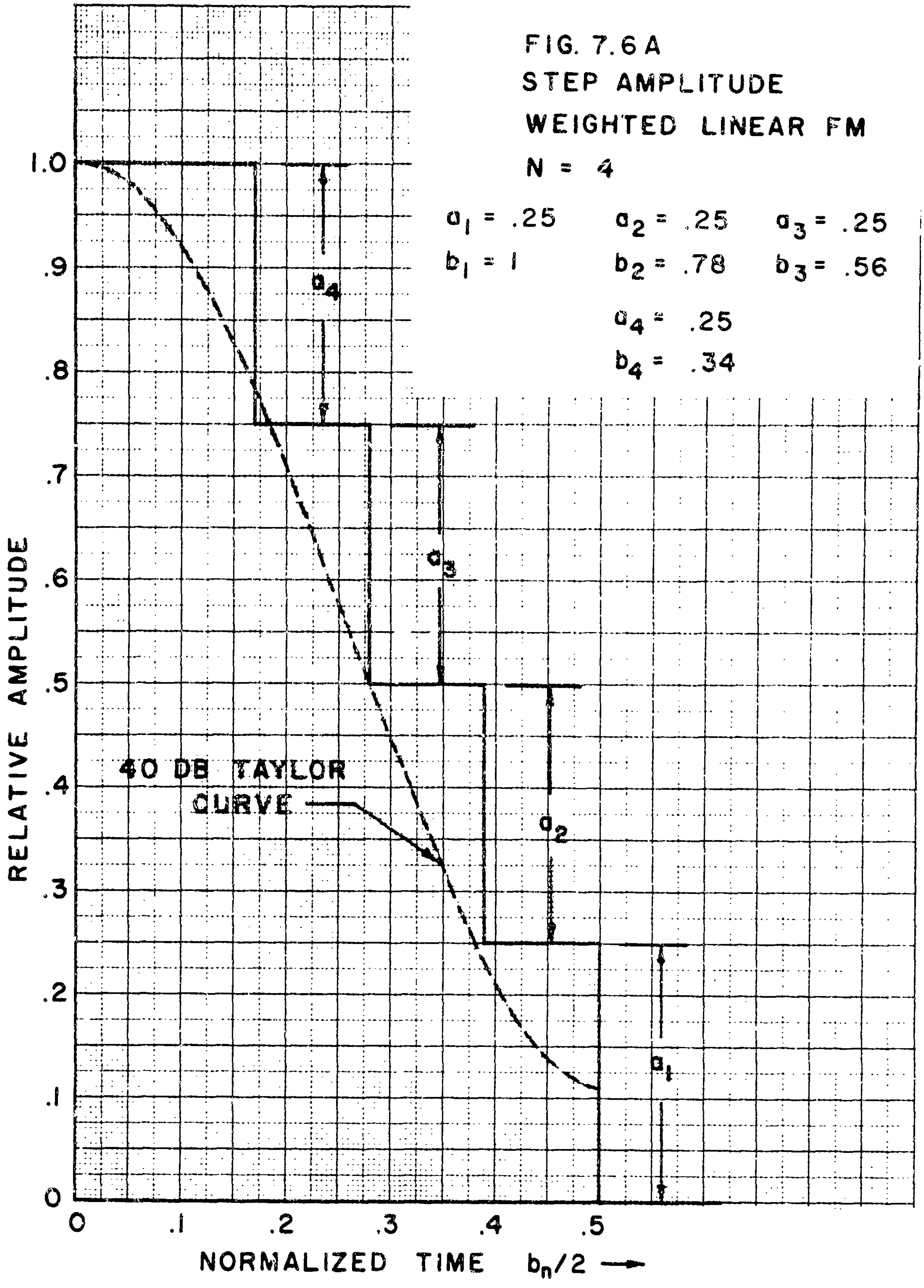


FIG. 7.6 A
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 4
a1 = .25
b1 = 1
a2 = .25
b2 = .78
a3 = .25
b3 = .56
a4 = .25
b4 = .34
40 DB TAYLOR CURVE
a4
a3
a2
a1
RELATIVE AMPLITUDE
NORMALIZED TIME bn/2
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0
.1
.2
.3
.4
.5

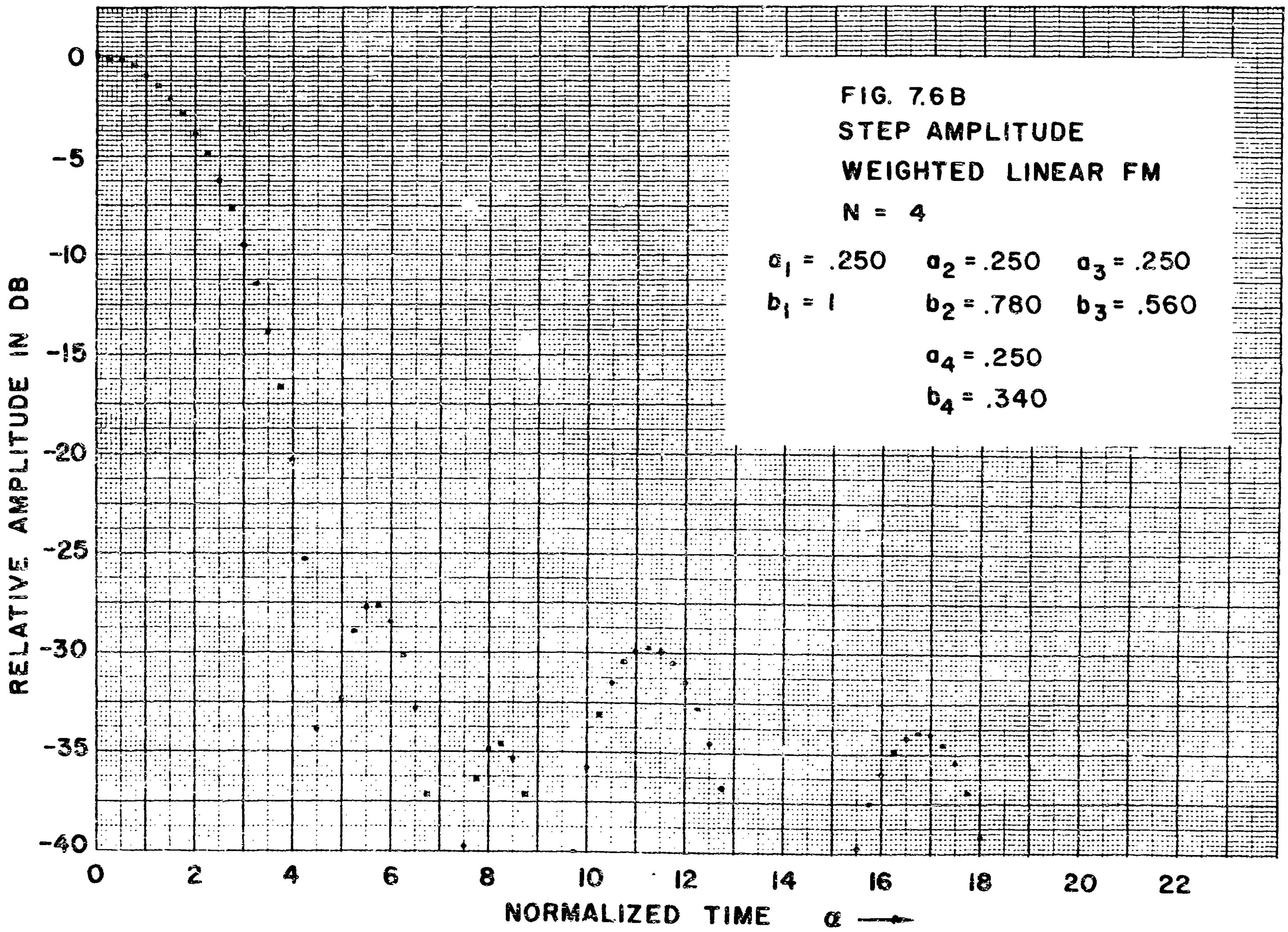


FIG. 7.6B
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 4
a_1 = .250
b_1 = 1
a_2 = .250
b_2 = .780
a_3 = .250
b_3 = .560
a_4 = .250
b_4 = .340
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
0
2
4
6
8
10
12
14
16
18
20
22
NORMALIZED TIME α

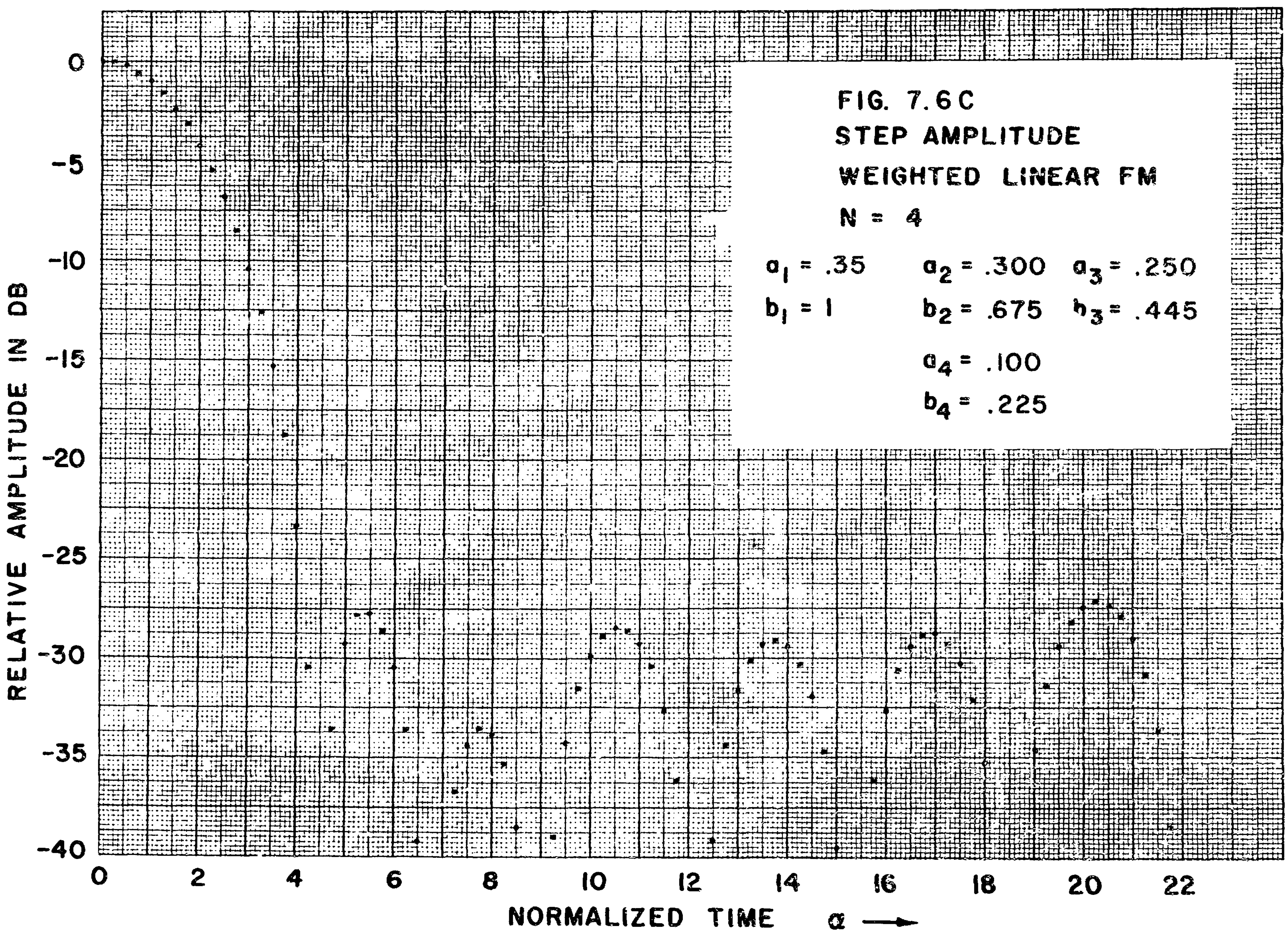


FIG. 7.6C
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 4
a1 = .35
b1 = 1
a2 = .300
b2 = .675
a3 = .250
h3 = .445
a4 = .100
b4 = .225
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
0
2
4
6
8
10
12
14
16
18
20
22
NORMALIZED TIME α

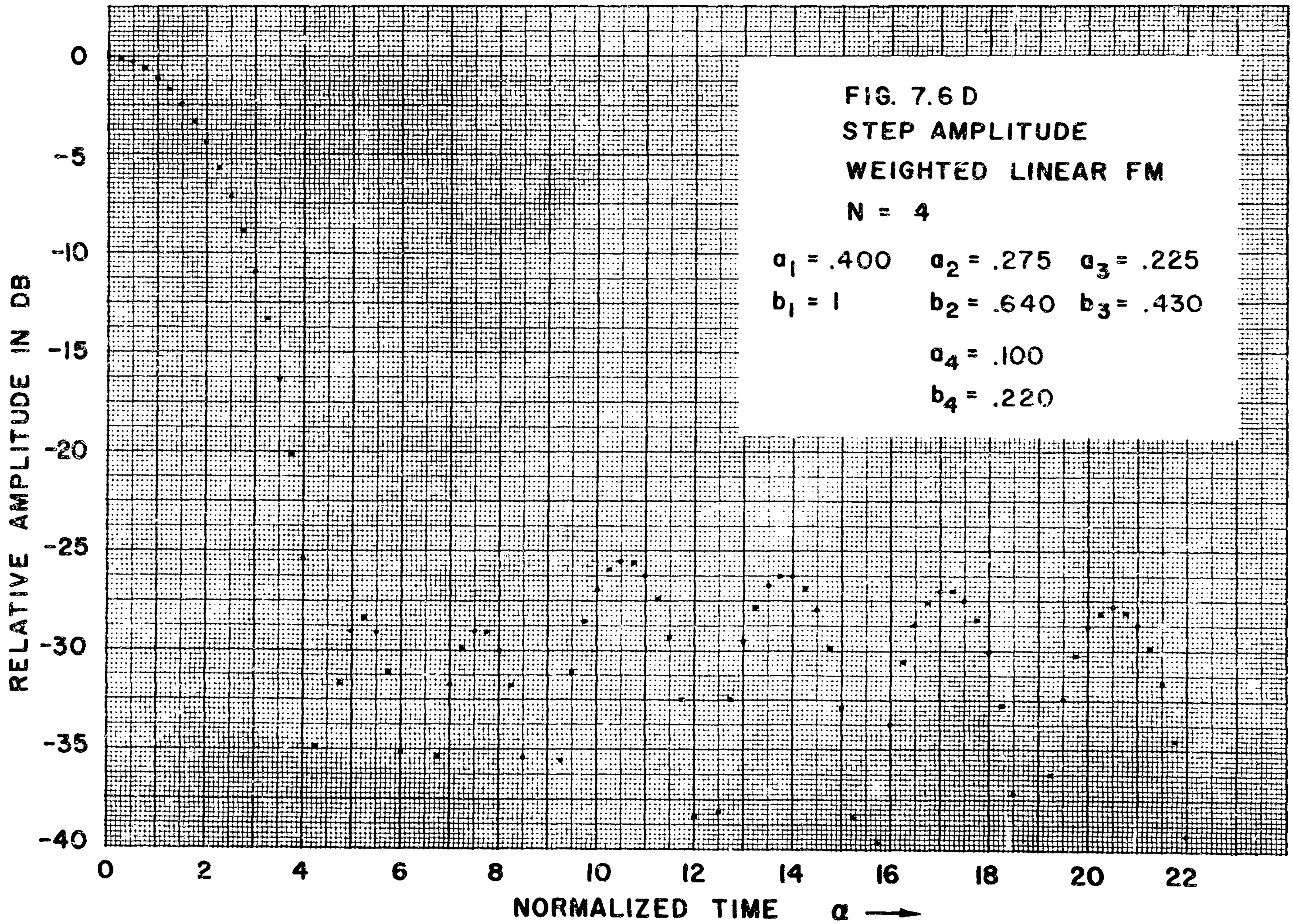


FIG. 7.6 D
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 4

$a_1 = .400$ $a_2 = .275$ $a_3 = .225$
$b_1 = 1$ $b_2 = .640$ $b_3 = .430$
$a_4 = .100$
$b_4 = .220$

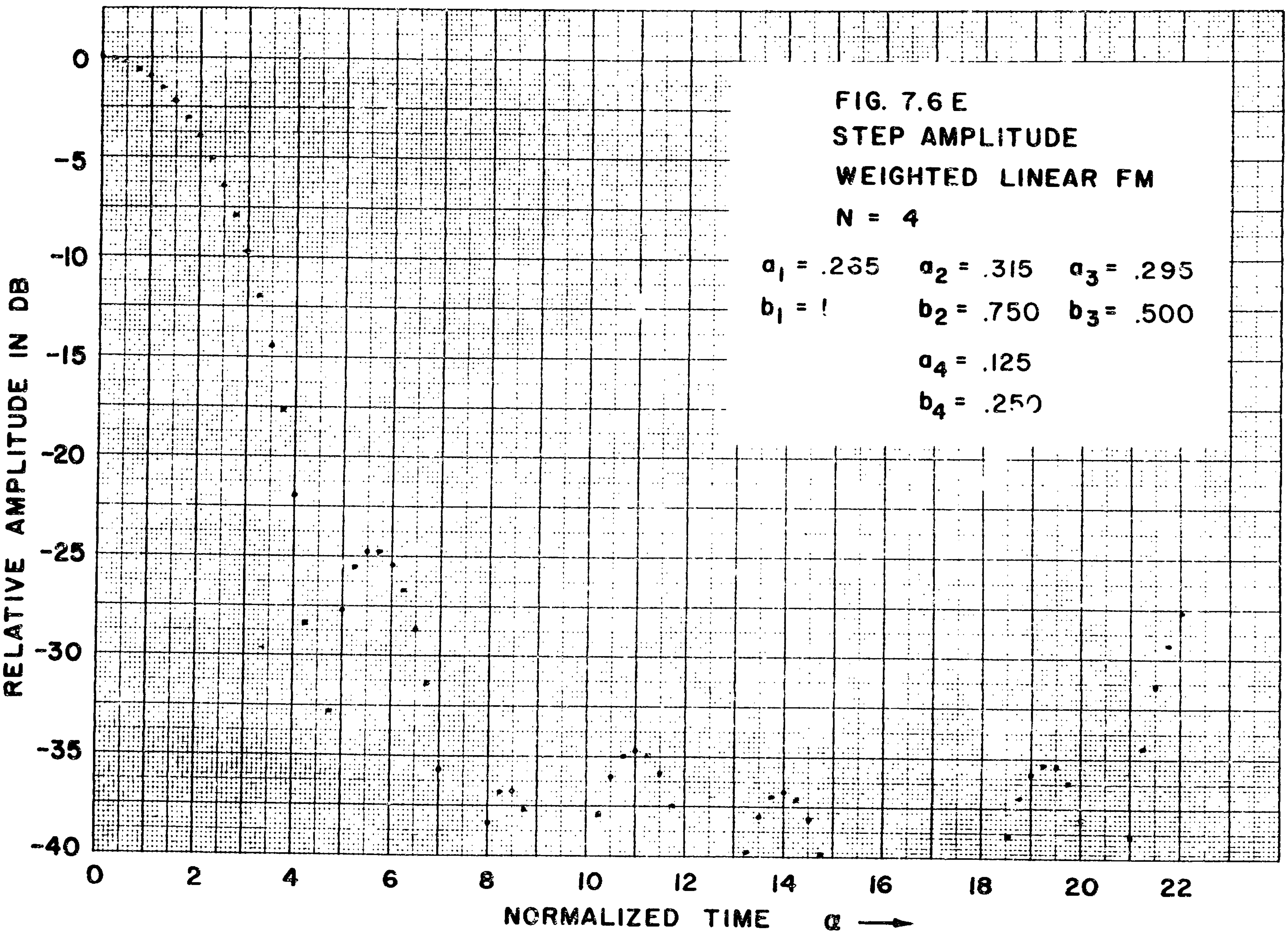


FIG. 7.6 E
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 4
a1 = .265
b1 = 1
a2 = .315
b2 = .750
a3 = .295
b3 = .500
a4 = .125
b4 = .250
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
NORMALIZED TIME α
0
2
4
6
8
10
12
14
16
18
20
22

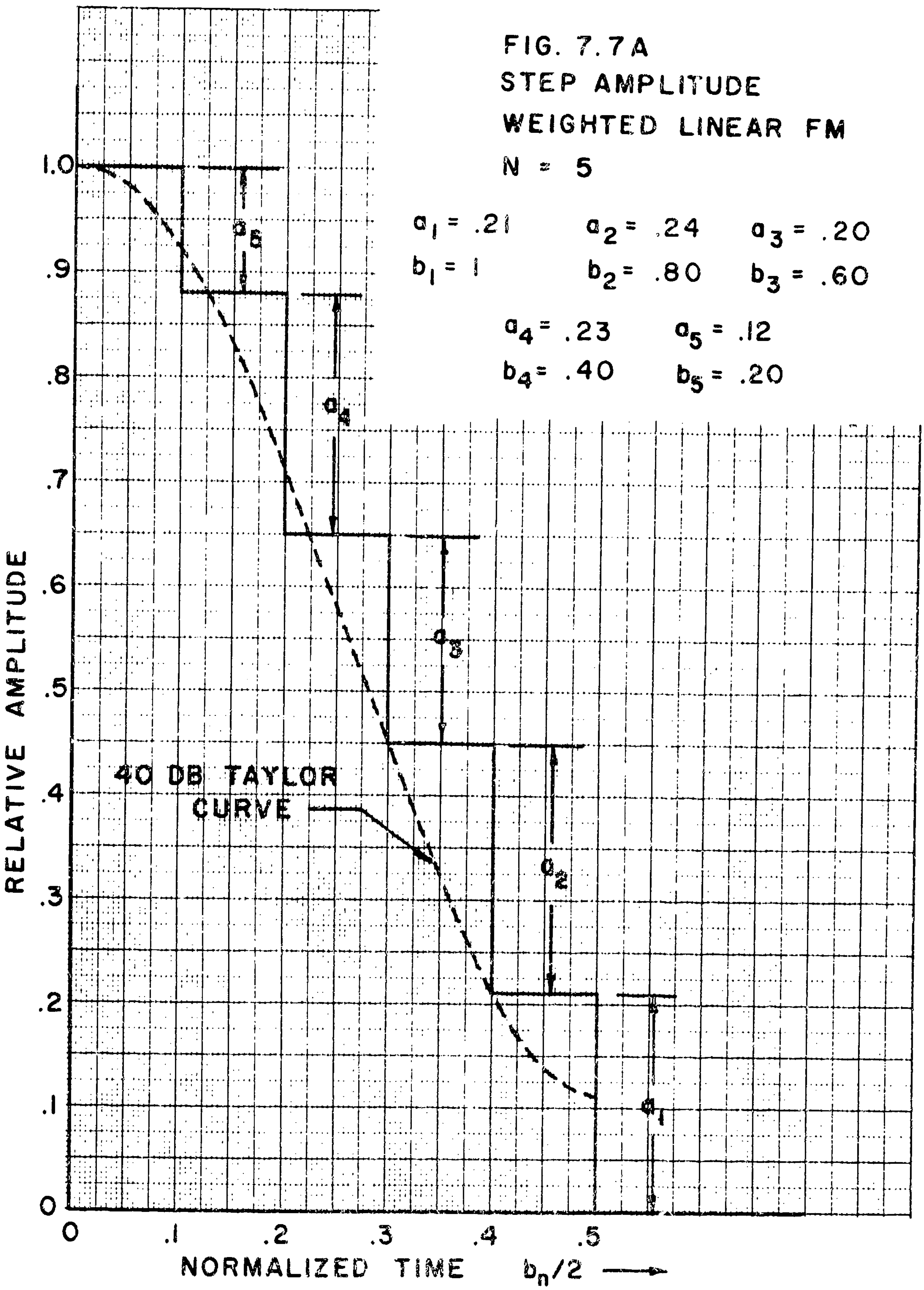


FIG. 7.7A
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 5

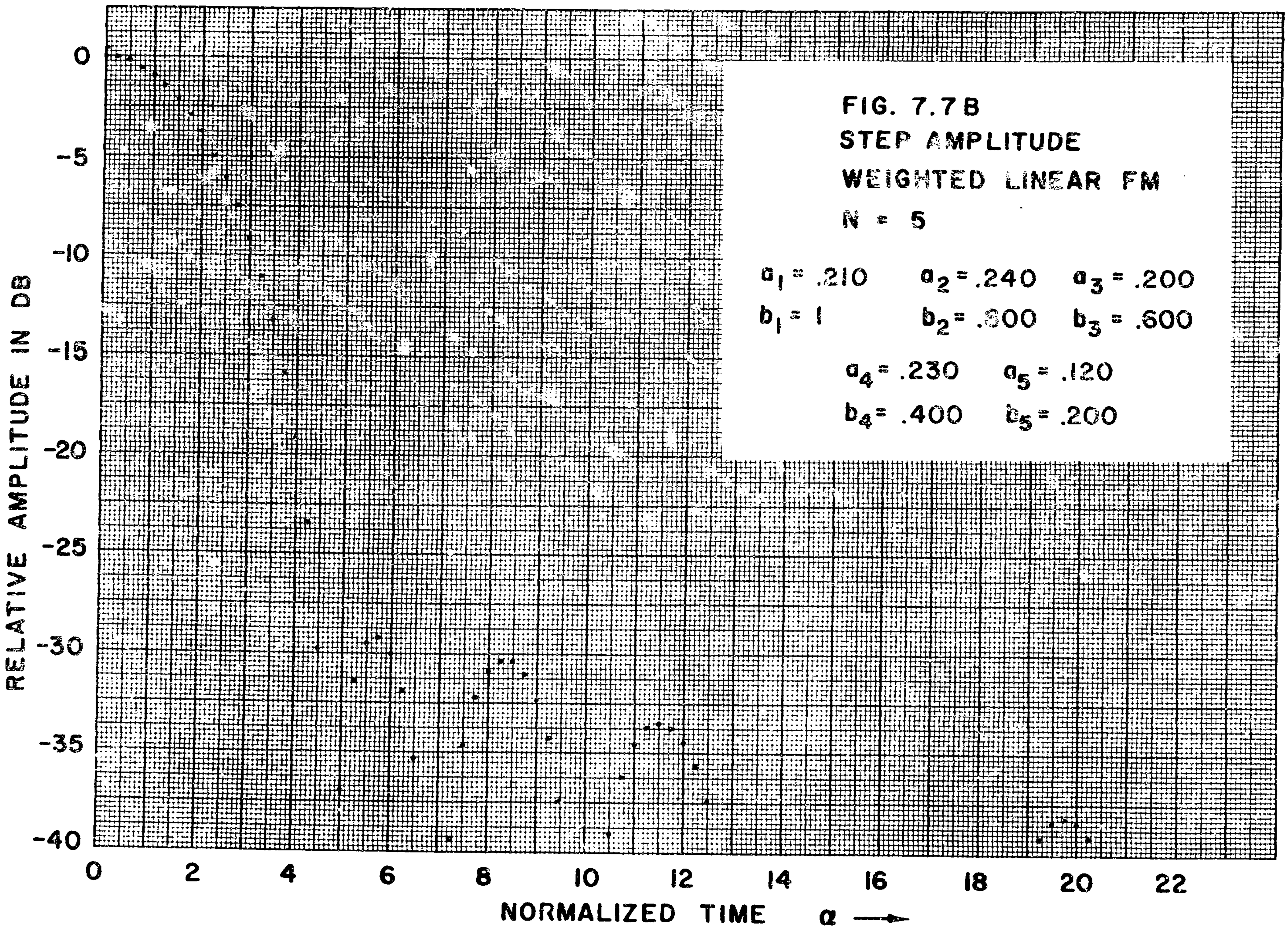


FIG. 7.7B
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 5
a1 = .210   a2 = .240   a3 = .200
b1 = 1   b2 = .800   b3 = .600
a4 = .230   a5 = .120
b4 = .400   b5 = .200
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
0
2
4
6
8
10
12
14
16
18
20
22
NORMALIZED TIME α
K•E 10 X 10 TO THE CENTIMETER 46 1513
KEUFFEL & ESSER CO

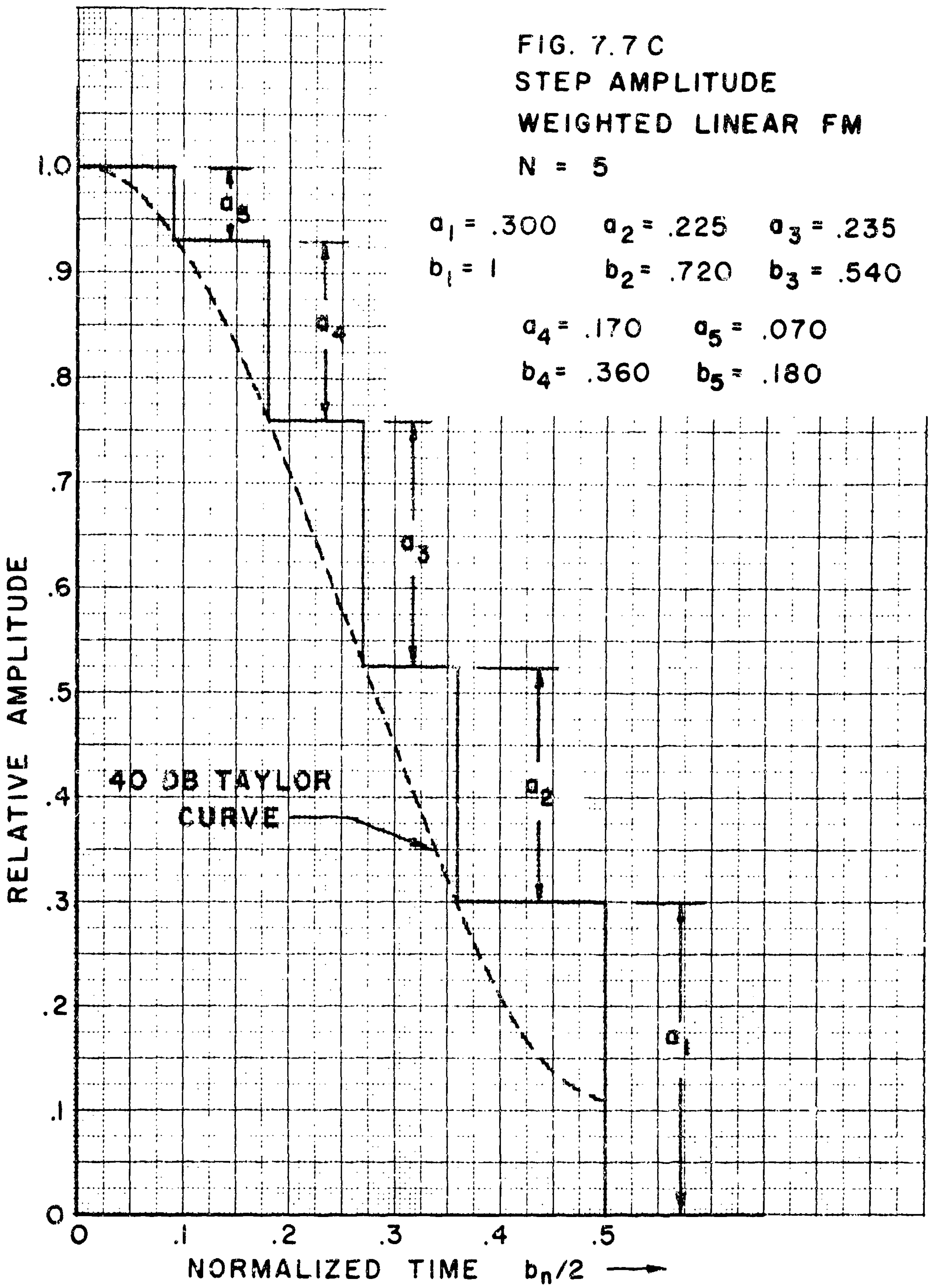


FIG. 7.7 C
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 5
a_1 = .300 a_2 = .225 a_3 = .235
b_1 = 1 b_2 = .720 b_3 = .540
a_4 = .170 a_5 = .070
b_4 = .360 b_5 = .180
a_5
a_4
a_3
a_2
a_1
40 DB TAYLOR CURVE
RELATIVE AMPLITUDE
NORMALIZED TIME $b_n/2$

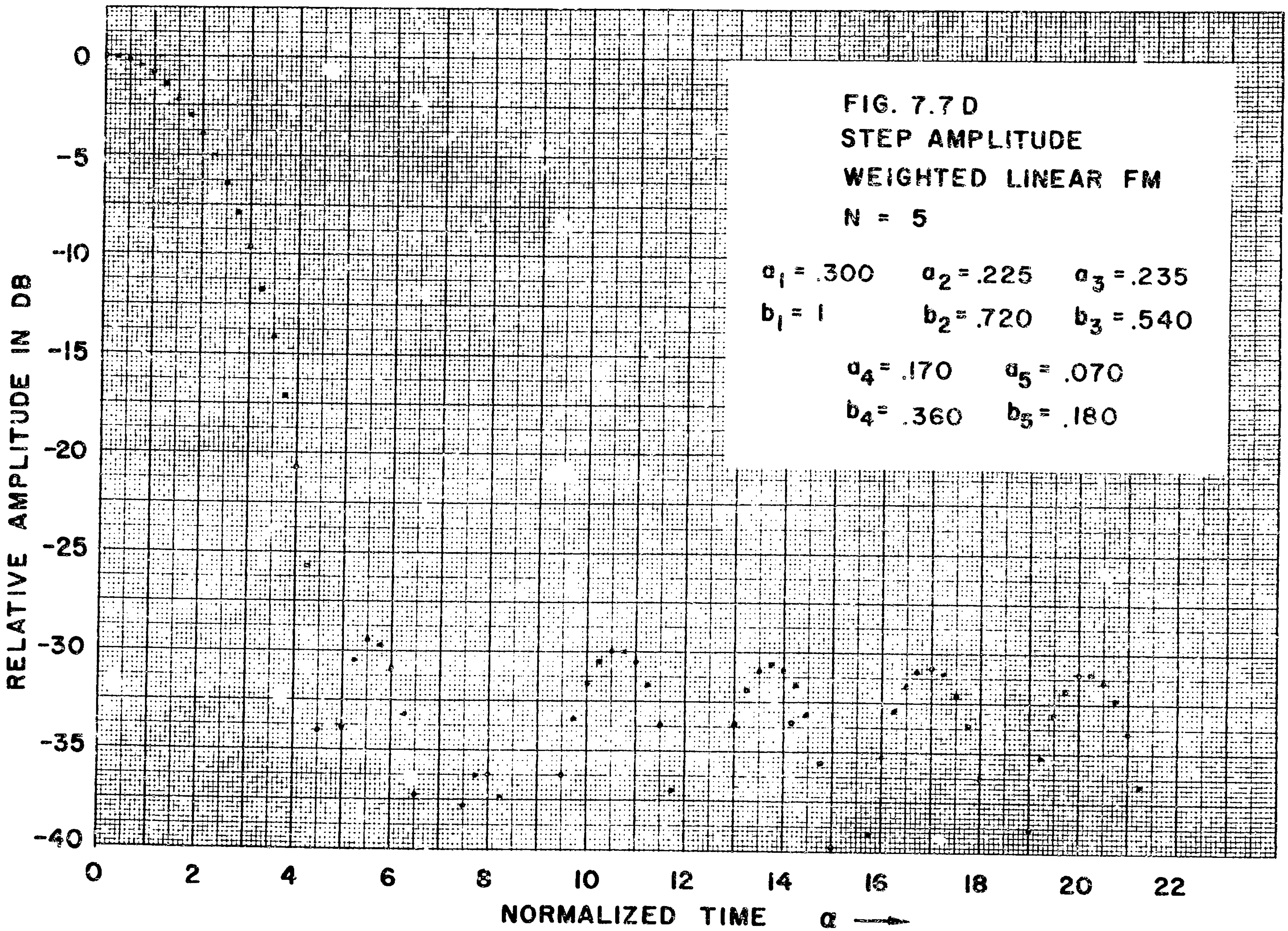


FIG. 7.7 D
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 5
a1 = .300 a2 = .225 a3 = .235
b1 = 1 b2 = .720 b3 = .540
a4 = .170 a5 = .070
b4 = .360 b5 = .180
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
0
2
4
6
8
10
12
14
16
18
20
22
NORMALIZED TIME α

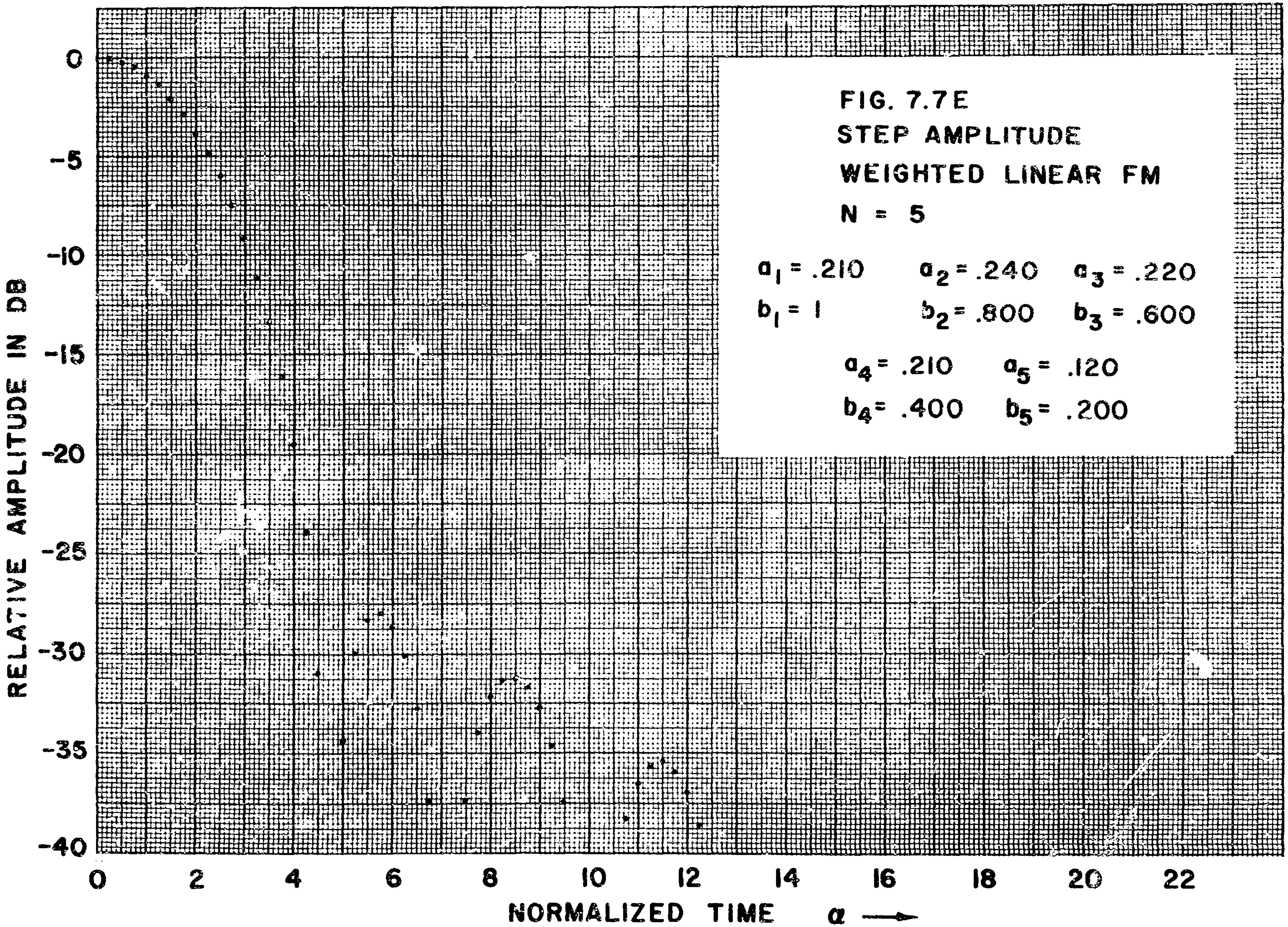


FIG. 7.7E
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 5
$a_1 = .210$ $a_2 = .240$ $a_3 = .220$
$b_1 = 1$ $b_2 = .800$ $b_3 = .600$
$a_4 = .210$ $a_5 = .120$
$b_4 = .400$ $b_5 = .200$
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
0 2 4 6 8 10 12 14 16 18 20 22
NORMALIZED TIME α →

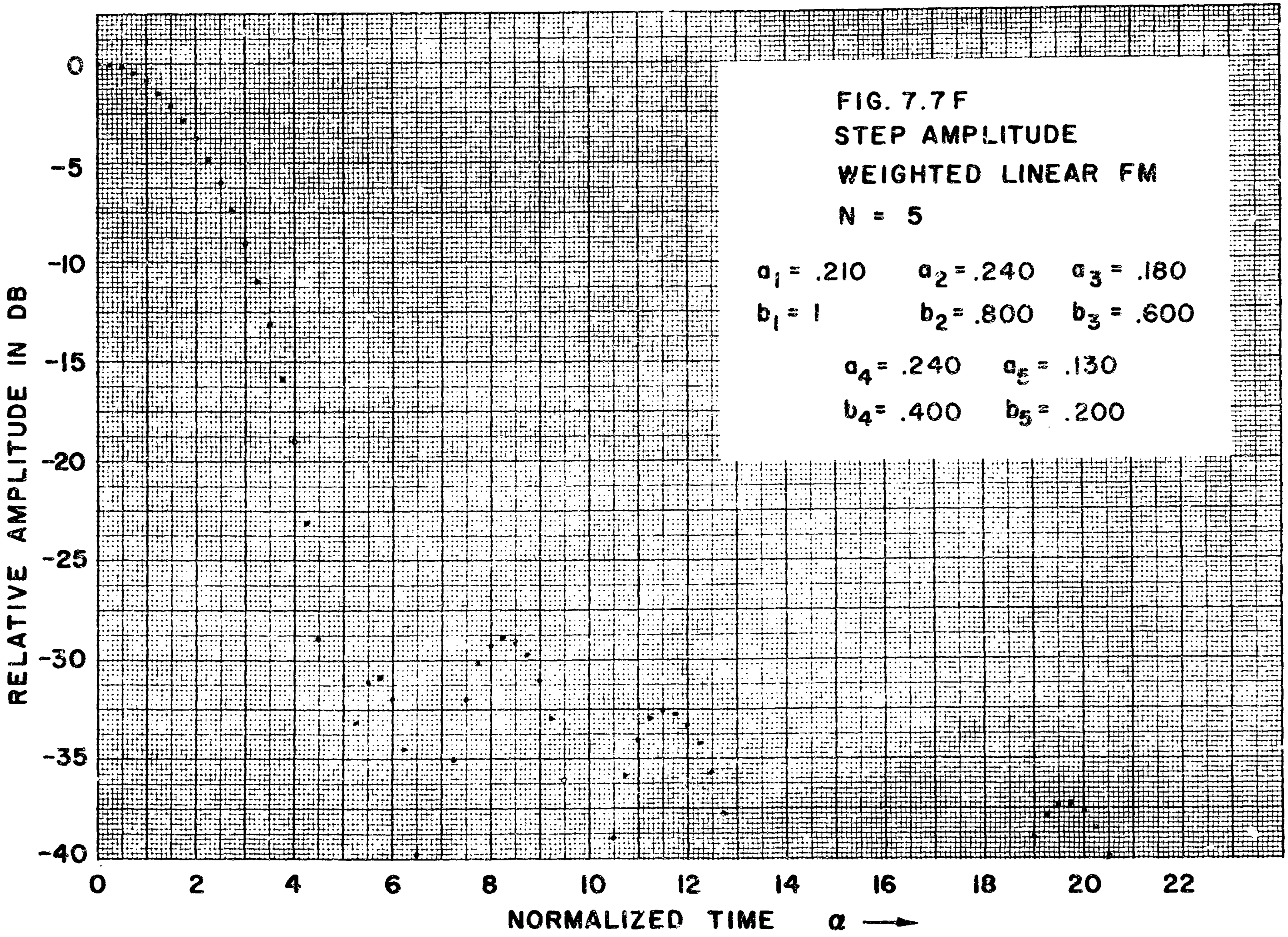


FIG. 7.7F
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 5
a1 = .210 a2 = .240 a3 = .180
b1 = 1 b2 = .800 b3 = .600
a4 = .240 a5 = .130
b4 = .400 b5 = .200
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
0 2 4 6 8 10 12 14 16 18 20 22
NORMALIZED TIME α

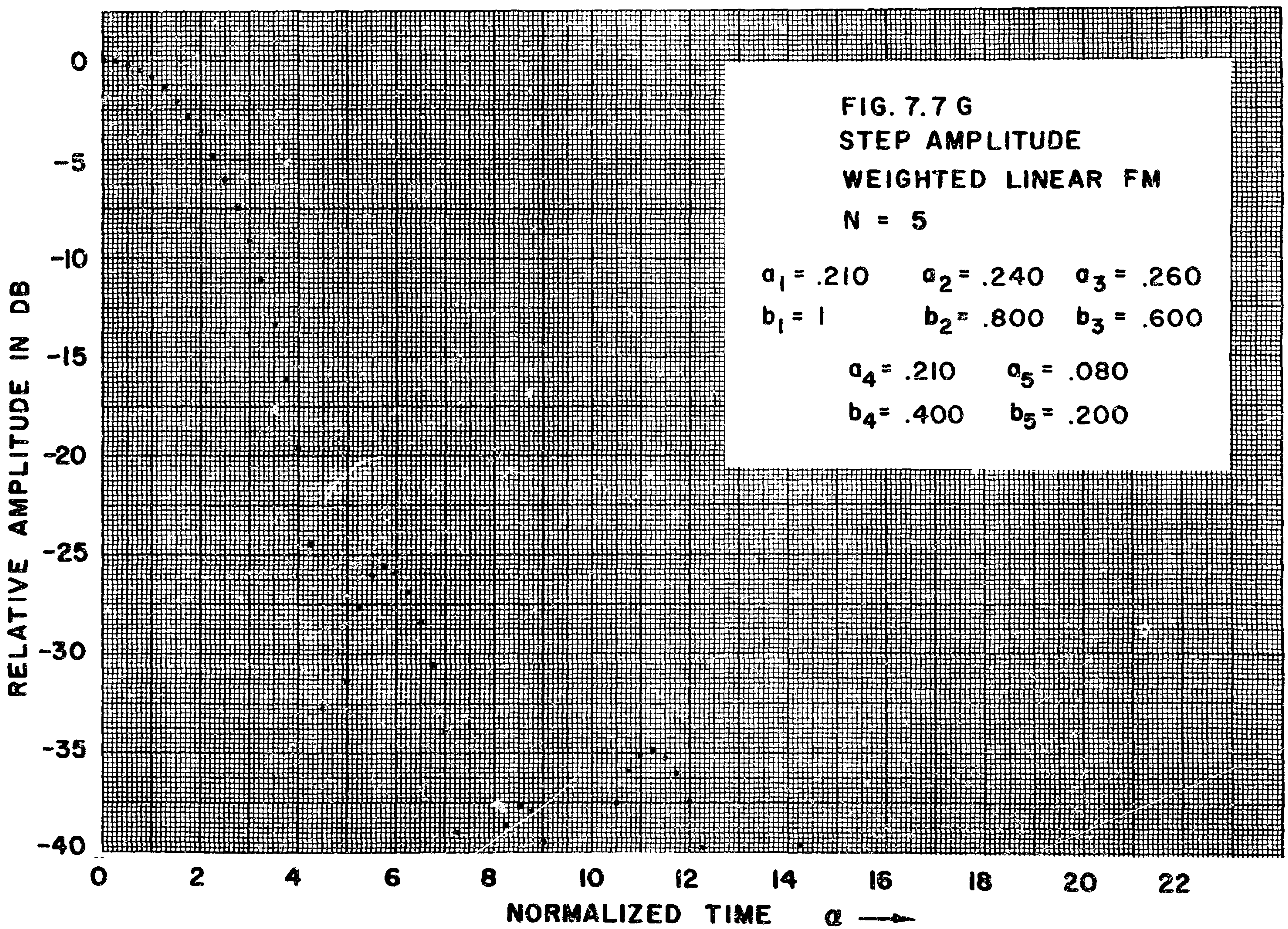


FIG. 7.7 G
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 5
a1 = .210 a2 = .240 a3 = .260
b1 = 1 b2 = .800 b3 = .600
a4 = .210 a5 = .080
b4 = .400 b5 = .200
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
0 2 4 6 8 10 12 14 16 18 20 22
NORMALIZED TIME α

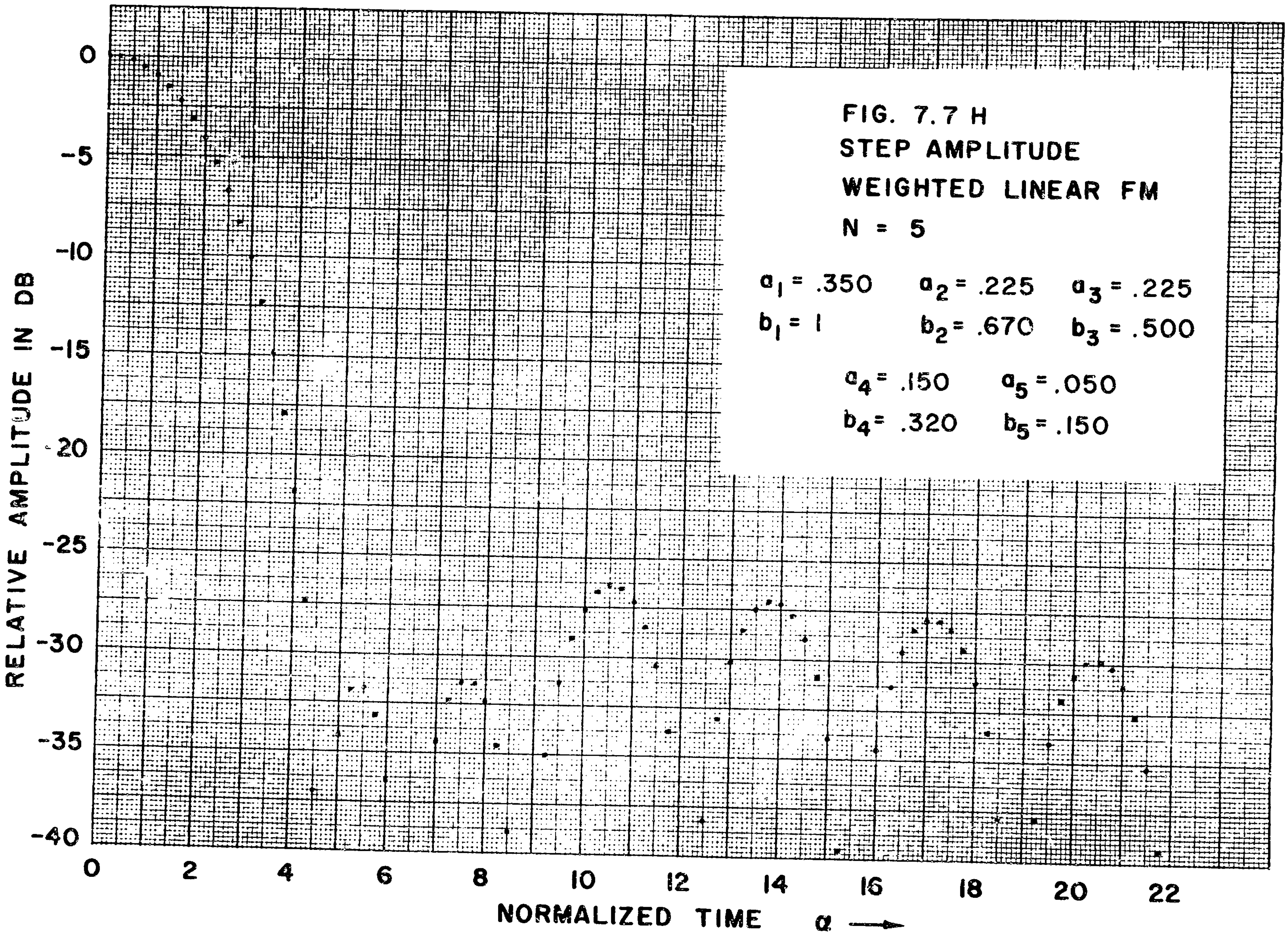


FIG. 7.7 H
STEP AMPLITUDE
WEIGHTED LINEAR FM
N = 5
a1 = .350 a2 = .225 a3 = .225
b1 = 1 b2 = .670 b3 = .500
a4 = .150 a5 = .050
b4 = .320 b5 = .150
RELATIVE AMPLITUDE IN DB
0
-5
-10
-15
-20
-25
-30
-35
-40
0
2
4
6
8
10
12
14
16
18
20
22
NORMALIZED TIME α

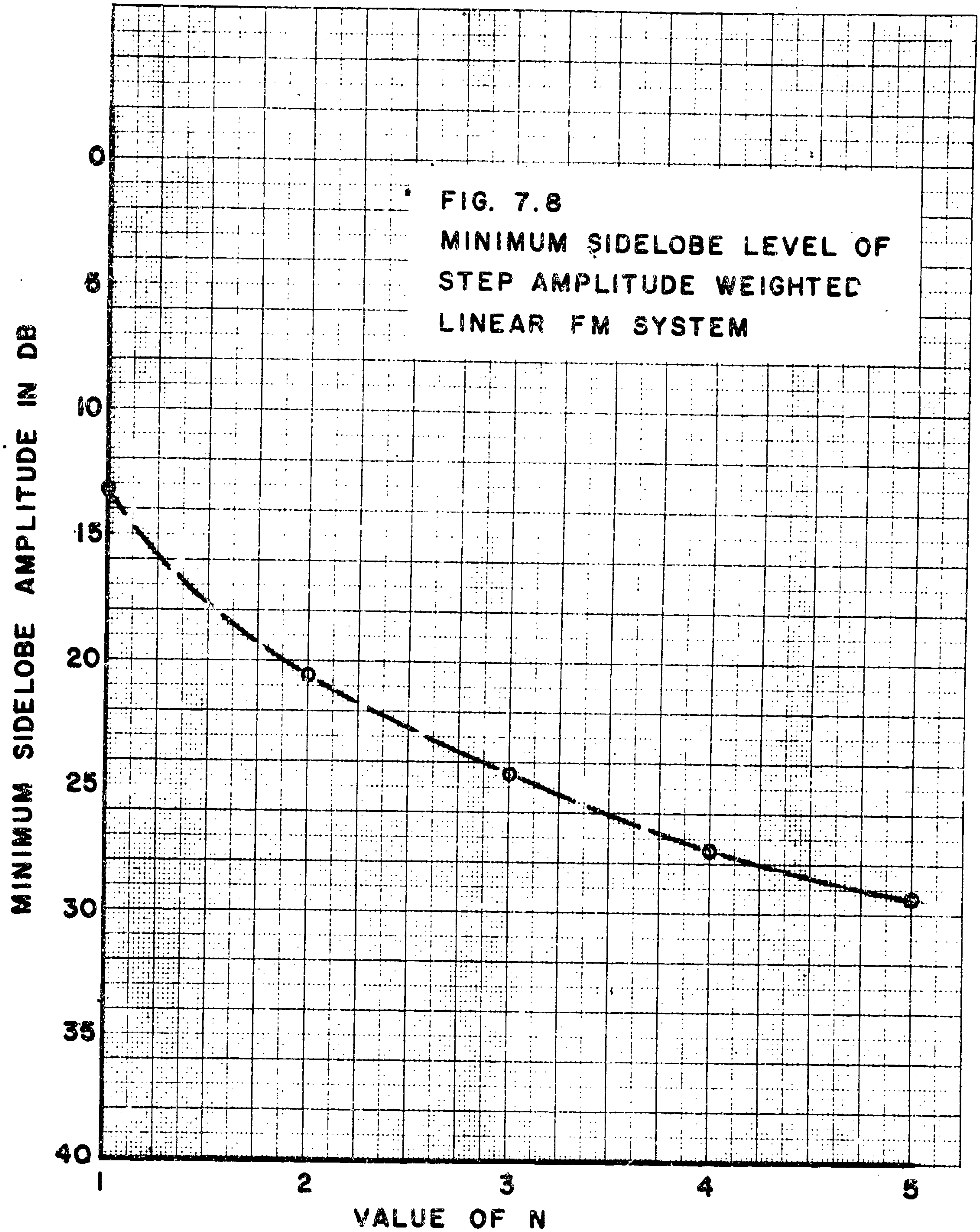


FIG. 7.8
MINIMUM SIDELOBE LEVEL OF
STEP AMPLITUDE WEIGHTED
LINEAR FM SYSTEM
MINIMUM SIDELOBE AMPLITUDE IN DB
0
5
10
15
20
25
30
35
40
1
2
3
4
5
VALUE OF N

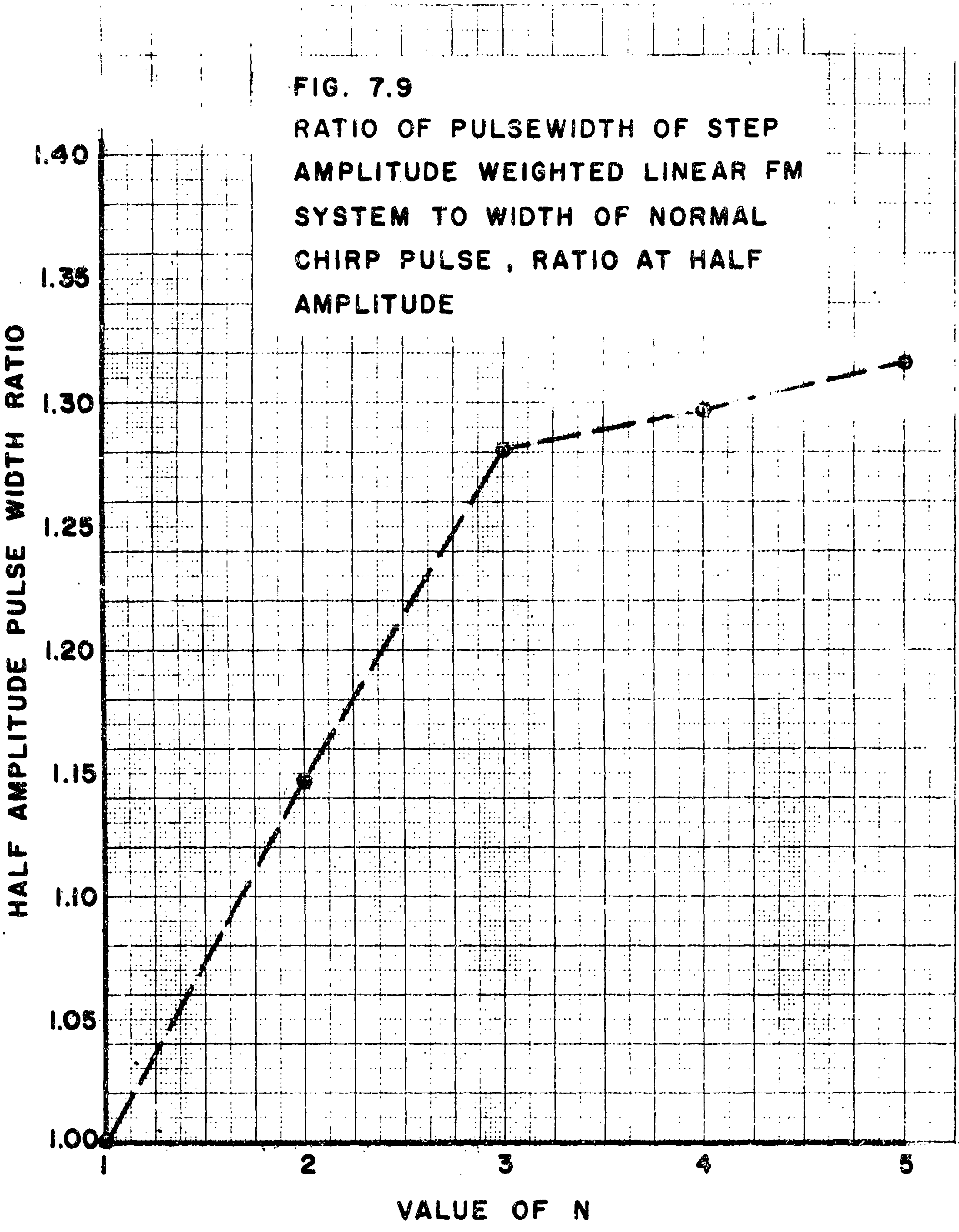


FIG. 7.9
RATIO OF PULSEWIDTH OF STEP AMPLITUDE WEIGHTED LINEAR FM SYSTEM TO WIDTH OF NORMAL CHIRP PULSE, RATIO AT HALF AMPLITUDE